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Existence of Periodic Solutions for Neutral-Type Neural Networks with Delays on Time Scales

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ABSTRACT

In this paper, we employ a fixed point theorem due to Krasnosel'skii to attain the existence of periodic solutions for neutral-type neural networks with delays on a periodic time scale. Some new sufficient conditions are established to show that there exists a unique periodic solution by the contraction mapping principle.

Keywords: Neutral-Type; Neural Networks; On Time Scales; Periodic Solution

1. Introduction

Recently, scholars and researchers have paid more attention to the discussion of neural networks described by neutral-type differential equations with delays (see [1-6]). Meanwhile, difference equations or discrete-time analogues of differential equations can preserve the convergence dynamics of their continuous time counterparts in some degree [7]. Due to their usage in applications, these discrete-type neural networks with or without delays have been discussed by [8,9] and references therein. It is interesting to study that neural systems on time scales can unify the continuous and discrete situations. The theory of time scales initiated by S. Hilger [10,11] has been incorporated to investigate neural networks [12,13] and so on.

However, few works have considered for neutral-type neural networks on time scales [14,15]. In this paper, we consider the existence of periodic solutions for the neutral networks with delays

$$\begin{aligned} x_i^{\Delta}(t) &= -a_i(t)x_i^{\sigma}(t) + \sum_{j=1}^{n} c_{ij}(t)x_j^{\Delta}(t-k) \\ &\quad + \sum_{j=1}^{n} b_{ij}(t)g_j(x_j(t-k)) \qquad (1) \\ &\quad + I_i(t), \quad i \in \mathrm{N} := \{1, 2, \cdots n\} \end{aligned}$$

For a review of dynamic equations o-time scales, we direct the reader to the monographs [9,10] and begin with a few definitions.

Definition 1.1. A time scale T is p-periodic if there exists $p > 0$ and $p \in \mathrm{T}$ such that if $t \in \mathrm{T}$ then $t \pm p \in \mathrm{T}$. For $\mathrm{T} \neq \mathbb{R}$, the smallest positive p is called the period of T.

Definition 1.2. Let $\mathrm{T} \neq \mathbb{R}$ be a p-periodic time scale. f: $\mathrm{T} \to \mathbb{R}$ is periodic with period T if there exists a natural number n such $\mathrm{T} = np, f(t+T) = f(t)$ for all $t \in \mathrm{T}$ and T is the smallest number such that $f(t+T) = f(t)$.

Without other statements, let T be a p-periodic time scale such that $0 \in \mathrm{T}$. We will show the existence of periodic solutions for (1) where $k = mp, m \in \mathbb{Z}.$

2. Preliminaries

Theorem 2.1. ([10,11]) Assume $\upsilon : \mathrm{T} \to \mathrm{T}$ is strictly increasing and $\tilde{\mathrm{T}} := \upsilon(\mathrm{T})$ is a time scale. Let $\omega : \tilde{\mathrm{T}} \to \mathbb{R}$. If $\upsilon^{\Delta}(t)$ and $\omega^{\tilde{\Delta}}(\upsilon(t))$ exist for $t \in \mathrm{T}^k$, then one has

$$(\omega \circ \upsilon)^{\Delta} = (\omega^{\tilde{\Delta}} \circ \upsilon)\upsilon^{\Delta}.$$

Theorem 2.2. ([10,11]) Assume $\upsilon : \mathrm{T} \to \mathrm{T}$ is strictly increasing and $\tilde{\mathrm{T}} := \upsilon(\mathrm{T})$ is a time scale. If $f : \mathrm{T} \to \mathbb{R}$ is a rd-continuous and υ is differentiable with rd-continuous derivative , then one gets

$$\int_a^b f(t)\upsilon^{\Delta}(t)\Delta t = \int_{\upsilon(a)}^{\upsilon(b)} \left(f \circ \upsilon^{-1}\right)(s)\tilde{\Delta}s, \ \forall a, b \in \mathrm{T}.$$

A function $p : \mathrm{T} \to \mathbb{R}$ is said to be regressive provided $1 + \mu(t)p(t) \neq 0$ for all $t \in \mathrm{T}^k$. The set of all regressive rd-continuous function $f : \mathrm{T} \to \mathbb{R}$ is devote by R while the set R^+ is given by

$$R^+ = \left\{ f \in R : 1 + \mu(t)p(t) > 0, \ \forall \mathrm{t} \in \mathrm{T} \right\}.$$

Let $p \in R$. The exponential function is defined by

$$e_p(t,s) = \exp\left(\int_s^t \xi_{\mu(\tau)}(p(\tau))\Delta\tau \right) \qquad (2)$$

where $\xi_h(z)$ is called cylinder transformation.

Lemma 2.1.

Let $p,q \in R$. One gets that

(i) $e_0(t,s) \equiv 1$ and $e_p(t,t) \equiv 1$;

(ii) $e_p(\sigma(t),s) = (1+\mu(t)p(t))e_p(t,s)$;

(iii) $\dfrac{1}{e_p(t,s)} = e_{\Theta p}(t,s)$, where $\Theta p = -\dfrac{p(t)}{1+\mu(t)p(t)}$;

(iv) $e_p(t,s) = 1/e_p(s,t) = e_{\Theta p}(s,t)$;

(v) $e_p(t,s)e_p(s,r) = e_p(t,r)$;

(vi) $e_p(t,s)e_q(t,s) = e_{p\oplus q}(t,s)$;

(vii) $\dfrac{e_p(t,s)}{e_q(t,s)} = e_{p\Theta q}(t,s)$;

(viii) $\left(\dfrac{1}{e_p(\cdot,s)}\right)^{\Delta} = -\dfrac{p(t)}{e_p^{\sigma}(\cdot,s)}$;

Finally, we state Krasnosel'skii fixed point theorem which enables us to prove the existence of periodic solutions.

Theorem 2.3.

([16]) Let M be a closed convex nonempty subset of a Banach space $(\mathrm{B}, \|\cdot\|)$. Suppose that A and B map M into B such that (i) $x,y \in \mathrm{M}$ imply $\mathrm{A}x + \mathrm{B}y \in \mathrm{M}$; (ii) A is compact and continuous; (iii) B is a contraction mapping; Then there exists $z \in \mathrm{M}$ with $z = \mathrm{A}z + \mathrm{B}z$.

3. Existence of Periodic Solutions

Let $T>0$, $T \neq k$, $T \in \mathrm{T}$ be fixed and if $\mathrm{T} \neq \mathbb{R}$, $\mathrm{T} = np$ for some $n \in \mathbb{N}$. By the notation $[a,b]$, we mean $[a,b] = \{t \in \mathrm{T} : a \le t \le b\}$. The intervals $[a,b)$, $(a,b]$ and (a,b) are defined similarly, Defined $P_T = \{\phi \in C(\mathrm{T},R^n) : \phi(t+T) = \phi(t)\}$, where $C(\mathrm{T},R^n)$ is the space of all real valued continuous functions. Then P_T is a Banach space when it is endowed with norm $\|x\| = \sup_{t\in[0,T]} |x(t)|$.

For each $i,j \in \mathrm{N}$, we make basic assumption (H_1):

• $a_j \in R^+$ is continuous, $a_i(t) > 0$ and $a_i(t+T) = a_i(t)$ for all $t \in T$.

• $c_{ij}(t+T) = c_{ij}(t)$, $I_i(t+T) = I_i(t)$ and $c_{ij}^{\Delta}(t)$ is continuous.

• $g_j(u)$ is continuous, $g_j(0) = 0$ and $|g_j(u) - g_j(v)| \le L_j |u-v|$ for some $L_j > 0$.

Lemma 3.1.

Assume that (H_1) holds. $\{x_i(t)\}_{i\in\mathrm{N}} \in P_T$ is a solution of (1) if and only if

$$\begin{aligned} x_i(t) &= \sum_{j=1}^{n} c_{ij}(t)x_j(t-k) + (1-e_{\Theta a_i}(t,t-T))^{-1} \\ &\times \int_{t-T}^{t} [-r_i(s)x_j^{\sigma}(s-k) \\ &+ \sum_{j=1}^{n} b_{ij}(t)g_j(x_j(s-k) + I_i(s)]\, e_{\Theta a_i}(t,s)\Delta s, \end{aligned} \tag{3}$$

where $r_i(s) := \sum_{j=1}^{n}\left[c_{ij}^{\sigma}(s)a_i(s) + c_{ij}^{\Delta}(s)\right]$ and $i \in \mathrm{N}$.

Proof. Let $\{x_i(t)\}_{i\in\mathrm{N}} \in P_T$ is a solution of (1). It follows from (1) that

$$\begin{aligned} x_i^{\Delta} + a_i(t)x_i^{\sigma}(t) &= \sum_{j=1}^{n} c_{ij}(t)x_j^{\Delta}(t-k) \\ &+ \sum_{j=1}^{n} b_{ij}(t)g_j(x_j(t-k)) + I_i(t), \end{aligned} \tag{4}$$

where $i \in \mathrm{N}$. Multiply both sides of (4) by $e_{a_i}(t,0)$ and integrate from $t-T$ to t, one obtain that

$$\begin{aligned} \int_{t-T}^{t} [e_{a_i}(s,0)x_i(s)]^{\Delta}\Delta s &= \int_{t-T}^{t} [\sum_{j=1}^{n} c_{ij}(s)x_j^{\Delta}(s-k) \\ &+ \sum_{j=1}^{n} b_{ij}(s)g_j(x_j(s-k)) + I_i(s)]\, e_{a_i}(s,0)\Delta s. \end{aligned}$$

Divide both sides of above equation by $e_{a_i}(t,0)$, due to $x_i(t) = x_i(t-T)$ and Lemma 2.1, we have

$$\begin{aligned} x_i(t)\left(1-e_{\Theta a_i}(t,t-T)\right) &= \int_{t-T}^{t} [\sum_{j=1}^{n} c_{ij}(s)x_j^{\Delta}(s-k) \\ &+ \sum_{j=1}^{n} b_{ij}(s)g_j(x_j(s-k) + I_i(s)]e_{\Theta a_i}(t,s)\Delta s. \end{aligned} \tag{5}$$

It follows from integration by parts and the periodicity of $c_{ij}(\cdot)$ and $x_j(\cdot)$, we get

$$\begin{aligned} &\int_{t-T}^{t} e_{\Theta a_i}(t,s)\sum_{j=1}^{n} c_{ij}(s)x_j^{\Delta}(s-k)\Delta s \\ &= \sum_{j=1}^{n} c_{ij}(t)x_j(t-k)[1-e_{\Theta a_i}(t,t-T)] \\ &- \int_{t-T}^{t}\sum_{j=1}^{n} x_j^{\sigma}(s-k)[e_{\Theta a_i}(t,s)c_{ij}(s)]^{\Delta s}\Delta s. \end{aligned} \tag{6}$$

Substitute (6) into (5) and simplify, we get (3). From Lemma 2.1, we get the desired result and the proof is complete.

Define the mapping $H:\ \mathrm{P}_T \to \mathrm{P}_T$ by

$$\begin{aligned} &(H\varphi)_i(t) \\ &= \sum_{j=1}^{n} c_{ij}(t)\varphi_j(t-k) + \left(1-e_{\Theta a_i}(t,t-T)\right)^{-1} \times \\ &\int_{t-T}^{t}\left[-r_i(s)\varphi_j^{\sigma}(s-k) + \sum_{j=1}^{n} b_{ij}(s)g_j(\varphi_j(s-k)) + I_i(s)\right] \\ &\times e_{\Theta a_i}(t,s)\Delta s, \end{aligned} \tag{7}$$

where $i \in \mathrm{N}$. Let $(H\varphi)_i(t) = (B\varphi)_i(t) + (A\varphi)_i(t)$, where A, B are given by

$$(B\varphi)_i(t) = \sum_{j=1}^{n} c_{ij}(t)\varphi_j(t-k), \tag{8}$$

and

$$(A\varphi)_i(t) = \left(1 - e_{\Theta a_i}(t, t-T)\right) \times \int_{t-T}^{t} \left[-r_i(s)\varphi_j^{\sigma}(s-k) + \sum_{j=1}^{n} b_{ij}(s) g_j(\varphi_j(s-k)) + I_i(s)\right] e_{\Theta a_i}(t, s)\Delta s, \quad (9)$$

where $r_i(s)$ is defined in Lemma 3.1 and $i \in \mathrm{N}$. Next, we will prove that A is compact and B is a contraction mapping in Lemma 3.2 and Lemma 3.3, respectively.

Lemma 3.2. Assume that (H_1) holds. $A: \mathrm{P}_T \to \mathrm{P}_T$ defined by (9) is compact.

Proof. We first show that A maps P_T into P_T. It follows from (9) that

$$(A\varphi)_i(t+T) = \left(1 - e_{\Theta a_i}(t+T, t)\right)^{-1} \times \int_{t-T}^{t} \left[-r_i(u+T)\varphi_j^{\sigma}(u-k) + \sum_{j=1}^{n} b_{ij}(u) g_j(\varphi_j(u-k)) + I_i(u)\right] e_{\Theta a_i}(t+T, u+T)\Delta u, \quad (10)$$

and

$$r_i(u+T) = \sum_{j=1}^{n}\left[c_{ij}^{\sigma}(u+T)a_i(u+T) + c_{ij}^{\Delta}(u+T)\right] = \sum_{j=1}^{n}\left[c_{ij}^{\sigma}(u)a_i(u) + c_{ij}^{\Delta}(u)\right] = r_i(u).$$

i.e., $r_i(u)$ is T-periodic. It follows from (2) and Theorem 2.2 that $e_{\Theta a_i}(t+T, u+T) = e_{\Theta a_i}(t, u)$ and $e_{\Theta a_i}(t+T, t) = e_{\Theta a_i}(t, t-T)$. Thus (10) becomes

$$(A\varphi)_i(t+T) = \left(1 - e_{\Theta a_i}(t, t-T)\right)^{-1} \times \int_{t-T}^{t} \left[-r_i(u)\varphi_j^{\sigma}(u-k) + \sum_{j=1}^{n} b_{ij}(u) g_j(\varphi_j(u-k)) + I_i(u)\right] e_{\Theta a_i}(t, u)\Delta u = (A\varphi)_i(t).$$

where $i \in \mathrm{N}$. That is, $\mathrm{A}: \mathrm{P}_T \to \mathrm{P}_T$.

Secondly, we will show that A is continuous. Let $\varphi, \psi \in P_T$ with $\|\varphi\| \le C$, $\|\psi\| \le C$ and define

$$\left.\begin{aligned} \eta_i &:= \max_{t\in[0,T]} \left|\left(1 - e_{\Theta a_i}(t, t-T)\right)^{-1}\right| \\ \gamma_i &:= \max_{u\in[t-T,t]} e_{\Theta a_i}(t, u) \\ \overline{b_{ij}} &:= \max_{t\in[t-T,t]} \left|b_{ij}(t)\right|, \quad \beta_i := \max_{t\in[0,T]} \left|r_i(t)\right| \\ \|\varphi - \psi\| &:= \max_{j\in \mathrm{N}} \|\varphi_j - \psi_j\| \end{aligned}\right\} \quad (11)$$

Given $\varepsilon > 0$ and take $\delta = \varepsilon/(nM)$ such that $\|\varphi - \psi\| < \delta$. By making use of Lipschitz inequality of (H_1), we get

$$\|(A\varphi)_i - (A\psi)_i\| \le \gamma_i \eta_i \int_{t-T}^{t} \sum_{j=1}^{n} (\beta_i \|\varphi_j - \psi_j\| + \overline{b_{ij}} L_j \|\varphi_j - \psi_j\|) du \le M\|\varphi - \psi\| < \varepsilon,$$

where $M = \max\{M_i\}$ and

$$M_i = \gamma_i \eta_i T \sum_{j=1}^{n} (n\beta_i + \overline{b_{ij}} L_j).$$

Hence, it follows that

$$\|A\varphi - A\psi\| = \max_{i\in \mathrm{N}} \|(A\varphi)_i - (A\psi)_i\| < \varepsilon,$$

That proves A is continuous.

Thirdly, we need to show A is compact. Consider the sequence $\{\varphi^n\} \subset P_T$ and assume that the sequence is uniformly bounded. Let $R > 0$ be such $\|\varphi^{(n)}\| \le R$ for all $n \in \mathrm{N}$. It is easy to estimate that

$$\begin{aligned} \left|(A\varphi^{(n)})_i(t)\right| &= \Big|\left(1 - e_{\Theta a_i}(t, t-T)\right)^{-1} \times \int_{t-T}^{t} \Big[-r_i(s)\varphi_j^{(n)\sigma}(s-k) + \sum_{j=1}^{n} b_{ij}(s) g_j(\varphi_j^{(n)}(s-k)) + I_i(s)\Big] e_{\Theta a_i}(t, s)\Delta s\Big| \\ &\le \gamma_i \eta_i \int_{t-T}^{t} (|r_i(s)| \|\varphi_j^{(n)}\| + \sum_{j=1}^{n} \overline{b_{ij}} L_j \|\varphi_j^{(n)}\| + \overline{I_i})\Delta s \\ &\le \gamma_i \eta_i [n\beta_i R + \sum_{j=1}^{n} \overline{b_{ij}} L_j R + \overline{I_i}] T \triangleq D, \end{aligned}$$

where $\overline{I_i} = \sup_{s\in R}\{|I_i(s)|\}$. Thus the sequence $\{A\varphi^{(n)}\}$ is uniformly bounded. Now, it can be easily checked that

$$(A\varphi^{(n)})_i^{\Delta}(t) = (\Theta a_i)(t)(A\varphi^{(n)})_i(t) + \frac{1}{1 + a_i(t)\mu_i(t)} \times \{[-\sum_{j=1}^{n} (c_{ij}^{\sigma}(t)a_i(t) + c_{ij}^{\Delta}(t))]\varphi_i^{(n)\sigma}(t-k) + \sum_{j=1}^{n} b_{ij}(t) g_j(\varphi_j^{(n)}(t-k)) + I_i(t)\},$$

which leads to

$$\left|(A\varphi^{(n)})_i^{\Delta}(t)\right| \le D\|a\| + n\beta_i R + \sum_{j=1}^{n} \overline{b_{ij}} L_j R + \overline{I_i}.$$

For all n .That is $\|(A\varphi^{(n)})^{\Delta}\| \le F$ for some positive constant F .Thus the sequence $\{A\varphi^{(n)}\}$ is uniformly bounded and equi-continuous. The Arzela-Ascoli theorem [16] implies that $\{A\varphi^{(n_k)}\}$ uniformly converges to a continuous T-periodic function φ^*. Thus A is compact.

Lemma 3.3. Let B is defined by (8) and assume that

$$(H_2): \sup_{t\in[0,T]} \left|c_{ij}(t)\right| < \xi_{ij}, \qquad \max_{i\in \mathrm{N}} \sum_{j=1}^{n} \xi_{ij} < \xi < 1.$$

Then $B: \mathrm{P}_T \to \mathrm{P}_T$ is a contraction.

Proof. Trivially, $B: \mathrm{P}_T \to \mathrm{P}_T$. For any $\varphi, \psi \in P_T$, we have

$$\|(B\varphi)_i-(B\psi)_i\|=\max_{t\in[0,T]}\left|\sum_{j=1}^{n}c_{ij}(\varphi_j(t-k)-\psi_j(t-k))\right|$$
$$\le\sum_{j=1}^{n}\xi_{ij}\|\varphi-\psi\|.$$

which leads to $\|B\varphi-B\psi\|<\xi\|\varphi-\psi\|$. Hence, B defines a contraction mapping with contraction constant ξ.

Theorem 3.1. Assume that (H_1) and (H_2) hold. If all solutions $\{x_i(t)\}\in P_T$ of (1) satisfy with $|x_i(t)|\le G$ and

$$\max_{i\in N}\left\{[\xi+\gamma_i\eta_iT(n\beta_i+\sum_{j=1}^{n}\overline{b_{ij}}L_j)]G+\overline{I_i}\gamma_i\eta_iT\right\}\le G$$

where $G>0$ and $\overline{I_i}=\sup_{s\in R}\{|I_i(s)|\}$, then (1) has a T-periodic solution.

Proof. Define $M=\{\varphi\in P_T:\|\varphi\|\le G\}$. Lemma 3.1 implies $A: P_T\to P_T$ and A is compact and continuous. Lemma 3.2 implies B is a contraction and $B: P_T\to P_T$. Now, we need to show that if $\varphi,\ \psi\in M$, we have $\|(A\varphi)_i+(B\psi)_i\|\le G$. Let $\varphi,\ \psi\in M$ with $\|\varphi\|,\|\psi\|\le G$. It follows from (8) and (9) that

$$\|(A\varphi)_i+(B\psi)_i\|\le\gamma_i\eta_i\int_{t-T}^{t}(|r_i(s)|\|\varphi_j\|$$
$$+\sum_{j=1}^{n}\overline{b_{ij}}L_j\|\varphi_j\|+I_i(s))\Delta s+\xi\|\psi\|$$
$$\le[\xi+\gamma_i\eta_iT(n\beta_i+\sum_{j=1}^{n}\overline{b_{ij}}L_j)]G$$
$$+\overline{I_i}\gamma_i\eta_iT\le G$$

All the conditions of Krasnosel'skii theorem are satisfied on the set M. Thus there exists a fixed point z in M such that $z=Az+Bz$. By Lemma 3.1, this fixed point is a solution of (1). Hence (1) has a T-periodic solution.

Theorem 3.2. Assume that (H_1) and (H_2) hold. Let β_i,γ_i,η_i be given by (11). If

$$\gamma_i\eta_iT(n\beta_i+\sum_{j=1}^{n}\overline{b_{ij}}L_j)<1-\xi$$

holds, then (1) has a unique T-periodic solution.

Proof. Let the mapping H be given by (7). For $\varphi,\ \psi\in P_T$, we have

$$\|(H\varphi)_i-(H\psi)_i\|\le\xi\|\varphi-\psi\|+\gamma_i\eta_i\int_{t-T}^{t}\sum_{j=1}^{n}(\beta_i\|\varphi_j-\psi_j\|$$
$$+\overline{b_{ij}}L_j\|\varphi_j-\psi_j\|)\Delta s$$
$$\le[\xi+\gamma_i\eta_iT(n\beta_i+\sum_{j=1}^{n}\overline{b_{ij}}L_j)]\|\varphi-\psi\|,$$

which leads to

$$\|H\varphi-H\psi\|=\max_{i\in N}\|(H\varphi)_i-(H\psi)_i\|$$
$$\le[\xi+\gamma_i\eta_iT(n\beta_i+\sum_{j=1}^{n}\overline{b_{ij}}L_j)]\|\varphi-\psi\|.$$

That is, H defines a contraction mapping and there exists a unique fixed point which is a T-periodic solution of (1). This completes the proof.

4. Conclusion

Due to time scales calculus theory and the fixed point theorem, we obtained some more generalized results to ensure the existence of the periodic solutions for neutral-type neural networks with delays. The conditions can be easily checked in practice by simple algebraic methods. The method in this paper can be applied to prove the existence of the periodic solutions of some other similar systems such as neutral-type networks with leakage-terms [17].

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