

# Development of a Bi-Material Representative Volume Element Using Damaged Homogenisation Approach

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Received September 2013

## ABSTRACT

Most civil engineering structures are formed using a number of materials that are bonded to each other with their surface-to-surface interaction playing key role on the overall response of the structure. Unfortunately these interactions are extremely variable; simplified and extremely detailed models trialed to date prove quite complex. Models that assume perfect interaction, on the other hand, predict unsafe behavior. In this paper a damage mechanics based interaction between two materials of different softening properties is developed using homogenisation approach. This paper describes the process of developing a bi-material representative volume element (RVE) using damaged homogenisation approach. The novelty in this paper is the development of non-local transient damage identification algorithm. Numerical examples prove the stability of the approach for a simplified RVE and encourage application to other shapes of RVEs.

**Keywords:** Transient Gradient Non-Local; Homogenisation; Damage Mechanics

## 1. Introduction

Civil engineering structures are formed using modules that are systematically and repetitively assembled in various shapes and forms for aesthetically pleasing buildings and infrastructure. The assembled structures often outperform the modules under similar loading conditions. For example, although an individual brick under compression is quite brittle, masonry arches encompassing systematic assembly of bricks are relatively more ductile [1]. Under lateral loads wall type structures that fail due to horizontal sliding are shown to have higher ductility compared to those that fail due to diagonal cracking although constructed of same materials [2]. No reliable and computationally efficient models are yet available. In an ongoing research with the objective of developing fundamentally sound, yet simplified models for reliable analysis of structures, a representative volume element encompassing two materials (Brick and Mortar) of strain softening properties has been developed using a transient, gradient enhanced non-local homogenisation incorporating damaged interface interaction of the two materials.

Numerical examples are provided throughout the paper to illustrate the process of formulating the RVE clearly. The results show the stability of the approach.

## 2. Continuum Damage Mechanics for Quasi-Brittle Material

### 2.1. Elasticity Based Damage Mechanics

In the classical continuum damage mechanics a scalar isotropic damage quantity  $\omega$  has been introduced in the stress-strain relationship in the following way [3]:

$$\sigma_{ij} = (1 - \omega) C_{ijkl} \varepsilon_{kl} \quad (1)$$

where  $\sigma_{ij}$  is the Cauchy stress,  $\varepsilon_{kl}$  is the linear strain and  $C_{ijkl}$  represents the elasticity matrix components. Einstein's summation convention applies (i, j, k, l = 1, 2, 3). The Poisson's ratio is assumed not to be affected by damage.

A damage loading function is introduced, in order to enable the model to predict damage growth, as follows [4]:

$$f(\tilde{\varepsilon}_{eq}, \kappa) = \tilde{\varepsilon}_{eq} - \kappa \quad (2)$$

where  $\tilde{\varepsilon}_{eq}$ , is the equivalent strain which depends on the current state of strain tensor and  $\kappa$ , is a history parameter which represents maximum equivalent strain that the material has experienced. For,  $f < 0$ , there will be no damage growth present and material would behave linearly elastic. Change in parameter  $\kappa$ , follows the Kuhn-Tucker relations [4]:

$$f \leq 0, \dot{\kappa} \geq 0, f \dot{\kappa} = 0 \quad (3)$$

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which implies that damage growth will happen when  $f = 0$ , where history parameter satisfies  $\kappa = \tilde{\varepsilon}_{eq}$ .

## 2.2. Definition of Equivalent Strain

Shape and size of the loading surface depends on the definition of equivalent strain  $\tilde{\varepsilon}_{eq}$ , which maps the strain tensor into a scalar value by weighting its components considering their different effects on cracking. In this paper, a strain based modified von Mises definition which is able to differentiate between tensile and compressive strains. This differentiation is needed in order to predict the behavior of quasi-brittle materials such as concrete and brick. This definition reads as follows [5]:

$$\tilde{\varepsilon}_{eq} = \frac{(k-1)}{2k(1-2\nu)} I_1 + \frac{1}{2k} \sqrt{\frac{(k-1)^2}{(1-2\nu)^2} I_1^2 + \frac{12k}{(1+\nu)^2} J_2} \quad (4)$$

where  $I_1$ , is the first invariant of strain tensor and  $J_2$ , is the second invariant of deviatoric strain tensor. Parameter  $k$  controls the sensitivity of the equivalent strain to tension and compression which is usually set to the ratio of compressive strength of material to its tensile strength. In this equation, Poisson's ratio has been represented by  $\nu$ .

## 2.3. Damage Evolution Law for Quasi-Brittle Material

Fracture in quasi-brittle material is not a result of growth of one dominant defect but a collective process of damage growth and nucleation in its microstructure. Quasi-brittle materials like concrete, brick and mortar, demonstrate a gradual loss of strength, instead of a sudden loss of deformation resistance like brittle fracture.

Several scalar damage evolution laws have been developed in the literature [6,7]. In engineering material softening is nonlinear which has a relatively steep stress drop when cracking starts and a moderate decrease afterwards. In this work, an exponential softening law for concrete has been used in the form of

$$\omega = 1 - \frac{\varepsilon_i}{\varepsilon} \left[ (1-\alpha) + \alpha e^{-\beta(\varepsilon-\varepsilon_i)} \right] \quad (5)$$

Due to crack bridging the experimentally obtained load displacement data has a long tail. Using this expression when  $\varepsilon \rightarrow \infty$ , stress approaches  $(1-\alpha)E\varepsilon_i$  which can represent this long tail. Parameter  $\beta$  controls the damage growth rate which depends on the tensile fracture energy of the material. When  $\beta$  is higher, model will show a faster crack growth and a more brittle response. This type of damage evolution law was used for both constituents.

## 3. Non-Local Model for Strain Softening Material

Damage growth is highly dependent on the microstructure of the material. In quasi-brittle material such as concrete, brick and mortar, cracks are bridged by aggregates. Therefore, the fracture process is directly related to the aggregates size and distribution. However, in classical damage mechanics models the scale of microstructure has not been included. This unrealistic shortcoming of classical damage models results in the damage localisation [8]. To overcome this localisation problem in simulation of strain softening material, we can introduce non-locality to the constitutive relation so that the growth of damage variable depends on the average deformation of the material in a certain region. Addition of this non-local concept to the damage model will result in a smooth damage growth depending on the length scale [9].

### 3.1. Gradient Enhanced Non-Local Model

Non-local strain can be introduced as the solution of the following partial differential equation

$$\bar{\varepsilon}_{eq} - c \nabla^2 \bar{\varepsilon}_{eq} = \tilde{\varepsilon}_{eq} \quad (6)$$

This means that the damage field variable should depend on a non-local equivalent strain  $\bar{\varepsilon}_{eq}$ , instead of local equivalent strain  $\tilde{\varepsilon}_{eq}$ . Gradient parameter  $c$ , is a constant related to the squared of the internal length parameter.  $\bar{\varepsilon}_{eq}$ , can now be implicitly calculated in terms of  $\tilde{\varepsilon}_{eq}$ , using a  $C^0$ -continues finite element domain. To solve this Helmholtz partial differential equation a natural boundary condition has been considered as proposed in [9].

$$\nabla \bar{\varepsilon}_{eq} \cdot n = 0 \quad (7)$$

in which  $n$ , is the unit normal to boundary  $\Gamma$ .

### 3.2. The Transient-Gradient Damage Model

Using a constant parameter  $c$ , in the gradient model leads to an increase of damage growth in and outside the localisation zone. This issue can be resolved by considering a transient value instead of a constant for the gradient parameter [10]. This modification transforms Equation (6) as follows

$$\bar{\varepsilon}_{eq} - \xi \nabla^2 \bar{\varepsilon}_{eq} = \tilde{\varepsilon}_{eq} \quad (8)$$

in which  $\xi$ , represents the transient gradient parameter and is defined as follows

$$\xi = \begin{cases} c \left( \frac{\varepsilon}{\varepsilon_\xi} \right)^{n_\xi} & \varepsilon \leq \varepsilon_\xi \\ c & \varepsilon > \varepsilon_\xi \end{cases} \quad (9)$$

In order to solve this new partial differential equation, an extra set of continuity equation needs to be added to the original gradient enhanced model. To avoid adding this extra continuity equation, Equation (9) can be divided by  $\xi \neq 0$ , which leads to the diffusion equation [11]

$$\frac{\bar{\varepsilon}_{eq}}{\xi} - \nabla^2 \bar{\varepsilon}_{eq} = \frac{\tilde{\varepsilon}_{eq}}{\xi} \quad (10)$$

which requires the same number of continuity equations as the original gradient enhanced model. The transient gradient parameter needs to be slightly changed to avoid division by zero into [11]

$$\xi = \begin{cases} c_0 + (c - c_0) \left( \frac{\varepsilon}{\varepsilon_\xi} \right)^{n_\xi} & \varepsilon \leq \varepsilon_\xi \\ c & \varepsilon > \varepsilon_\xi \end{cases} \quad (9)$$

in which  $c_0$ , is considered to be an arbitrary positive value so that non-local interaction is prevented at the beginning of the analysis.

#### 4. Computational Homogenisation

To derive an enhanced constitutive material model for a complex composite like masonry computational homogenisation can be used so that we can derive the global behaviour of the masonry from its constituents such as concrete block and mortar. Uniform loading and periodic geometry for masonry has been assumed and thus, homogenisation theory for periodic media which was adopted in [12,13] seems suitable to use. Computations have been performed on a single representative volume element (RVE) which contains the information of the entire mesostructure. A boundary value problem has been solved on the RVE using finite element method. Based on homogenisation theory for periodic media, strains should be compatible and the stresses should be anti-periodic on two opposite sides of the RVE. This will ensure that two neighbouring RVEs fit together.

##### 4.1. Strain-Periodic Displacement Field

The strain-periodic displacement field has the form

$$\bar{u}(\bar{x}) = \varepsilon \cdot \bar{x} + \bar{w}(\bar{x}) \quad (11)$$

where  $\varepsilon$ , is the macroscopic strain tensor,  $\bar{x}$ , is the position vector and  $\bar{w}(\bar{x})$ , is a mesoscopic displacement fluctuation field which distinguishes the real mesostructural displacement field from the linear  $\varepsilon \cdot \bar{x}$ , field [13]. The fluctuation field is assumed to be periodic. The volume average of the mesoscopic strain field resulting from equation (11) is given by

$$\frac{1}{\Omega_{RVE}} \int_{\Omega_{RVE}} \varepsilon(\bar{u}) d\Omega = \frac{1}{\Omega_{RVE}} \int_{\Omega_{RVE}} \nabla(\varepsilon \cdot \bar{x} + \bar{w}) d\Omega = \varepsilon \quad (12)$$

which shows the volume average of mesoscopic strain field is equal to macroscopic strain  $\varepsilon$ . By using the Hill-Mandel work equivalence the total macro-stress can be determined as

$$\sigma = \frac{1}{\Omega_{RVE}} \int_{\Omega_{RVE}} \sigma^m d\Omega \quad (13)$$

in which  $\sigma$ , and  $\sigma^m$ , represent macro and meso stress, respectively.

##### 4.2. Mesoscopic Representative Volume Element (RVE)

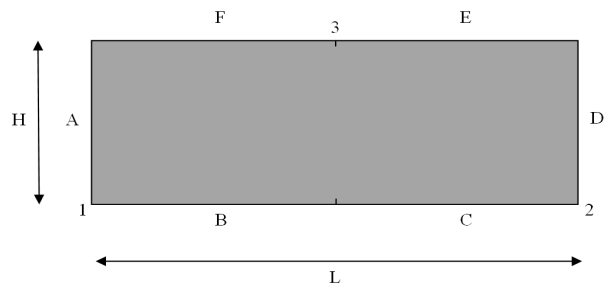
In order to minimize the computational cost at the mesoscopic scale and also capture all possible failure mechanisms, the RVE should be chosen carefully. It is important to note that if the average behaviour remains unique any periodic RVE predicts equivalent results, i.e. in an infinitesimal strain setting and before localisation happens. For masonry, due to periodicity of the initial mesostructure, the RVE is chosen as the smallest periodic element. Based on the assumption that arrangement of the constituent materials is the main cause of average stiffness degradation, the initial and damage induced anisotropy will be captured correctly using this RVE [13].

##### 4.3. RVE's Boundary Conditions

Periodic boundary conditions have been applied on three controlling nodes (see **Figure 1**) of the RVE as indicated in [13] and justified in [12].

The periodicity conditions for edges can then be formulated in terms of the controlling nodes as

$$\begin{aligned} \bar{\varepsilon}_{eq,F} &= \bar{\varepsilon}_{eq,F} \\ \bar{\varepsilon}_{eq,E} &= \bar{\varepsilon}_{eq,B} \\ \bar{\varepsilon}_{eq,D} &= \bar{\varepsilon}_{eq,A} \\ \bar{u}_F &= \bar{u}_C + \bar{u}_3 - \bar{u}_2 \\ \bar{u}_E &= \bar{u}_B + \bar{u}_3 - \bar{u}_1 \\ \bar{u}_D &= \bar{u}_A + \bar{u}_2 - \bar{u}_1 \end{aligned} \quad (14)$$



**Figure 1. Controlling nodes and periodicity conditions on a typical masonry RVE.**

This means both displacement and damage will be periodic in the RVE. These periodicity conditions lead to periodic stresses and strains mesoscopic fields. Loading is applied on the RVE by means of the three controlling points as illustrated in **Figure 2**. In these figures W, H, and L, are width, height and length of the RVE, respectively. Moreover,  $\sigma_{xx}$ ,  $\sigma_{yy}$ , and,  $\sigma_{xy}$ , are stress parallel to bed joint, stress perpendicular to bed joint and shear stress, respectively.

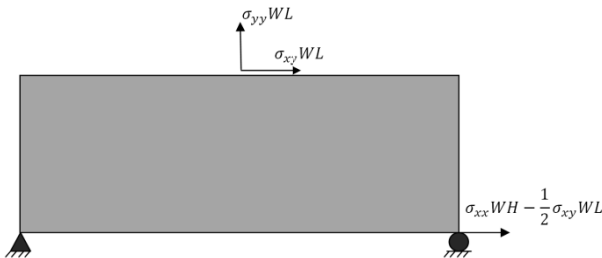
### 5. Numerical Examples

After implementing the process explained in previous sections, a RVE has been constructed with 240 mm length, 60 mm height and an out-of-plane thickness of 110 mm. The RVE consists of a 230\*50 clay brick and a 5 mm mortar around all its sides. The finite element discretisation which was used for the analysis has been illustrated in **Figure 3**. These geometric configurations have been considered, in order to simulate the biaxial experimental results on conventional masonry panels obtained in [14].

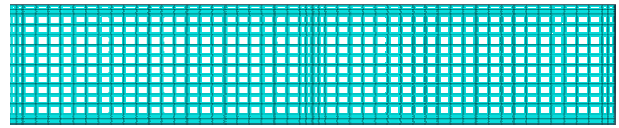
Mesoscopic material properties used for the analysis can be found in **Table 1**. Elasticity based damage mechanics has been considered for both materials.

Quadratic interpolation functions have been considered for the displacement fields and linear interpolation functions have been considered for the non-local field. Parameters for the transient gradient enhanced non-local model are shown in **Table 2**. Conventional load control method is employed since the experimental results have been obtained under load control tests.

Two loading cases have been considered for analysis; compression parallel to bed joint and compression perpendicular to bed joint. These loading configurations have been applied on the RVE as illustrated in **Figure 2**. Evolution of damage under compression load perpendicular and parallel to bed joint can be seen in **Figures 4** and **5**, respectively. In case of compression perpendicular to bed joint, damage growth initiates on the corner of the brick-mortar interface and continues into mortar bed joint. In the second case which is under compression load parallel to bed joint, damage growth initiates from the same spot and continues to grow into the perpend joints.



**Figure 2.** Loading modes applied on the RVE.



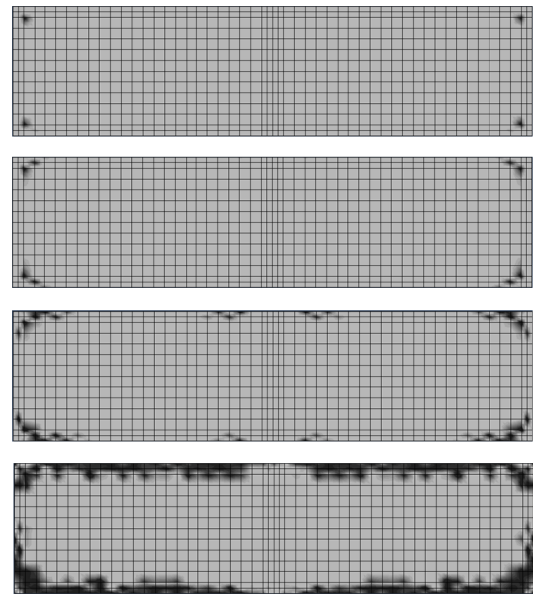
**Figure 3.** Typical finite element mesh used in computations.

**Table 1.** Material parameters for RVE computations.

Material	E	$\nu$	$\alpha$	$\beta$	$\kappa$	$\epsilon_i$
Brick	14000	0.2	1.0	800	10	0.0002
Mortar	3000	0.2	1.0	100	10	0.0003

**Table 2.** Parameters for transient gradient model.

Material	$c$	$c_0$	$n_\xi$	$e_\xi$
Brick	2	0.01	1.0	0.002
Mortar	2	0.01	1.0	0.003

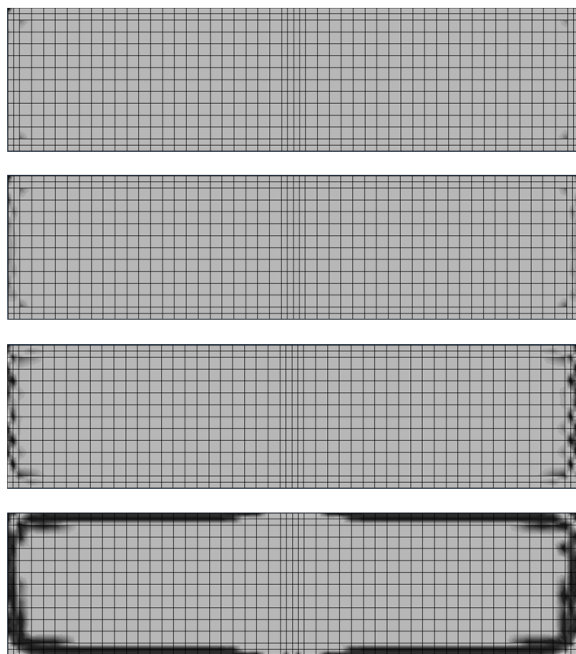


**Figure 4.** Evolution of damage in the RVE for compression perpendicular to bed joint.

Damage evolution patterns obtained from the analysis have a good agreement with failure patterns seen in experimental investigations. Mesh sensitivity of the transient gradient model has been investigated in [10,11].

### 6. Conclusion

In this contribution, the interaction between two materials with different softening properties has been investigated using a homogenisation approach. To avoid spurious damage growth and mesh sensitivity, a transient gradient enhanced non-local model has been introduced to the model. Influence of the choice of this model on the



**Figure 5. Evolution of damage in the RVE for compression parallel to bed joint.**

evolution of damage under two loading cases has been investigated. The overall RVE behaviour using the transient gradient model indicates that calibration of the transient parameter under different loading conditions needs to be investigated.

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