

Determination of the Structural Constant of the Atom

Milan Perkovic*

The First Technical School TESLA, Zagreb, Croatia
Email: milan@drivesc.com

Received December 11, 2013; revised January 10, 2014; accepted January 17, 2014

Copyright © 2014 Milan Perkovic. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited. In accordance of the Creative Commons Attribution License all Copyrights © 2014 are reserved for SCIRP and the owner of the intellectual property Milan Perkovic. All Copyright © 2014 are guarded by law and by SCIRP as a guardian.

ABSTRACT

The equations for energy, momentum, frequency, wavelength and also Schrödinger equation of the electromagnetic wave in the atom are derived using the model of atom by analogy with the transmission line. The action constant $A_0 = (\mu_0/\epsilon_0)^{1/2} s_0^2 e^2$ is a key term in the above mentioned equations. Besides the other well-known quantities, the only one unknown quantity in the last expression is a structural constant s_0 . Therefore, this article is dedicated to the calculation of the structural constant of the atoms on the basis of the above mentioned model. The structural constant of the atoms $s_0 = 8.277\ 56$ shows up as a link between macroscopic and atomic world. After calculating this constant we get the theory of atoms based on Maxwell's and Lorentz equations only. This theory does not require Planck constant h , which once was introduced empirically. Replacement for h is the action constant A_0 , which is here theoretically derived, while the replacement for fine structure constant α is $1/(2s_0^2)$. In this way, the structural constant s_0 replaces both constants, h and α . This paper also defines the stationary states of atoms and shows that the maximal atomic number is equal to $2s_0^2 = 137.036$, i.e., as integer should be $Z_{\max}=137$. The presented model of the atoms covers three of the four fundamental interactions, namely the electromagnetic, weak and strong interactions.

KEYWORDS

Action Constant; Fine Structure Constant; Lecher's Line; Phase Velocity; Planck's Constant; Stability of Atoms; Stationary States; Structural Coefficient; Structural Constant; Transmission Line

1. Introduction

Although for a long time classical physics and Maxwell's equations were pushed out of research in the field of atomic phenomena, the article [1] showed that the equations of atom theory can be derived with the help of Maxwell's and Lorentz equations in the classical way. Thus, we derived equations for energy, momentum, frequency and wavelength of an electromagnetic wave in an atom, which coincide with the corresponding equations in quantum mechanics. Moreover, we can derive Schrödinger wave equation as well. To achieve that, a new approach based on the classical theory is used. In that approach, an analogy between the electromagnetic wave in the atom and the wave of voltage and electric current in the transmission line plays a crucial role. The basic novelty of this approach is the introduction of a new concept of the so-called structural constant s_0 , which brings together both the parameters of transmission line (macro world) and the charge of the atoms (micro world). This article focuses on the calculation of a structural constant of the atom s_0 .

2. Model of an Atom

2.1. Atom and Transmission Line

According to [1], using Maxwell equations, electromagnetic wave in an atom is described by two partial differential equations of second order, the so-called wave equations [2]:

*Corresponding author.

$$\nabla^2 \mathbf{E} - \frac{1}{u_{\text{em}}^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0; \quad \nabla^2 \mathbf{H} - \frac{1}{u_{\text{em}}^2} \frac{\partial^2 \mathbf{H}}{\partial t^2} = 0, \quad (1)$$

where vector \mathbf{E} is the electric field strength, vector \mathbf{H} is the magnetic field strength, ∇^2 is del-squared (*i.e.*, Laplace operator), t is time, and $u_{\text{em}} = \lambda v$ is phase velocity of the electromagnetic wave in the atom, where λ is the wavelength and v is the frequency of electromagnetic wave in the atom.

The same form of wave equations as (1), but only in one dimension, is also present on the parallel-wire transmission line (Lecher's line), consisting of a pair of ideal conductive nonmagnetic parallel wires of radius ρ , separated by δ , where the ratio $\delta/\rho = \chi$, [3]:

$$\frac{\partial^2 u}{\partial z^2} - L'C' \frac{\partial^2 u}{\partial t^2} = 0; \quad \frac{\partial^2 i}{\partial z^2} - L'C' \frac{\partial^2 i}{\partial t^2} = 0, \quad (2)$$

where u is voltage at the entrance of the element dz of Lecher's line, i is electric current at the entrance of the element dz of Lecher's line, $C' = \pi\epsilon / \ln \left[\chi/2 + (\chi^2/4 - 1)^{1/2} \right]$ is capacitance of Lecher's line per unit length,

$L' = \mu (\ln \chi + 1/4) / \pi$ is inductance of Lecher's line per unit length, [4], ϵ is the permittivity of space in the atom, or in space of Lecher's line, and μ is the permeability of the space in the atom, or in the space of Lecher's line.

Systems with the same differential equations behave equally. Hence, in [1] it was concluded that the electric field in the atom behaves as the voltage on Lecher's line, and the magnetic field in the atom behaves as the current in Lecher's line. Furthermore, [1] shows that the Lecher's line may be represented as the inductive-capacitive network (so-called *LC* network), which finally makes the oscillatory circuit (*LC* circuit). The natural frequency of *LC* circuit $\nu = \left[2\pi(LC)^{1/2} \right]^{-1}$, where C is the sum of all small capacitors of the *LC* network on the open end of the network, and L is sum of all small inductances of the *LC* network on the short-circuited of the network [5].

2.2. Electromagnetic Energy in the Atom

According to the model presented in [1], electromagnetic energy in the atom E_{em} can be described as the electromagnetic energy of *LC* circuit, *i.e.* $E_{\text{em}} = \Theta^2 / (2C)$, where Θ is maximal charge on the said capacitor C , [6]. On the other hand, from the balance of forces in the atom follows [6]:

$$\frac{mv^2}{r\sqrt{1-\beta^2}} = \frac{|qQ|}{4\pi\epsilon r^2}; \quad r = \frac{|qQ|}{4\pi\epsilon mc^2} \frac{\sqrt{1-\beta^2}}{\beta^2}, \quad (3)$$

where r is the radius of the circular orbit of the electron, q is the charge of the electron ($q = -e$), e is elementary charge, Q is the charge of the nucleus ($Q = Ze$), Z is atomic number (which theoretically is not in integral domain), m is the electron rest mass, $c = 1/(\mu_0\epsilon_0)^{1/2}$ is the speed of light in vacuum, $\beta = v/c$, where v is the velocity of the electron, $\epsilon = \epsilon_r\epsilon_0$, as stated above is permittivity, ϵ_r is relative permittivity, ϵ_0 is permittivity of free space, $\mu = \mu_r\mu_0$, as stated above, is permeability, μ_r is relative permeability, μ_0 is permeability of free space, the transverse mass of the electron is $m/(1-\beta^2)^{1/2}$, [7]. Increase of transverse mass of the electron is

$$\Delta m = m \left[(1-\beta^2)^{-1/2} - 1 \right].$$

The kinetic energy of the electron, [6], is $K = \Delta mc^2$, **Figure 1**. Using Equation (3) and noting that an electron holds an opposite charge to the nucleus, the potential energy of electron, [3,6], is

$U = qQ / (4\pi\epsilon r) = -mc^2 \beta^2 / (1-\beta^2)^{1/2}$. The total mechanical energy of an electron, [6], is the sum of its kinetic and potential energies, $W = K + U = -mc^2 \left[1 - (1-\beta^2)^{1/2} \right]$. According to the law of conservation of energy, this

energy is equal to the negative emitted electromagnetic energy of the atom, $E_{\text{em}} = -W = eV$, where V is the potential difference through which the electron passes to get an equal energy as electromagnetic energy E_{em} . Out of the equation $E_{\text{em}} = mc^2 \left[1 - (1-\beta^2)^{1/2} \right]$ comes $(1-\beta^2)^{1/2} = 1 - E_{\text{em}}/mc^2$ and $\beta^2 = 2E_{\text{em}}(1 - E_{\text{em}}/2mc^2) / mc^2$,

so the radius r in Equation (3) through arranging leads to:

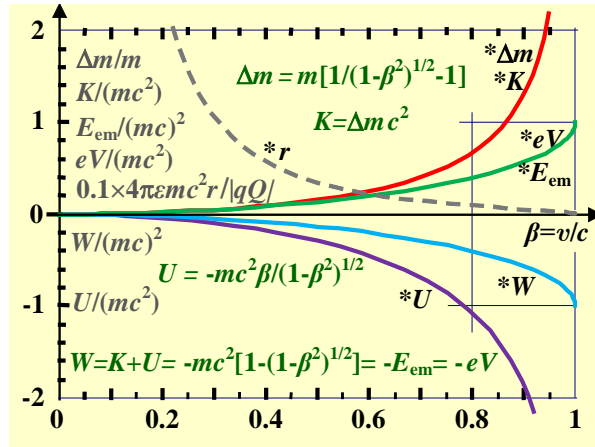


Figure 1. Energy relations in an atom (these relationships are valid for both classical and quantum physics; please note that the normalized quantities are marked with *). Radius of the circular orbit of the electron $*r = r/[10|qQ|/(4\pi\epsilon mc^2)]$, kinetic energy of the electron $*K = K/(mc^2)$, potential energy of the electron $*U = U/(mc^2)$, total mechanical energy of the electron $*W = *K + *U$, electromagnetic energy of the atom $*E_{em} = -*W = *eV$; all versus normalized velocity of the electron $\beta = v/c$.

$$r = \frac{|qQ|}{8\pi\epsilon E_{em}} \frac{1 - E_{em}/mc^2}{1 - E_{em}/2mc^2}, \quad (4)$$

or

$$E_{em} = \frac{1}{2} \frac{|qQ|}{4\pi\epsilon r} \frac{1 - E_{em}/mc^2}{1 - E_{em}/2mc^2} = \frac{1}{2} |U| \frac{1 - E_{em}/mc^2}{1 - E_{em}/2mc^2}. \quad (5)$$

On the other hand, electromagnetic energy is $E_{em} = \Theta^2/(2C)$, which means that Equation (5) we can write as:

$$\frac{1}{2} \frac{|qQ|}{4\pi\epsilon r} \frac{1 - E_{em}/mc^2}{1 - E_{em}/2mc^2} = \frac{1}{2} \frac{\Theta^2}{C}. \quad (6)$$

The single Equation (6) has two unknowns, *i.e.*, parameter C and variable Θ . By using Diophantine equations we get one of the many solutions: $C = 4\pi\epsilon r$, and $\Theta^2 = |qQ|(1 - E_{em}/mc^2)/(1 - E_{em}/2mc^2)$ [8].

2.3. Natural Frequency of the Electromagnetic Wave in the Atom

Equation (6), which represents the electromagnetic energy in an atom, can be written like this [1]:

$$E_{em} = \frac{1}{2} \frac{\Theta^2}{C} = \frac{1}{2} \frac{\pi}{\pi} \frac{\Theta^2}{\sqrt{C}\sqrt{C}} \frac{\sqrt{L}}{\sqrt{L}} = \pi \sqrt{\frac{L}{C}} \Theta^2 \frac{1}{2\pi\sqrt{LC}} = \pi Z_{LC} \Theta^2 \nu = \pi Z_{LC} |qQ| \frac{1 - E_{em}/mc^2}{1 - E_{em}/2mc^2} \nu = A\nu, \quad (7)$$

where

$$A = \pi Z_{LC} |qQ| \frac{1 - E_{em}/mc^2}{1 - E_{em}/2mc^2} \quad (8)$$

is the action of the electromagnetic oscillator, and

$$Z_{LC} = \sqrt{\frac{L}{C}} = \sqrt{\frac{L dz}{C' dz}} = \sqrt{\frac{L'}{C'}} = \sqrt{\frac{\mu}{\epsilon}} \sqrt{\frac{\left[\ln\left(\chi/2 + \sqrt{\chi^2/4 - 1}\right) \right] (\ln \chi + 1/4)}{\pi}} = \sqrt{\frac{\mu}{\epsilon}} \frac{\sigma(\chi)}{\pi} \quad (9)$$

is the characteristic impedance of Lecher's line, while

$$\sigma(\chi) = \sqrt{\left[\ln\left(\chi/2 + \sqrt{\chi^2/4 - 1}\right) \right] (\ln \chi + 1/4)} \quad (10)$$

is the structural coefficient of Lecher's line.

From Equation (7), in fact from $E_{\text{em}} = \Theta^2 / (2C) = \pi Z_{\text{LC}} \Theta^2 \nu$, furthermore from Equations (9) and $C = 4\pi\epsilon r$, follows the natural frequency of Lecher's line, *i.e.*, follows the natural frequency of electromagnetic wave in an atom connected with an electron which is located on the orbit of radius r :

$$\nu = \frac{1}{2\pi Z_{\text{LC}} C} = \frac{1}{8\pi\sqrt{\mu\epsilon}\sigma(\chi)r}. \quad (11)$$

Therefore in an atom with multiple electrons there are multiple natural frequencies ν . It should be noted that the same expression as (11) came from simultaneously multiplying and dividing the right side of the original expression $\nu = [2\pi(LC)^{1/2}]^{-1}$ with $C^{1/2}$, and then using the expression $C = 4\pi\epsilon r$ and Equation (9).

Equation (11) shows that the natural frequency ν does not depend on the amount of charge in the atom, but depends on the properties of the space (μ , ϵ), structural coefficient of Lecher's line $\sigma(\chi)$, (which for its part depends only on the parameters of Lecher's line δ and ρ), and the radius r of the circular orbit. Indeed, electromagnetic oscillations in the atom require a charge, but amount of the charge does not affect the amount of the frequency ν .

If Equation (4) is inserted in Equation (11) we obtain natural frequency of electromagnetic wave in an atom:

$$\nu = \frac{E_{\text{em}}}{\sqrt{\mu/\epsilon}\sigma(\chi)|qQ|} \frac{1 - E_{\text{em}}/2mc^2}{1 - E_{\text{em}}/mc^2}. \quad (12)$$

2.4. Structural Constant of the Atom

In the Equation (12) charges q and Q appear in the form of the product $|qQ|$. In order to satisfy the condition that the frequency ν is independent of the charge, and to avoid direct or indirect involvement of the charge, the total product $(\mu/\epsilon)^{1/2}\sigma(\chi)|qQ| = (\mu/\epsilon)^{1/2}\sigma(\chi)Ze^2$ in Equation (12) must be a constant, *i.e.*, only a product $\sigma(\chi)Z$ must be constant, because $\mu/\epsilon = \mu_0/\epsilon_0$ and e are already considered as constants. This product, in the form

$$s_0 = \sqrt{\sigma(\chi)Z} = \text{const.}, \quad (13)$$

is called the structural constant of the atom. In the article [1] the value of structural constant has already been estimated ($s_0 = 8.27756$). However, the present paper shows accurate procedure for calculating this constant.

2.5. Action Constant of the Atom

From Equations (8), (9) and (13) follows

$$A = \sqrt{\frac{\mu}{\epsilon}} s_0^2 e^2 \frac{1 - E_{\text{em}}/mc^2}{1 - E_{\text{em}}/2mc^2} = A_0 \frac{1 - E_{\text{em}}/mc^2}{1 - E_{\text{em}}/2mc^2}, \quad \frac{A}{A_0} = \frac{1 - E_{\text{em}}/mc^2}{1 - E_{\text{em}}/2mc^2}, \quad (14)$$

where

$$A_0 = \pi Z_{\text{LC}} |qQ| = \sqrt{\frac{\mu}{\epsilon}} s_0^2 e^2 = \sqrt{\frac{\mu_r \mu_0}{\epsilon_r \epsilon_0}} s_0^2 e^2 = \sqrt{\frac{\mu_0}{\epsilon_0}} s_0^2 e^2, \quad (15)$$

is action constant, which implies that $\mu_r = \epsilon_r$. Electromagnetic energy in Equation (7), *i.e.*, $E_{\text{em}} = A\nu$, using Equation (14), we can now write as:

$$E_{\text{em}} = A_0 \nu \frac{1 - E_{\text{em}}/mc^2}{1 - E_{\text{em}}/2mc^2}. \quad (16)$$

By solving this equation we obtain:

$$\begin{aligned} E_{\text{em}} &= A_0 \nu + mc^2 - \sqrt{(A_0 \nu)^2 + (mc^2)^2}, \\ A &= A_0 + mc^2/\nu - \sqrt{A_0^2 + (mc^2/\nu)^2}. \end{aligned} \quad (17)$$

Taking into account that $E_{\text{em}} = eV$ it is as follows:

$$eV/mc^2 = A_0 v/mc^2 + 1 - \sqrt{(A_0 v/mc^2)^2 + 1}. \quad (18)$$

The extended Duane–Hunt's law we get from Equations (14) and (16) using $E_{\text{em}} = eV$ [8], and using $(1 - \beta^2)^{1/2} = 1 - E_{\text{em}}/mc^2$ and $\beta^2 = 2E_{\text{em}}(1 - E_{\text{em}}/2mc^2)/mc^2$ we get also a portion $A_0 v$:

$$v = \frac{eV}{A_0} \frac{1 - eV/2mc^2}{1 - eV/mc^2}; \quad v = \frac{eV}{A}; \quad A_0 v = \frac{1}{2} \frac{m}{\sqrt{1 - \beta^2}} v^2. \quad (19)$$

From Equations (5) and (16) follows:

$$v = \frac{|U|}{2A_0}. \quad (20)$$

2.6. Phase Velocity of the Electromagnetic Wave in the Atom

Thanks to the analogy between the electromagnetic wave in the atom and the wave of voltage and current at the Lecher's line, according to Equations (1) and (2), and $C' = \pi \varepsilon / \ln \left[\chi/2 + (\chi^2/4 - 1)^{1/2} \right]$, $L' = \mu (\ln \chi + 1/4)/\pi$, the phase velocity is, [3]:

$$u_{\text{em}} = \lambda v = \frac{1}{\sqrt{L'C'}}, \quad (21)$$

i. e., in normalized form [9]:

$$u_{\text{em}} \sqrt{\varepsilon \mu} = \sqrt{\ln \left(\frac{\chi}{2} + \sqrt{\frac{\chi^2}{4} - 1} \right)} / \left(\ln \chi + \frac{1}{4} \right) = F(\chi), \quad (22)$$

where $(\varepsilon \mu)^{1/2} = (\varepsilon_r \mu_r)^{1/2} / c$. In case $\chi \gg 1$ expression under the square root in Equation (22) then it is approximately equal to one as described in [1]. In this paper, however, we use the integral Equation (22) and analyze it in the area around $\chi=2$, which is suitable for calculating the structural constant s_0 . Now we have a system of two equations with two unknowns, ε and μ ,

$$\sqrt{\mu/\varepsilon} = \sqrt{\mu_0/\varepsilon_0}, \quad u_{\text{em}} \sqrt{\mu \varepsilon} = F(\chi), \quad (23)$$

for which the solutions are (Figure 2):

$$\varepsilon = \varepsilon_r \varepsilon_0 = \sqrt{\frac{\varepsilon_0}{\mu_0}} \frac{F(\chi)}{u_{\text{em}}}, \quad \mu = \mu_r \mu_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{F(\chi)}{u_{\text{em}}}, \quad \varepsilon_r = \mu_r = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \frac{F(\chi)}{u_{\text{em}}} = \frac{c}{u_{\text{em}}} F(\chi). \quad (24)$$

2.7. Wavelength and Momentum of Electromagnetic Wave in the Atom

The momentum of the electromagnetic wave in the atom is equal to the momentum of a photon [1], $p_{\text{em}} = E_{\text{em}}/u_{\text{em}} = E_{\text{em}}/(\lambda v)$, Figure 2,

$$p_{\text{em}} = \frac{E_{\text{em}}}{\lambda v} = \frac{A_0}{\lambda} \frac{1 - eV/mc^2}{1 - eV/2mc^2} = \frac{A}{\lambda}. \quad (25)$$

In accordance to the law of conservation of momentum, this momentum is equal to the linear momentum of the electron, [1],

$$\frac{mv}{\sqrt{1 - \beta^2}} = \frac{mc\beta}{\sqrt{1 - \beta^2}} = \frac{A}{\lambda}. \quad (26)$$

By applying the expressions $(1 - \beta^2)^{1/2} = 1 - E_{\text{em}}/mc^2$ and $\beta^2 = 2E_{\text{em}}(1 - E_{\text{em}}/2mc^2)/mc^2$ to Equation (26) we obtain

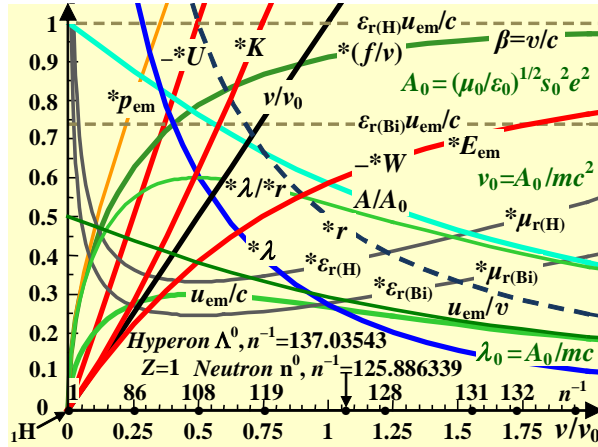


Figure 2. (Please note that the normalized quantities are marked with *). Radius of the circular orbit of the electron $*r = r/[|qQ|/(4\pi\epsilon mc^2)]$, normalized velocity of the electron $\beta = v/c$, kinetic energy of the electron $*K = K/(mc^2)$, potential energy of the electron $*U = U/(mc^2)$, total mechanical energy of the electron $*W = *K + *U$, electromagnetic energy of the atom $*E_{em} = -*W$, momentum of electromagnetic wave in the atom $*p_{em} = p_{em}/(mc)$, wavelength of the electromagnetic wave in the atom $*\lambda = \lambda/[A_0/(mc^2)]$, normalized phase velocity of the electromagnetic wave in the atom u_{em}/c and u_{em}/v , normalized action of the electromagnetic oscillator A/A_0 , relative permittivity and relative permeability of the space in the hydrogen atom $\epsilon_{r(H)} = *\mu_{r(H)} = \epsilon_{r(H)}/10 = \mu_{r(H)}/10$ and also relative permeability and relative permeability of the space within atom of bismuth $\epsilon_{r(Bi)} = *\mu_{r(Bi)} = \epsilon_{r(Bi)}/10 = \mu_{r(Bi)}/10$, product of relative permittivity and normalized phase velocity $\epsilon_{r(H)}\mu_{em}/c = \mu_{r(H)}\mu_{em}/c$ in the case of hydrogen, product of relative permittivity and normalized phase velocity $\epsilon_{r(Bi)}\mu_{em}/c = \mu_{r(Bi)}\mu_{em}/c$ in the case of bismuth, the ratio of wavelength and radius $*\lambda^*r = (\lambda r)/(4\pi\epsilon c A_0 |qQ|) = 2u_{em}/c$, the ratio of the frequency f of rotation of the electron orbiting atom and the frequency ν of the electromagnetic wave $*(f/\nu) = (f/\nu)/(4\pi\epsilon c A_0 |qQ|)$, all versus normalized frequency of the electromagnetic wave in the atom ν/ν_0 , where $\nu_0 = A_0/mc^2$; $n^{+1} = 1, 2, 3, \dots$ is ordinal number of stationary orbits in the atom, $n^{-1} = 1/1, 1/2, 1/3, \dots 1/n$, for hyperon Ξ^0 $n^{-1} = 137.03587$.

$$\lambda = \frac{A_0}{\sqrt{2meV}} \frac{(1 - eV/mc^2)^2}{\sqrt{(1 - eV/2mc^2)^3}}, \quad (27)$$

and using Equation (19) it becomes

$$\lambda = \sqrt{\frac{A_0}{2mv}} \frac{\sqrt{(1 - eV/mc^2)^3}}{1 - eV/2mc^2}. \quad (28)$$

Phase velocity of the electromagnetic wave in an atom is obtained by multiplying Equations (19) and (27):

$$u_{em} = \lambda \nu = \sqrt{\frac{eV}{2m}} \frac{1 - eV/mc^2}{\sqrt{1 - eV/2mc^2}}. \quad (29)$$

From (25) and (27) follows

$$p_{em} = \sqrt{2meV} \frac{\sqrt{1 - eV/2mc^2}}{1 - eV/mc^2}. \quad (30)$$

The ratio of the wavelength of the electromagnetic wave in the atom and the atom radius are obtained from Equations (4) and (27) using $E_{em} = eV$ and Equation (29), **Figure 2**:

$$\frac{\lambda}{r} = \frac{8\pi\epsilon A_0}{|qQ|} \frac{\sqrt{eV}}{\sqrt{2m}} \frac{1 - eV/mc^2}{1 - eV/2mc^2} = \frac{8\pi\epsilon A_0}{|qQ|} u_{em}. \quad (31)$$

This expression, with $|U| = |qQ|/(4\pi\epsilon r)$, leads to:

$$\lambda = 2A_0 \frac{u_{em}}{|U|}. \quad (32)$$

A minimum of two separate oscillating processes are performed simultaneously within an atom, *i.e.*, the circular motion of electrons around the nucleus and oscillation of electromagnetic wave energy [10]. The time period of one circular tour of electrons around the nucleus is $T_e = 2r\pi/v = 1/f$, where f is the frequency of circulation of electrons around the nucleus. The duration of the period of the electromagnetic wave is $T_{em} = 1/\nu$. Hence, $\nu/f = 2\pi vr/v$. Using Equation (31), as well as $v/c = \beta$ and $\lambda\nu = u_{em}$ follows, **Figure 2**:

$$\frac{T_e}{T_{em}} = \frac{\nu}{f} = \frac{|qQ|}{4\varepsilon A_0 v}, \quad \frac{f}{\nu} = \frac{4\varepsilon A_0 c}{|qQ|} \beta. \quad (33)$$

2.8. Stationary States of the Atoms

Long term existence of the rotation of electrons and long term existence of the electromagnetic wave in the atom (stationary state) is only possible if there is synchronism between them (synchronously stationary state) [10,11]. To be coherent with the active power of the electromagnetic wave in an atom, the electron needs to oscillate (*i.e.*, rotate) with dual frequency of the wave, because the active power of wave oscillates with dual frequency $2\omega = 2(2\pi\nu)$, (this will be further discussed in Sub-Heading 2.9). This means that in the synchronously stationary state of the atom, the time period of electron rotation T_e is a half period of T_{em} (or, for reasons of synchronism, is $n^{\pm 1}$ -multiple of a half period of T_{em}), *i.e.*, $T_e = n^{\pm 1}T_{em}/2$, where $n = 1, 2, 3, \dots$ is ordinal number of stationary orbits in the atom (or $n^{-1} = 1/1, 1/2, 1/3, \dots 1/n$). Equation (33) gives the speed of electron in a synchronously stationary state ([11], compare with [6]):

$$v_n = \frac{1}{n^{\pm 1}} \frac{|qQ|}{2\varepsilon A_0}. \quad (34)$$

The Equations (3) and (34) give the radius of the electron orbits in the synchronously stationary states:

$$r_n = (n^{\pm 1})^2 \frac{\varepsilon A_0^2 \sqrt{1-\beta^2}}{\pi m |qQ|}. \quad (35)$$

From Equations (33), (34) and (35) follows [11]:

$$f_n = \frac{1}{(n^{\pm 1})^3} \frac{|qQ|^2 m}{4\varepsilon^2 A_0^3 \sqrt{1-\beta^2}}, \quad (36)$$

$$\nu_n = \frac{1}{(n^{\pm 1})^2} \frac{|qQ|^2 m}{8\varepsilon^2 A_0^3 \sqrt{1-\beta^2}}, \quad (37)$$

and, [10],

$$f_n = 2 \left(\frac{\nu_n}{n^{\pm 1}} \right). \quad (38)$$

The total mechanical energy of an electron $W_n = -E_{em(n)}$ follows from Equations (17) and (37):

$$W_n = -\frac{1}{(n^{\pm 1})^2} \frac{|qQ|^2 m}{8\varepsilon^2 A_0^2 \sqrt{1-\beta^2}} - mc^2 + \sqrt{\left(\frac{1}{(n^{\pm 1})^2} \frac{|qQ|^2 m}{8\varepsilon^2 A_0^2 \sqrt{1-\beta^2}} \right)^2 + (mc^2)^2}. \quad (39)$$

For energies much smaller than mc^2 :

$$W_n \approx -\frac{1}{(n^{\pm 1})^2} \frac{|qQ|^2 m}{8\varepsilon^2 A_0^2 \sqrt{1-\beta^2}}. \quad (40)$$

If assume the maximum speed of electron is equal to the speed of light in a given medium, *i.e.*, according to Equation (22) $v_{max} = u_{em} = F(\chi)/(\mu\varepsilon)^{1/2}$ (to increase the speed of electron should be $n^{\pm 1} = 1/n_{max}$) from Equations (15) and (34) we get:

$$n_{\max} = \frac{2s_0^2}{Z} F(\chi). \quad (41)$$

From Equation (41) follows the greatest possible atomic number Z_{\max} when n_{\max} is minimal and $F(\chi)$ is maximal, really when $n_{\max} = 1$ and $F(\chi) = 1$, *i.e.*,

$$Z_{\max} = \frac{2s_0^2}{n_{\max}} F(\chi) = 2s_0^2. \quad (42)$$

2.9. Wave Equations of the Electromagnetic Wave in the Atom

Wave equations of electromagnetic wave in an atom are expressed by Equation (1). If we insert phase velocity u_{em} in these equation from Equation (29), *i.e.*,

$$u_{\text{em}}^2 = \frac{eV (1 - eV/mc^2)^2}{2m (1 - eV/2mc^2)}, \quad (43)$$

we obtain

$$\nabla^2 \mathbf{E} - \frac{2m}{eV} \frac{1 - eV/2mc^2}{(1 - eV/mc^2)^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0, \quad \nabla^2 \mathbf{H} - \frac{2m}{eV} \frac{1 - eV/2mc^2}{(1 - eV/mc^2)^2} \frac{\partial^2 \mathbf{H}}{\partial t^2} = 0. \quad (44)$$

Wave Equations (1) and (44) have a lot of solutions. We will apply the solutions that correspond to the transmission line, *i.e.*, to the *LC* network. These solutions are standing waves [12,13]:

$$E_x(z, t) = E_0 \sin\left(\frac{2\pi z}{\lambda}\right) \cos(\omega t), \quad H_y(z, t) = -\frac{E_0}{\sqrt{\mu/\varepsilon}} \cos\left(\frac{2\pi z}{\lambda}\right) \sin(\omega t), \quad (45)$$

where E_0 is the maximum value, *i.e.*, the amplitude of electric field strength E , $E_x(z, t)$ is the x -component of the electric field strength dependent on the z -axis and the time t , and $H_y(z, t)$ is the y -component of the magnetic field strength H dependent on the z -axis and the time t , $\omega = 2\pi\nu$. All mathematical operations we perform for the y -component of the magnetic field $H_y(z, t)$ can be performed for the x -component of the electric field $E_x(z, t)$ in the same way.

In the standing waves (45) the energy oscillates between the electric and magnetic form. The electrical energy is maximum when the magnetic energy is zero, and vice versa. Furthermore, the standing wave transfers no energy through the space because the average active power of the wave is equal to zero. The current value of the active power oscillates in both directions, + and - of z axis, with dual frequency 2ω from point to point of z axis [12]. As already mentioned, this is why (for the maintenance of stationary state of the atom) the electron has to rotate twice as fast compared to the lower harmonics (n^+), or twice as fast compared to the upper harmonics (n^-), *i.e.*, $f = 2(\nu/n^{\pm})$ in accordance with Equation (38).

If we use the second derivative with respect to z of the y -component $H_y(z, t)$ of the magnetic field strength in Equation (45), we get: $\partial^2 H_y(z, t)/\partial z^2 + (2\pi/\lambda)^2 H_y(z, t) = 0$. After inclusion of the wavelength λ from Equation (27) we obtain:

$$\frac{\partial^2 H_y(z, t)}{\partial z^2} + \frac{8\pi^2 m e V (1 - eV/2mc^2)^3}{A_0^2 (1 - eV/mc^2)^4} H_y(z, t) = 0, \quad (46)$$

If $eV/mc^2 \ll 1$, then $eV \approx K = W - U$, and Equation (46) becomes

$$\frac{\partial^2 H_y(z, t)}{\partial z^2} + \frac{8\pi^2 m}{A_0^2} (W - U) H_y(z, t) = 0. \quad (47)$$

The second derivative of $H_y(z, t)$ with respect to t gives:

$\partial^2 H_y(z, t)/\partial t^2 + \omega^2 H_y(z, t) = 0$. After inclusion of the angular frequency $\omega = 2\pi\nu$ from Equation (19) we obtain:

$$\frac{\partial^2 H_y(z, t)}{\partial t^2} + \left(\frac{2\pi}{A_0} eV \frac{1 - eV/2mc^2}{1 - eV/mc^2} \right)^2 H_y(z, t) = 0. \quad (48)$$

If $eV/mc^2 \ll 1$, then $eV \approx K = W - U$, and Equation (48) becomes

$$\frac{\partial^2 H_y(z, t)}{\partial t^2} + \frac{4\pi^2}{A_0^2} (W - U)^2 H_y(z, t) = 0. \quad (49)$$

3. Calculation of the Structural Constant s_0

Structural constant of the atom s_0 can be determined in several ways, *e.g.*, by measuring two quantities, the voltage V and frequency ν and calculating the action constant A_0 by Duane–Hunt’s law, *i.e.*, using Equations (15) and (19), [8]. However, here we will use a more direct theoretical calculation, with only one empirical item necessary.

Namely, the increase of the nuclear charge in the atom increases atomic number Z . In accordance with Equation (13), the value of structural coefficient $\sigma(\chi) = s_0^2/Z$ is assigned to each atom. So, greater atomic number means a lower structural coefficient $\sigma(\chi)$.

On the other hand, there is a critical nuclear charge which ensures stability of the atom [2,14]. In other words, to reduce $\sigma(\chi)$ means to grow instability of the atom. In general, the higher atomic number means the less stability (*i.e.*, the less half–life, or $t_{1/2}$) of the atom, starting from bismuth ${}_{83}\text{Bi}$ ($Z = 83$, $t_{1/2} = 6 \times 10^{26}$ s, [15]) to ununoctium ${}_{118}\text{Uuo}$ ($Z = 118$, $t_{1/2} = 5$ ms), [exceptions are atoms of technetium (${}_{43}\text{Tc}$, $Z = 43$, $t_{1/2} = 1.3 \times 10^{14}$ s) and promethium (${}_{63}\text{Pm}$, $Z = 63$, $t_{1/2} = 5.6 \times 10^8$ s)].

For the calculation of structural constant s_0 , it is enough to find only one associated pair of $\sigma(\chi)$ and Z . The curve $\sigma(\chi)$ has no extremes, **Figure 3**. Thus it is not easy to find a mentioned pair of $\sigma(\chi)$ and Z . In that sense, a better situation is with the phase velocity u_{em} , specifically with the normalized phase velocity $u_{\text{em}}(\mu\epsilon)^{1/2} = F(\chi)$ of electromagnetic wave in the atom, **Figure 3**, [9]. Neither of these two curves have extremes, but there is a sharp knee on $F(\chi)$ which can be used to determine the structural constant s_0 .

Although there is no theory about the connection between the phase velocity of electromagnetic waves in the atom and the stability of the atoms, it is still possible to use this mathematical benefit of sharp knee for those atoms, in which there is the lower phase velocity of electromagnetic waves that exhibit greater instability. Use of this result will be discussed just a little bit later.

The nuclear binding energy per nucleon slightly decreases with increase the atomic number (starting from the first radioactive element bismuth, ${}_{83}\text{Bi}$, 7.848 MeV, to the unoctium, ${}_{118}\text{Uuo}$, 7.074 MeV, about 0.31% decrease for each 35 atoms in that area [15]). Physically this means that the boundary between stable and unstable areas is not emphasized. Mathematically it allows that between the two areas set up so–called transition area, **Figure 3**. This is, at the moment, the most accurate way to determine the boundary between stable atoms and the others. Indeed, the first unstable atom must be located on that border. This is a key fact to determine the structural constant s_0 of the atom.

Before calculating, we observe the first derivative $F'(\chi)$ of the curves of normalized phase velocity $F(\chi)$ of electromagnetic waves in an atom (**Figure 3**). When this derivative is greater than 1, it means that the phase velocity rapidly decline, it is a zone of unstable atoms ($2 < \chi < 2.129\ 531\ 7$). It should be noted that the situation $\chi \leq 2$ is theoretically impossible because then there is no Lecher’s line.

When the second derivative $F''(\chi)$ of the normalized phase velocity $F(\chi)$ is greater than 1, it means that the phase velocity starts to rapidly decline (**Figure 3**), this is a transition zone ($2.129\ 531\ 7 < \chi < 2.382\ 788$).

The border crossing from the transition zone to the stable zone (*i.e.*, $\chi_0 = 2.382\ 788$), in accordance with the experiments, [15], is closest to the bismuth atom. Bismuth atom (${}_{83}\text{Bi}$) is the first unstable atom, in the entire chain of stable atoms, which ends with lead (${}_{82}\text{Pb}$). The corresponding value of the structural coefficient in that place is $\sigma(\chi_0) = 0.825\ 402$, **Figure 3**. Bismuth is a chemical element with atomic number $Z = 83$, with half–life more than a billion times the estimated age of the universe. Even though charges in reality take discrete values (e , $2e$, $3e$, ... Ze), theoretical value of Z in Equation (13) can be within the range $Z_{\text{th}} = 83 \pm 1/2$. Thus, according to Equation (13) we get the structural constant of the atom $[0.825\ 402 \times (83 \pm 1/2)]^{1/2}$, *i.e.*, $8.252 < s_0 < 8.302$, with a mean value 8.277 and with sample standard deviation $\pm 0.035\ 355$ or as a percentage $s_0 = 8.277 \pm 0.43\%$.

4. Conclusion

In classical physics, the use of Maxwell’s equations and Lorentz theory of electrons gives us equations that describe the behavior of atom. For this description, usual classical physical constants are sufficient. The only new necessary physical constant is the structural constant of the atom s_0 . This new physical constant was calculated in the framework of a unified theory based on the model of the atom by analogy with the transmission line. To

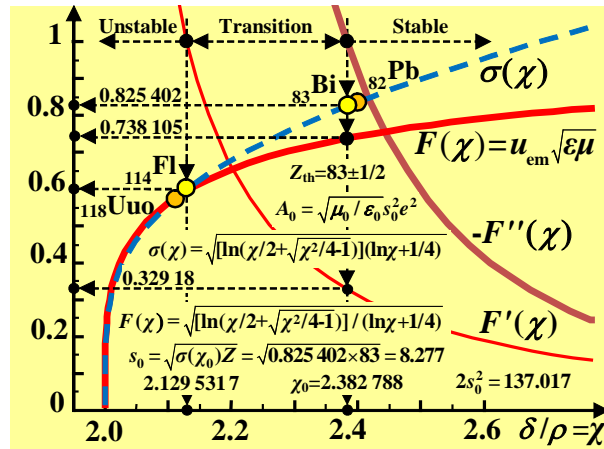


Figure 3. Structural coefficient of Lecher's line $\sigma(\chi)$, normalized phase velocity of the electromagnetic wave in the atom $F(\chi) = u_{\text{em}}(\mu\epsilon)^{1/2}$, the first derivative of the normalized phase velocity $F'(\chi)$, inverted second derivative of the normalized phase velocity $-F''(\chi)$, all versus ratio $\delta/\rho = \chi$ of the transmission Lecher's line, consisting of a pair of ideal conducting nonmagnetic parallel wires of radius ρ separated by δ , which represents a model of an atom.

calculate it, we need only one empirical item, *i.e.*, the atomic number of the first unstable atom in periodic table of elements. It is known, that it is bismuth, with atomic number $Z = 83$. From this data, the structural constant s_0 is calculated. Thus, we get $s_0 = 8.277 \pm 0.43\%$. Comparing with the fine structure constant we get $s_0 = 8.27756$, which is consistent with the calculation performed here (the relative difference is less than 0.0068%). This constant, thanks to the expressions $(\mu_0/\epsilon_0)^{1/2}s_0^2e^2$ and $(2s_0^2)^{-1}$, can replace two existing constants, Planck's constant h and the fine structure constant α . We get also the stationary states of the atoms and the maximal atomic number $Z = 2s_0^2 = 137.036$, *i.e.*, as integer should be $Z_{\text{max}} = 137$. The difference between $2s_0^2$ and Z_{max} we call the slipping. This indicates that, in addition to integer synchronously stationary states, non-integer (asynchronously) stationary states in atoms are also possible. Now, just a mention, as it seems to asynchronously stationary states can produce a continuous spectrum of atoms. This, however, requires more detailed research. This model of the atoms covers the energies of the electromagnetic, weak and strong fundamental interactions [16].

Acknowledgements

Wolfram Research, Inc. *Mathematica* software is used by courtesy of *Systemcom Ltd.*, Zagreb, Croatia. The author thanks Ms. Erica Vesic for editing this article in English, Mr. Damir Vuk for the useful discussions and *Prvomajska TZR Ltd.*, Zagreb, Croatia, www.prvomajska-tzr.hr, for the financial support.

REFERENCES

- [1] M. Perkovac, "Model of an Atom by Analogy with the Transmission Line," *Journal of Modern Physics*, Vol. 4, No. 7, 2013, pp. 899-903. <http://dx.doi.org/10.4236/jmp.2013.47121>.
- [2] H. Hänsel and W. Neumann, "Physik," Spektrum Akademischer Verlag, Heidelberg, 1995.
- [3] Czichos and Association Hütte, "Die Grundlagen der Ingenieurwissenschaften," Springer-Verlag, Berlin, 1989.
- [4] T. Bosanac, "Teoretska elektrotehnika 1," Tehnicka knjiga, Zagreb, 1973.
- [5] R. Rüdberg, "Elektrische Schaltvorgänge," Julius Springer, Berlin, 1923.
- [6] D. C. Giancoli, "Physics for Scientists and Engineers," Prentice Hall, Englewood Cliffs, 1988.
- [7] L. Page and N. I. Adams, "Electrodynamics," D. Van Nostrand Company, Inc., New York, 1940.
- [8] M. Perkovac, "Statistical Test of Duane—Hunt's Law and Its Comparison with an Alternative Law," 2010. <http://arxiv.org/abs/1010.6083>.
- [9] M. Perkovac, "Maxwell Equations for Nanotechnology," *Proceedings of the 35th International Convention of the IEEE MI-PRO*, Opatija, 21-25 May 2012, pp. 429-436. http://ieeexplore.ieee.org/xpl/freeabs_all.jsp?arnumber=6240683.
- [10] M. Perkovac, "Quantization in Classical Electrodynamics," *Physics Essays*, Vol. 15, No. 1, 2002, pp. 41-60. <http://dx.doi.org/10.4006/1.3025509>.
- [11] M. Perkovac, "Absorption and Emission of Radiation by an Atomic Oscillator," *Physics Essays*, Vol. 16, 2003, pp. 162-173.

<http://dx.doi.org/10.4006/1.3025572>.

- [12] Z. Haznadar and Z. Stih, "Elektromagnetizam," Skolska knjiga, Zagreb, 1997.
- [13] J. D. Jackson, "Classical Electrodynamics," John Wiley & Sons, Inc., New York, 1999.
- [14] R. D. Benguria, M. Loss and H. Siedentop, "Stability of Atoms and Molecules in an Ultrarelativistic Thomas-Fermi-Weizsäcker Model," *Journal of Mathematical Physics*, Vol. 49, 2007, Article ID: 012302. <http://dx.doi.org/10.1063/1.2832620>.
- [15] <http://periodictable.com/Isotopes/001.1/index.html>.
- [16] <http://www.physics.upenn.edu/~pgl/e27/E27.pdf>.