

Random Attractor of the Stochastic Strongly Damped for the Higher-Order Nonlinear Kirchhoff-Type Equation

Guoguang Lin, Ling Chen, Wei Wang*

Department of Mathematics, Yunnan University, Kunming, China

Email: gglin@ynu.edu.cn, chenl800828@163.com, *wangw2641@163.com

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Abstract

In this paper, we consider the stochastic higher-order Kirchhoff-type equation with nonlinear strongly dissipation and white noise. We first deal with random term by using Ornstein-Uhlenbeck process and establish the wellness of the solution, then the existence of global random attractor are proved.

Keywords

Random Dynamical System, Random Attractor, Strongly Dissipation, White Noise

2010 Mathematics Classification: 35K10, 35K25, 35K35

1. Introduction

In this paper, we consider the following stochastic strongly damped higher-order nonlinear Kirchhoff-type equation with white noise:

$$\begin{aligned} du_t + \left[(-\Delta)^m u_t + \phi \left(\|\nabla^m u\|^2 \right) (-\Delta)^m u + g(u) \right] dt \\ = f(x) dt + q(x) dW(t), \quad x \in \Omega, \quad m > 1, \end{aligned} \quad (1.1)$$

with the Dirichlet boundary condition

$$u(x, t) = 0, \quad \frac{\partial^i u}{\partial v^i} = 0, \quad i = 1, 2, \dots, m-1, \quad x \in \partial\Omega, \quad (1.2)$$

and the initial value conditions

$$u(x, 0) = u_0(x) \in H^m(\Omega), \quad u_t(x, 0) = u_1(x) \in L^2(\Omega), \quad (1.3)$$

where Ω is a bounded domain of R^n , with a smooth boundary $\partial\Omega$, Δ is the Laplacian with respect to the variable $x \in \Omega$, $u = u(x, t)$ is a real function of $x \in \Omega$ and $t \geq 0$, ϕ is the damping coefficient, f is a given external force, v

is the outer norm vector, $g(u)$ is a nonlinear forcing, their respectively satisfies the following conditions:

- 1) $g(u) \leq c_0(1+|u|^p), 0 < p \leq \frac{2n}{n-2m}, n \geq 3;$
- 2) $\liminf_{t \rightarrow \infty} \frac{sg(s) - c_1G(s)}{s^2} \geq 0;$
- 3) $G(s) \geq c_2|u|^{p+1} - K_0, K_0 > 0;$
- 4) $\phi(s) \geq m_0;$

where c_0, c_1, c_2, m_0 are positive constants.

As well as we known, the study of stochastic dynamical is more and more widely the attention of scholars, and the study of random attractor has become an important goal. In a sense, the random attractor is popularized for classic determine dynamical system of the global attractor. Global attractor of Kirchhoff-type equations have been investigated by many authors, see, e.g., [1] [2] [3] [4], however, the existence random attractor has also been studied by many authors, in [5], Zhaojuan Wang, Shengfan Zhou and Anhui Gu, they study the asymptotic dynamics for a stochastic damped wave equation with multiplicative noise defined on unbounded domains, and investigate the existence of a random attractor, they overcome the difficulty of lacking the compactness of Sobolev embedding in unbounded domains by the energy equation. In [6], Guigui Xu, Libo Wang and Guoguang Lin study the long time behavior of solution to the stochastic strongly damped wave equation with white noise, in this paper, they use the method introduced in [7], so that they needn't divide the equation into two parts. In [8], Zhaojuan Wang, Shengfan Zhou and Anhui Gu study the asymptotic dynamics of the stochastic strongly damped wave equation with homogeneous Neuman boundary condition, and prove the existence of a random attractor. The other long time behavior of solution of evolution equations, we can see [9]-[19].

In this work, we deal with random term by using Ornstein-Uhlenbeck process, the key is to handle the nonlinear terms and strongly damped $(-\Delta)^m u$, and $\phi(\|\nabla^m u\|^2)$ is also difficult to be conducted. So far as we know, there were no result on random attractor for the stochastic higher-order Kirchhoff-type equation with nonlinear strongly dissipation and white noise. It is therefore important to investigate the existence of random attractor on (1.1)-(1.3).

This paper is organized as follows: In Section 2, we recall many basic concepts related to a random attractor for general random dynamical system. In Section 3, we introduce O-U process and deal with random term. In Section 4, we prove the existence of random attractor of the random dynamical system.

2. Preliminaries

In this section, we collect some basic knowledge about general random dynamical system ([9] [10] [11]).

Let $(X, \|\cdot\|_X)$ be a separable Hilbert space with Borel σ -algebra $B(X)$. Let $(\Omega, F, P, (\theta_t)_{t \in R})$ be the metric dynamical system on the probability space

(Ω, F, P) .

Definition 2.1. (see [9] [10]). A continuous random dynamical system on X over $(\Omega, F, P, (\theta_t)_{t \in \mathbb{R}})$ is a $(B(\mathbb{R}^+) \times F \times B(X), B(X))$ -measurable mapping $\varphi: \mathbb{R}^+ \times \Omega \times X \rightarrow X, (t, \omega, u) \mapsto \varphi(t, \omega, u)$. Such that the following properties hold (1)

- 1) $\varphi(0, \omega, \cdot)$ is the identity on X ;
- 2) $\varphi(t+s, \omega, \cdot) = \varphi(t, \theta_s \omega, \varphi(s, \omega, \cdot))$ for all $s, t \geq 0$;
- 3) $\varphi(t, \omega, \cdot): X \rightarrow X$ is continuous for all $t \geq 0$.

Definition 2.2. (see [10])

1) A set-valued mapping $D(\omega): \Omega \rightarrow 2^X, \omega \rightarrow D(\omega)$, is said to be a random set if the mapping $\omega \rightarrow d(u, D(\omega))$ is measurable for any $u \in X$. If $D(\omega)$ is also closed (compact) for each $\omega \in \Omega$, $D(\omega)$ is called a random closed (compact) set. A random set $D(\omega)$ is said to be bounded if there exist $u_0 \in X$ and a random variable $R(\omega) > 0$ such that

$$D(\omega) \subset u \in X : \|u - u_0\|_X \leq R(\omega) \text{ for all } \omega \in \Omega.$$

2) A random set $D(\omega)$ is called tempered provided for P -a.e. $\omega \in \Omega$, $\lim_{t \rightarrow \infty} e^{-\beta t} d(D(\theta_{-t} \omega)) = 0$ for all $\beta > 0$, where $d(D) = \sup \|b\|_X : b \in D$.

Let Y be the set of all random tempered sets in X .

3) A random set $B(\omega)$ is said to be a random absorbing set if for any tempered random set $D(\omega)$, and P -a.e. $\omega \in \Omega$, there exists $t_0(\omega)$ such that $\varphi(t, \theta_{-t} \omega, D(\theta_{-t} \omega)) \subset B(\omega)$ for all $t \geq t_0(\omega)$.

4) A random set $B_1(\omega)$ is said to be a random attracting set if for any tempered random set $D(\omega)$, and P -a.e. $\omega \in \Omega$, we have

$$\lim_{t \rightarrow \infty} d_H(\varphi(t, \theta_{-t} \omega, D(\theta_{-t} \omega)), B_1(\omega)) = 0,$$

where d_H is the Hausdorff semi-distance given by

$$d_H(E, F) = \sup_{u \in E} \inf_{v \in F} \|u - v\|_X \text{ for any } E, F \subset X.$$

5) φ is said to be asymptotically compact in X if for P -a.e. $\omega \in \Omega, \varphi(t_n, \theta_{-t_n} \omega, x_n)_{n=1}^\infty$ has a convergent subsequence in X whenever $t_n \rightarrow +\infty$, and $x_n \in B(\theta_{-t_n} \omega)$ with $B(\omega) \in Y$.

6) A random compact set $A(\omega)$ is said to be a random attractor if it is a random attracting set and $\varphi(t, \omega, A(\omega)) = A(\theta_t \omega)$ for P -a.e. $\omega \in \Omega$ and all $t \geq 0$.

Theorem 2.1. ([10]) Let φ be a continuous random dynamical system with state space X over $(\Omega, F, P, (\theta_t)_{t \in \mathbb{R}})$. If there is a closed random absorbing set $B(\omega)$ of φ and φ is asymptotically compact in X , then $A(\omega)$ is a random attractor of φ , where

$$A(\omega) = \bigcap_{t > 0} \overline{\bigcup_{\tau \geq t} \varphi(\tau, \theta_{-\tau} \omega, B(\theta_{-\tau} \omega))}, \omega \in \Omega.$$

Moreover, $A(\omega)$ is the unique random attractor of φ .

3. O-U Process and Stochastic Dynamical System

Let

$$(u, v) = \int_{\Omega} uv dx, \quad \|u\|_0 = (u, u)^{\frac{1}{2}}, \quad \forall u, v \in L^2(\Omega),$$

$$(u, v)_{2r} = (A^r u, A^r v), \quad \|u\|_{2r} = (A^r u, A^r u)^{\frac{1}{2}}, \quad \forall u, v \in V_{2r} = D(A^r), \quad A = -\Delta.$$

Let $E = H_0^m(\Omega) \times L^2(\Omega)$, and define a weighted inner product and norm in E

$$(y_1, y_2)_E = (\nabla^m u_1, \nabla^m u_2) + (v_1, v_2), \quad \|y\|_E^2 = \|\nabla^m u\|^2 + \|v\|^2,$$

$$\forall y_i = (u_i, v_i)^T, \quad y = (u, v)^T \in E, i = 1, 2.$$

3.1. O-U Process

O-U process is given by Wiener process on the metric system $(\Omega, F, P, (\theta_t)_{t \in R})$, we can see ([11] [12] [13]).

Let $z(\theta, \omega) = -\alpha \int_{-\infty}^0 e^{\alpha \tau} \theta, \omega(\tau) d\tau$, where $t \in R$, for $\forall t \geq 0$, $z(\theta_t, \omega)$ meet Itô equation: $dz + \alpha z dt = dW(t)$. And there is a probability measure P , θ_t -invariant set $\Omega_0 \subset \Omega$; so that stochastic process $z(\theta_t, \omega) = -\alpha \int_{-\infty}^0 e^{\alpha \tau} \theta, \omega(\tau) d\tau$ meet the following properties:

- 1) For $\forall \omega \in \Omega_0$, mapping $s \rightarrow z(\theta_s, \omega)$ for continuous mapping;
- 2) Random variable $\|z(\omega)\|$ is called tempered;
- 3) Exist temper set $r(\omega) > 0$, such that

$$\|z(\theta_t, \omega)\| + \|z(\theta_t, \omega)\|^2 \leq r(\theta_t, \omega) \leq r(\omega) e^{\frac{\alpha}{2}|t|};$$

- 4) $\lim_{t \rightarrow +\infty} \frac{1}{t} \int_0^t |z(\theta_t, \omega)|^2 d\tau = \frac{1}{2\alpha}$;
- 5) $\lim_{t \rightarrow +\infty} \frac{1}{t} \int_0^t |z(\theta_t, \omega)| d\tau = \frac{1}{\sqrt{\pi\alpha}}$.

3.2. Stochastic Dynamical System

For convenience, we rewrite the Question (1.1)-(1.3):

$$\begin{cases} du = u_t dt, \\ du_t + \left(A^m u_t + \phi \left(\left\| A^{\frac{m}{2}} u \right\|^2 \right) A^m u + g(u) \right) dt \\ = f(x) dt + q(x) dW(t), t \in [0, +\infty), \\ u(x, 0) = u_0(x), u_t(x, 0) = u_1(x), x \in \Omega. \end{cases} \tag{3.2.1}$$

Let $\psi = (u, y)^T$, $y = u_t + \mu u$, and $\mu = \varepsilon$ (ε defined in [20]), then (3.2.1) has the following simple matrix form

$$\begin{cases} d\psi + L\psi dt = F(\theta_t, \omega, \psi), \\ \psi_0(\omega) = (u_0, u_1 + \mu u_0)^T. \end{cases} \tag{3.2.2}$$

where

$$\psi = \begin{pmatrix} u \\ y \end{pmatrix}, \quad L = \begin{pmatrix} \mu I & -I \\ ((\phi - \mu) A^m + \mu^2) I & (A^m - \mu) I \end{pmatrix},$$

$$F(\theta_t, \omega, \psi) = \begin{pmatrix} 0 \\ (-g(u) + f(x))dt + q(x)dW(t) \end{pmatrix}.$$

Let $v = y - qz(\theta_t, \omega)$, then (3.2.1) can be rewritten as the equivalent system:

$$\begin{cases} \gamma_t + L\gamma = \bar{F}(\theta_t, \omega, \gamma), \\ \gamma_0(\omega) = (u_0, u_1 + \mu u_0 - qz(\theta_t, \omega))^T. \end{cases} \tag{3.2.3}$$

where

$$\gamma = \begin{pmatrix} u \\ v \end{pmatrix}, L = \begin{pmatrix} \mu I & -I \\ ((\phi - \mu)A^m + \mu^2)I & (A^m - \mu)I \end{pmatrix},$$

$$\bar{F}(\theta_t, \omega, \gamma) = \begin{pmatrix} qz(\theta_t, \omega) \\ -g(u) + f(x) - (A^m - \mu - 1)qz(\theta_t, \omega) \end{pmatrix}.$$

In [14] [15] they have proven that the operator L of (3.2.3) is the infinitesimal generation operator of C_0 semigroup e^{Lt} in Hilbert space E , $\bar{F}(\theta_t, \omega, \gamma): [0, +\infty) \times E \rightarrow E$ is continuous in t and globally Lipschitz continuous in γ for each $\omega \in \Omega$. By the classical theory concerning the existence and uniqueness of the solutions [14] [16] [17], so we have the following theorem.

Theorem 3.2.1. Consider (3.2.3). For each $\omega \in \Omega$ and initial value $\gamma_0 = (u_0, u_1 + \mu u_0 - qz(\theta_t, \omega))^T \in E$, there exists a unique function γ such that satisfies the integral equation

$$\gamma(t, \omega) = e^{-Lt} \gamma_0 + \int_0^t e^{Ls} \bar{F}(\omega, \gamma(s)) ds,$$

and

$$\gamma \in C([0, T]; H_0^m(\Omega)) \times C([0, T]; L^2(\Omega)), \forall T > 0.$$

For $\forall t \geq 0, a.e. \omega \in \Omega$, let the solution mapping of $E \rightarrow E$

$$\tilde{S}(t, \omega): \gamma(0, \omega) = (u_0(x), v_0(x))^T \mapsto \gamma(t, \omega) = (u(x, t, \omega), v(x, t, \omega))^T$$

generates a random dynamical system.

Define two isomorphic mapping:

$$T_\mu(\theta_t, \omega): (y_1, y_2)^T \mapsto (y_1, y_2 - \mu y_1 + qz(\theta_t, \omega))^T,$$

$$R_\mu: (y_1, y_2)^T \mapsto (y_1, y_2 - \mu y_1).$$

And inverse isomorphic mapping:

$$T_\mu^{-1}(\theta_t, \omega): (y_1, y_2)^T \mapsto (y_1, y_2 + \mu y_1 - qz(\theta_t, \omega))^T,$$

$$R_\mu^{-1}: (y_1, y_2)^T \mapsto (y_1, y_2 + \mu y_1).$$

Then the mapping $S_1(t, \omega) = T_\mu(\theta_t, \omega) \tilde{S}(t, \omega) T_\mu^{-1}(\theta_t, \omega)$ generates a random dynamical system associated with (1.1)-(1.3); and mapping

$S(t, \omega) = R_\mu S_1(t, \omega) R_\mu^{-1}$ generates a random dynamical system associated with (3.2.2).

Notice that all of the above random dynamical system $S_1(t, \omega)$, $S(t, \omega)$ are equivalent. Hence we only need to consider the random dynamical system

$S(t, \omega)$.

4. The Existence of Random Attractor

First, we prove the random dynamical system $S(t, \omega)$ exists a bounded random absorb set, hence we let $D(E)$ be all temper subsets in E .

Lemma 4.1. (Lemma 3.1 of [20]) Let $V = H_0^m(\Omega) \times L^2(\Omega)$, for any $y = (y_1, y_2)^T \in V$, we have

$$(Ly, y)_E \geq k_1 \|y\|_E^2 + k_2 \|\nabla^m y_2\|_0^2 \geq k_1 \|y\|_E^2 + k_3 \|y_2\|_0^2. \tag{4.1}$$

where k_1, k_2 are determined in [20], $k_3 = \lambda_1^m k_2$, λ_1 is first eigenvalues of (1.1).

Lemma 4.2. Let ψ is a solve of (3.2.2), then there is a bounded random compact set $\tilde{B}_0(\omega) \in D(E)$, such that for arbitrarily random set $B(\omega) \in D(E)$, existence a random variable $T_{B(\omega)} > 0$, so that

$$\psi(t, \theta_{-t}\omega)B(\theta_{-t}\omega) \subset B_0(\omega), \forall t \geq T_B(\omega), \omega \in \Omega. \tag{4.2}$$

Proof. Let γ is a solve of (3.2.3), applying the inner product of the equation (3.2.3) with $\gamma = (u, v)^T \in E$, we discover that

$$\frac{1}{2} \frac{d}{dt} \|\gamma\|_E^2 + (L\gamma, \gamma)_E = (\bar{F}(\theta_t\omega, \gamma), \gamma), \tag{4.3}$$

where

$$(\bar{F}(\theta_t\omega, \gamma), \gamma) = (\nabla^m q(x)z(\theta_t\omega), \nabla^m u) + (-g(u) + f(x) - (A^m - \mu - 1)q(x)z(\theta_t\omega), v)$$

$$(\nabla^m q(x)z(\theta_t\omega), \nabla^m u) \leq \frac{1}{2} \|\nabla^m u\|^2 + \frac{1}{2} \|\nabla^m q(x)\|^2 \|z(\theta_t\omega)\|_0^2, \tag{4.4}$$

$$(f(x) + \mu q(x)z(\theta_t\omega), v) \leq \frac{k_3}{2} \|v\|_0^2 + \frac{1}{2k_3} (\|f(x)\|^2 + \mu^2 \|q(x)\|_0^2 \|z(\theta_t\omega)\|_0^2), \tag{4.5}$$

$$(-(A^m - 1)q(x)z(\theta_t\omega), v) \leq \frac{k_3}{2} \|v\|_0^2 + \frac{1}{2k_3} (\|A^m q(x)\|^2 + \|q(x)\|_0^2 \|z(\theta_t\omega)\|_0^2), \tag{4.6}$$

$$(-g(u), v) = (-g(u), u_t + \mu u - q(x)z(\theta_t\omega)), \tag{4.7}$$

$$(g(u), u_t) = \int_{\Omega} g(u)u_t dx = \frac{d}{dt} \int_{\Omega} G(u) dx, \tag{4.8}$$

$$\mu(g(u), u) = \mu \int_{\Omega} g(u)u dx \geq \mu c_1 \int_{\Omega} G(u) dx - k_0, k_0 \geq 0, \tag{4.9}$$

$$\begin{aligned} (g(u), q(x)z(\theta_t\omega)) &= \int_{\Omega} g(u)q(x)z(\theta_t\omega) dx \\ &\leq c_0 \int_{\Omega} |q(x)z(\theta_t\omega)| dx + c_0 \int_{\Omega} |u|^p |q(x)z(\theta_t\omega)| \\ &\leq c_0 \sqrt{\|\Omega\|} \|q(x)\|_0 \|z(\theta_t\omega)\|_0 + c_0 \left(\int_{\Omega} |u|^{p+1} dx \right)^{\frac{p}{p+1}} \|q(x)\|_{L^{p+1}(\Omega)} \|z(\theta_t\omega)\|_{00} \\ &\leq C_1 + C_2 \|q(x)\|_0^2 \|z(\theta_t\omega)\|_0^2 + C_3 \left(\int_{\Omega} G(u) dx \right)^{\frac{p}{p+1}} \|q(x)\|_{L^{p+1}(\Omega)} \|z(\theta_t\omega)\|_{00} \\ &\leq C_1 + C_2 \|q(x)\|_0^2 \|z(\theta_t\omega)\|_0^2 + \frac{\mu C_1}{2} \int_{\Omega} G(u) dx + C_4 \|q(x)\|^{p+1} \|z(\theta_t\omega)\|_{00}^{p+1}. \end{aligned} \tag{4.10}$$

According to (4.1) and (4.4)-(4.10), we have

$$\begin{aligned} & \frac{d}{dt} \left(\|\gamma\|_E^2 + 2 \int_{\Omega} G(u) dx \right) + \eta \left(\|\gamma\|_E^2 + 2 \int_{\Omega} G(u) dx \right) \\ & \leq C_5 + M \|z(\theta_t \omega)\|_0^2 + N \|z(\theta_t \omega)\|_0^{p+1}, \end{aligned} \tag{4.11}$$

where

$$\begin{aligned} \eta &= \min \left\{ 2k_1, \frac{\mu c_1}{2} \right\}, \\ M &= \left(\frac{4\mu^2 + 4}{k_3} + 2C_2 \right) \|q(x)\|_0^2 + \frac{4}{k_3} \|A^m q(x)\|_0^2, \\ N &= 2C_4 \|q(x)\|_0^{p+1}. \end{aligned}$$

According to Gronwall inequation, $Pa.e.\omega \in \Omega$, we have

$$\begin{aligned} & \|\gamma(t, \omega)\|_E^2 + 2 \int_{\Omega} G(u) dx \\ & \leq e^{-\eta t} \left(\|\gamma(0, \omega)\|_E^2 + 2 \int_{\Omega} G(u_0) dx \right) \\ & \quad + \int_0^t e^{-\eta(t-r)} \left(C_5 + M \|z(\theta_r \omega)\|_0^2 + N \|z(\theta_r \omega)\|_0^{p+1} \right) dr. \end{aligned} \tag{4.12}$$

Because $z(\theta_t \omega)$ is tempered, and $z(\theta_t \omega)$ is continuous about t , according to [21], we can get a temper random variables $r_1 : \Omega \rightarrow R^+$, such that $\forall t \in R, \omega \in \Omega$, we have

$$\|z(\theta_t \omega)\|_0^2 \leq r_1(\theta_t \omega) \leq e^{\frac{\eta}{2} t} r_1(\theta_t \omega). \tag{4.13}$$

Substituting ω by $\theta_{-t} \omega$ in (4.12), we know

$$\begin{aligned} & \|\gamma(t, \theta_{-t} \omega)\|_E^2 + 2 \int_{\Omega} G(u) dx \\ & \leq e^{-\eta t} \left(\|\gamma(0, \theta_{-t} \omega)\|_E^2 + 2 \int_{\Omega} G(u_0) dx \right) \\ & \quad + \int_0^t e^{-\eta(t-r)} \left(C_5 + M \|z(\theta_{r-t} \omega)\|_0^2 + N \|z(\theta_{r-t} \omega)\|_0^{p+1} \right) dr, \end{aligned} \tag{4.14}$$

where

$$\begin{aligned} & \int_0^t e^{-\eta(t-r)} \left(C_5 + M \|z(\theta_{r-t} \omega)\|_0^2 + N \|z(\theta_{r-t} \omega)\|_0^{p+1} \right) dr \\ & = \int_{-t}^0 e^{\eta t} \left(C_5 + M \|z(\theta_{\tau} \omega)\|_0^2 + N \|z(\theta_{\tau} \omega)\|_0^{p+1} \right) d\tau \\ & \leq \frac{C_5}{\eta} + \frac{2}{\eta} M r_1(\omega) + \frac{2}{\eta} N r_1^{\frac{p+1}{2}}(\omega). \end{aligned} \tag{4.15}$$

Because $\psi_0(\theta_{-t} \omega) \in B(\theta_{-t} \omega)$ is tempered, and $\|z(\theta_{-t} \omega)\|_0$ is also tempered, hence we let

$$R_0^2(\omega) = \frac{C_5}{\eta} + \frac{2}{\eta} M r_1(\omega) + \frac{2}{\eta} N r_1^{\frac{p+1}{2}}(\omega), \tag{4.16}$$

then $R_0^2(\omega)$ is also tempered, $\hat{B}_0 = \{\gamma \in E : \|\gamma\|_E \leq R_2(\omega)\}$ is called a random

absorb set, and because of

$$\tilde{S}(t, \theta_{-t}\omega)\gamma_0(\theta_{-t}\omega) = \psi(t, \theta_{-t}\omega)\left(\gamma_0(\theta_{-t}\omega) + (0, q(x)z(\theta_{-t}\omega))^T\right) - (0, q(x)z(\theta_t\omega))^T,$$

so let

$\tilde{B}_0(\omega) = \{\psi \in E : \|\psi\|_E \leq R_0(\omega) + \|q(x)z(\theta_t\omega)\| = \overline{R}_0(\omega)\}$, then $\tilde{B}_0(\omega)$ is a random absorb set of $\psi(t, \omega)$, and $\tilde{B}_0(\omega) \in D(E)$.

Next, we will prove the random dynamical system $S(t, \omega)$ has a compact absorb set

Lemma 4.3. For $\forall B(\omega) \in D(E)$, let $\psi(t)$ be a solve of (3.2.2), initial value $\psi_0 = (u_0, u_1 + \mu u_0)^T \in B$, we decompose $\psi = \psi^1 + \psi^2$, where ψ^1, ψ^2 satisfy

$$\begin{cases} d\psi^1 + L\psi^1 dt = 0, \\ \psi_0^1(\omega) = (u_0, u_1 + \mu u_0)^T, \end{cases} \tag{4.17}$$

$$\begin{cases} d\psi^2 + L\psi^2 dt = 0, \\ \psi_0^2(\omega) = 0. \end{cases} \tag{4.18}$$

Then

$$\|\psi^1(t, \theta_{-t}\omega)\|_E^2 \rightarrow 0 (t \rightarrow \infty), \forall \psi_0(\theta_{-t}\omega) \in B(\theta_{-t}\omega), \tag{4.19}$$

and exist a temper random radius $R_1(\omega)$, such that $\forall \omega \in \Omega$, satisfy

$$\left\| A^{\frac{m}{4}} \psi^2(t, \theta_{-t}\omega) \right\|_E \leq R_1(\omega). \tag{4.20}$$

Proof. Let $\gamma = \gamma^1 + \gamma^2 = (u^1, u_t^1 + \mu u^1)^T + (u^2, u_t^2 + \mu u^2 - qz(\theta_t\omega))^T$ be a solve of (3.2.3), according to (4.17) and (4.18), we know γ^1, γ^2 meet separately

$$\begin{cases} d\gamma_t^1 + L\gamma_t^1 dt = 0, \\ \gamma_0^1(\omega) = (u_0, u_1 + \mu u_0 - qz(\theta_t\omega))^T, \end{cases} \tag{4.21}$$

$$\begin{cases} d\gamma_t^2 + L\gamma_t^2 dt = \bar{F}(\theta_t\omega, \gamma), \\ \gamma_0^2(\omega) = 0. \end{cases} \tag{4.22}$$

Taking inner product (4.21) with $\gamma^1 = (u^1, u_t^1 + \mu u^1)^T$, we have

$$\frac{1}{2} \frac{d}{dt} \|\gamma^1\|_E^2 + (L\gamma^1, \gamma^1) = 0,$$

according to Lemma 4.1 and Gronwall inequality, we have

$$\|\gamma^1(t, \omega)\|_E^2 \leq e^{-2\xi t} \|\gamma^1(0, \omega)\|_E^2, \tag{4.23}$$

substituting ω by $\theta_{-t}\omega$, and $z(\theta_{-t}\omega) \in B$ is tempered, then

$$\|\gamma^1(t, \theta_{-t}\omega)\|_E^2 \leq e^{-2\xi t} \|\gamma^1(0, \theta_{-t}\omega)\|_E^2 \rightarrow 0 (t \rightarrow \infty), \forall \gamma(0, \theta_{-t}\omega) \in B.$$

So, (4.19) is hold. Taking inner product (4.22) with

$A \frac{m}{2} \gamma^2 = A \frac{m}{2} (u^2, u_t^2 + \mu u^2 - q(x)z(\theta_t\omega))^T$, we have

$$\frac{1}{2} \frac{d}{dt} \left\| A^{\frac{m}{4}} \gamma^2 \right\|_E^2 + \left(L\gamma^2, A \frac{m}{2} \gamma^2 \right) = \left(\bar{F}(\gamma, \theta_t\omega), A \frac{m}{2} \gamma^2 \right), \tag{4.24}$$

according to Lemma 4.1, Lemma 4.2, (4.24) and Young inequality, we have

$$\begin{aligned} & \frac{d}{dt} \left(\left\| A^{\frac{m}{4}} \gamma^2 \right\|_E^2 + 2 \int_{\Omega} A^{\frac{m}{2}} G(u^2) dx \right) + \eta \left(\left\| A^{\frac{m}{4}} \gamma^2 \right\|_E^2 + 2 \int_{\Omega} A^{\frac{m}{2}} G(u^2) dx \right) \\ & \leq C_5 R_0^2(\theta_t \omega) + M_1 \|z(\theta_t \omega)\|_0^2 + N_2 \|z(\theta_t \omega)\|_0^{p+1}, \forall t \geq T_{B(\omega)}, \end{aligned}$$

where $T_{B(\omega)}, R_0(\omega)$ are given by Lemma 4.2, and

$$M_1 = \left(\frac{4\mu^2 + 4}{k_3} + 2C_2 \right) \left\| A^{\frac{m}{4}} q(x) \right\|^2 + \frac{4}{k_3} \|A^m q(x)\|^2,$$

$$N_1 = C_4 \left\| A^{\frac{m}{4}} q(x) \right\|^{p+1}. \text{ Due to Gronwall inequality, and substituting } \omega \text{ by}$$

$\theta_{-t} \omega$, we have

$$\begin{aligned} & \left\| A^{\frac{m}{4}} \gamma^2(t, \theta_{-t} \omega) \right\|_E^2 + 2 \int_{\Omega} A^{\frac{m}{2}} G(u^2) dx \\ & \leq \int_0^t e^{-\eta(t-r)} \left(C_5 R_0^2(\theta_{r-t} \omega) + M_1 \|z(\theta_{r-t} \omega)\|_0^2 + N_2 \|z(\theta_{r-t} \omega)\|_0^{p+1} \right) dr, \forall t \geq T_{B(\omega)}. \end{aligned}$$

According to (4.14) and (4.16), then

$$\begin{aligned} & \int_0^t e^{-\eta(t-r)} \left(C_5 R_0^2(\theta_{r-t} \omega) + M_1 \|z(\theta_{r-t} \omega)\|_0^2 + N_2 \|z(\theta_{r-t} \omega)\|_0^{p+1} \right) dr \\ & = \int_{-t}^0 \left(C_5 R_0^2(\theta_{\tau} \omega) + M_1 \|z(\theta_{\tau} \omega)\|_0^2 + N_2 \|z(\theta_{\tau} \omega)\|_0^{p+1} \right) d\tau \\ & = \frac{C_5^2}{\eta^2} + \left(\frac{2}{\eta} M_1 + \frac{4C_5}{\eta^2} M \right) r_1(\omega) + \left(\frac{2}{\eta} N_1 + \frac{8}{(p+1)\eta} N \right) r_1^{\frac{p+1}{2}}(\omega). \end{aligned}$$

Let

$$\bar{R}_1^2(\omega) = \frac{C_5^2}{\eta^2} + \left(\frac{2}{\eta} M_1 + \frac{4C_5}{\eta^2} M \right) r_1(\omega) + \left(\frac{2}{\eta} N_1 + \frac{8}{(p+1)\eta} N \right) r_1^{\frac{p+1}{2}}(\omega).$$

Then $\bar{R}_1^2(\omega)$ is tempered, and because

$$\tilde{S}(t, \theta_{-t} \omega) \gamma_0(\theta_{-t} \omega) = \psi(t, \theta_{-t} \omega) \left(\gamma_0(\theta_{-t} \omega) + (0, q(x)z(\theta_{-t} \omega))^T \right) - (0, q(x)z(\theta_t \omega))^T,$$

hence, we set

$$\begin{aligned} R_1(\omega) &= \bar{R}_1(\omega) + \left\| A^{\frac{m}{4}} q(x) z(\theta_t \omega) \right\|, \text{ then, for } \forall \omega \in \Omega, \text{ we have} \\ \left\| A^{\frac{m}{4}} \psi^2(t, \theta_{-t} \omega) \right\|_E &\leq R_1(\omega), \text{ and } R_1(\omega) \text{ is tempered.} \end{aligned}$$

Lemma 4.4. (3.2.2) the identified stochastic dynamical system $S(t, \omega), t \geq 0$, while $t = 0, Pa.e. \omega \in \Omega$ exist a compact attracting set $K(\omega) \subset E$.

Proof. Let $K(\omega)$ be a closed ball, radius $R_1(\omega)$ in space

$$D\left(A^{\frac{3}{4}m}\right) \times D\left(A^{\frac{m}{4}}\right), \text{ because } D\left(A^{\frac{3}{4}m}\right) \times D\left(A^{\frac{m}{4}}\right) \mapsto E, \text{ so } K(\omega) \text{ is a compact}$$

set in E , for arbitrarily temper random set $B(\omega)$, for $\forall \psi(t, \theta_{-t} \omega) \in B$, according to Lemma 4.3, $\psi^2 = \psi - \psi^1 \in K(\omega)$, so for $\forall t \geq T_{B(\omega)} > 0$, we have

$$\begin{aligned}
& d_E(S(t, \theta_{-t}\omega)B(\theta_{-t}\omega), K(\omega)) \\
&= \inf_{\beta(t) \in K(\omega)} \|\psi(t, \theta_{-t}\omega) - \beta(t)\|_E^2 \leq \|\psi^1(t, \theta_{-t}\omega)\|_E^2 \\
&\leq e^{-2\eta t} \|\psi_0^1(t, \theta_{-t}\omega)\|_E^2 \rightarrow 0, (t \rightarrow \infty).
\end{aligned}$$

■

Theorem 4.1. The random dynamical system $S(t, \omega), t \geq 0$ has a unique random attractor $A(\omega)$ in E , where

$$A(\omega) = \bigcap_{t > 0} \overline{\bigcup_{\tau \geq t} S(t, \theta_{-\tau}\omega, K(\theta_{-\tau}\omega))}, \omega \in \Omega,$$

in which $K(\omega)$ is a tempered random compact attracting for $S(t, \omega)$.

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