International Journal of Modern Nonlinear Theory and Application, 2016, 5, 122-131
http://www.scirp.org/journal/ijmnta
ISSN Online: 2167-9487
ISSN Print: 2167-9479

Chaos in a Fractional-Order Single-Machine Infinite-Bus Power System and Its Adaptive Backstepping Control

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How to cite this paper: Liang, Z.H. and Gao, J.F. (2016) Chaos in a Fractional-Order Single-Machine Infinite-Bus Power System and Its Adaptive Backstepping Control. *International Journal of Modern Nonlinear Theory and Application*, **5**, 122-131.
http://dx.doi.org/10.4236/ijmnta.2016.53013

Received: August 18, 2016
Accepted: September 24, 2016
Published: September 27, 2016

Abstract

This paper has numerically studied the dynamical behaviors of a fractional-order single-machine infinite-bus (FOSMIB) power system. Periodic motions, period-doubling bifurcations and chaotic attractors are observed in the FOSMIB power system. The existence of chaotic behavior is affirmed by the positive largest Lyapunov exponent (LLE). Based on the fractional-order backstepping method, an adaptive controller is proposed to suppress chaos in the FOSMIB power system. Numerical simulation results demonstrate the validity of the proposed controller.

Keywords

Power System, Fractional Calculus, Chaos, Backstepping Method

1. Introduction

As a mathematical branch with a history of over 300 years, fractional calculus and its applications to physics and engineering have attracted increasing attentions in recent years [1] [2]. Fractional calculus provides a good instrument to describe the memory, hereditary, non-locality and self-similarity properties of various materials and processes. Many chaotic systems, such as Lorenz system [3], Chua's system [4], Duffing system [5], Rössler system [6], Chen system [7] and so on, still remain chaotic when their equations become fractional.

Chaotic phenomena have been observed in power systems during the past few decades [8]-[13]. Chaos causes electromechanical oscillations to behave randomly, which are harmful to the secure and stable operation of power systems, and even produce undesired negative consequences, such as angle divergence, voltage collapse and system splitting [14]. So far, almost all the studies of dynamics of power systems are concerned with the integer-order models, and there are little research results on fractional modeling and control design of power systems. Tan *et al.* studied the dynamics of a fractional-order interconnected power system and found that the system became chaotic when the fractional order is no less than 0.88 [15]. Sun and Li investigated the chaotic and bifurcation phenomena in a fractional-order three-bus power system and the existence of chaos was demonstrated for different orders [16].

In this paper, we numerically investigate the chaotic dynamics of a fractional-order single-machine infinite-bus (FOSMIB) power system. Period-doubling bifurcation and chaos are observed in FOSMIB power system and the existence of chaos is confirmed by evaluating the largest Lyapunov exponent (LLE). Based on the fractional-order backstepping method, an adaptive controller is presented to suppress chaos in the FOSMIB power system, and the effectiveness of the proposed controller is proved by the numerical simulation results.

The rest of the paper is organized as follows. Some definitions and lemmas about fractional calculus are introduced in Section 2. The dynamics of the FOSMIB power system are analyzed in Section 3. An adaptive controller is designed using the fractional-order backstepping method to suppress chaos in the FOSMIB power system in Section 4. Finally, conclusions are addressed in Section 5.

2. Preliminaries

There are several different definitions of fractional derivatives. The most appropriate one for practical problems is the Caputo definition. The Caputo fractional derivative is given by

$$D_t^q x(t) = \begin{cases} \dfrac{1}{\Gamma(m-q)} \displaystyle\int_0^t (t-\tau)^{(m-q-1)} x^{(m)}(\tau)\,\mathrm{d}\tau,\ m-1<q<m, \\ \dfrac{\mathrm{d}^m}{\mathrm{d}t^m} x(t),\ q=m, \end{cases} \tag{1}$$

where m is integer and $\Gamma(\cdot)$ is the Gamma function.

The Caputo fractional derivative satisfies the following properties:

$$\begin{aligned} &D_t^q C = 0, \\ &D_t^q\left(k_1 x(t) + k_2 y(t)\right) = k_1 D_t^q x(t) + k_2 D_t^q y(t), \end{aligned} \tag{2}$$

where C, k_1 and k_2 are real constants.

Lemma 1. [17]-[19] *Consider the fractional-order system*

$$D_t^q x(t) = f\left(x(t)\right), \tag{3}$$

where $q \in (0,1)$ *and* $x \in R^n$. *The equilibrium point* x^* *of system* (3) *is locally asymptotically stable if all the eigenvalues* λ *of the Jacobian matrix* $J = \partial f / \partial x|_{x^*}$ *satisfy*

$$\left|\arg(\lambda)\right| > \frac{q\pi}{2}. \tag{4}$$

Lemma 2. [20] *Let* $x(t) \in \mathbb{R}$ *be a continuous differentiable function. Then, at any* *instant the following inequality holds*

$$\frac{1}{2}D_t^q x^2(t) \le x(t)D_t^q x(t), \forall q \in (0,1). \tag{5}$$

A continuous function $\alpha:[0,t) \to [0,+\infty)$ is referred as class-K if it is strictly increasing and $\alpha(0)=0$ [21].

Lemma 3. (*Fractional-order extension of Lyapunov direct method* [22]) *Let* $x=0$ *be an equilibrium point of the nonautonomous fractional-order system*

$$D_t^q x(t) = f(t, x(t)) \tag{6}$$

with initial condition $x(0)$. *Assume that* $V(t,x(t))$ *is a Lyapunov candidate and* $\alpha_i(i=1,2,3)$ *are class-K functions. Then* $x=0$ *is asymptotically stable if the following conditions hold*

$$\alpha_1(\|x\|) \le V(t,x(t)) \le \alpha_2(\|x\|), \tag{7}$$

$$D_t^q V(t,x(t)) \le -\alpha_3(\|x\|), \tag{8}$$

where $q \in (0,1)$ *and* $\|\cdot\|$ *denotes an arbitrary norm.*

3. The FOSMIB Power System

In [12] Chen *et al.* analyzed the angle dynamics of the classical single-machine infinite-bus (SMIB) power system, which is governed by the so-called swing equation

$$M\ddot{\theta} + D\dot{\theta} + P_{\max}\sin\theta = P_m, \tag{9}$$

where M is the moment of inertia, D is the damping constant, $P_{\max}$ is the maximum power of generator and $P_m = A\sin\omega t$ is the power of the machine.

Let $x=\theta$ and $y=\dot{\theta}$, then Equation (9) can be rewritten as

$$\begin{cases} \dot{x} = y, \\ \dot{y} = -cy - \beta\sin x + f\sin\omega t, \end{cases} \tag{10}$$

where $c = D/M$, $\beta = P_{\max}/M$ and $f = A/M$ are positive constant parameters. When $c=0.5$, $\beta=1$, $\omega=1$ and $f=2.41$, the SMIB power system is chaotic.

Here, we consider the fractional-order single-machine infinite-bus (FOSMIB) power system

$$\begin{cases} D_t^q x = y, \\ D_t^q y = -cy - \beta\sin x + f\sin\omega t, \end{cases} \tag{11}$$

where $0 < q \le 1$ is the fractional order. When $q=1$, system (11) is the original integer-order SMIB power system.

The autonomous system (11) (as $f=0$) has two equilibrium points: $O(0,0)$ and $E(\pi,0)$. For the equilibrium point O, the Jacobian matrix is

$$J = \begin{bmatrix} 0 & 1 \\ -\beta & -c \end{bmatrix}, \tag{12}$$

and its eigenvalues are

$$\lambda_{1,2} = \begin{cases} \dfrac{-c \pm \sqrt{c^2 - 4\beta}}{2}, \beta \le \dfrac{c^2}{4}, \\ \dfrac{-c \pm i\sqrt{4\beta - c^2}}{2}, \beta > \dfrac{c^2}{4}. \end{cases} \tag{13}$$

In both cases, $\left|\arg\left(\lambda_{1,2}\right)\right| > q\pi/2$. According to Lemma 1, O is asymptotically stable.

For the equilibrium point E, the Jacobian matrix is

$$J = \begin{bmatrix} 0 & 1 \\ \beta & -c \end{bmatrix}, \tag{14}$$

and its eigenvalues are

$$\lambda_{1,2} = \frac{-c \pm \sqrt{c^2 + 4\beta}}{2}. \tag{15}$$

It can be seen that $\lambda_1 > 0$ and $\left|\arg\left(\lambda_1\right)\right| = 0 < q\pi/2$. In accordance with Lemma 1, E is unstable.

4. Dynamic Analysis of the FOSMIB Power System

In this section, we use the Adams-Bashforth-Moulton predictor-corrector algorithm proposed by Diethelm *et al.* in [22]-[24] to solve the FOSMIB power system (11). The dynamics are numerically analyzed by means of bifurcation diagrams, phase portraits and Lyapunov exponents. In the following simulations, parameter f is chosen as bifurcation parameter and the other parameters are fixed at $c = 0.5$, $\beta = 1$, $\omega = 1$. The initial conditions are selected as $x(0) = 1$, $y(0) = -0.3$.

First, let $q = 0.95$, and vary f from 2.4 to 3.5. The corresponding bifurcation diagram is plotted in **Figure 1(a)**, from which a period-doubling route to chaos can be found. To confirm chaos, the largest Lyapunov exponent (LLE) is calculated using Wolf

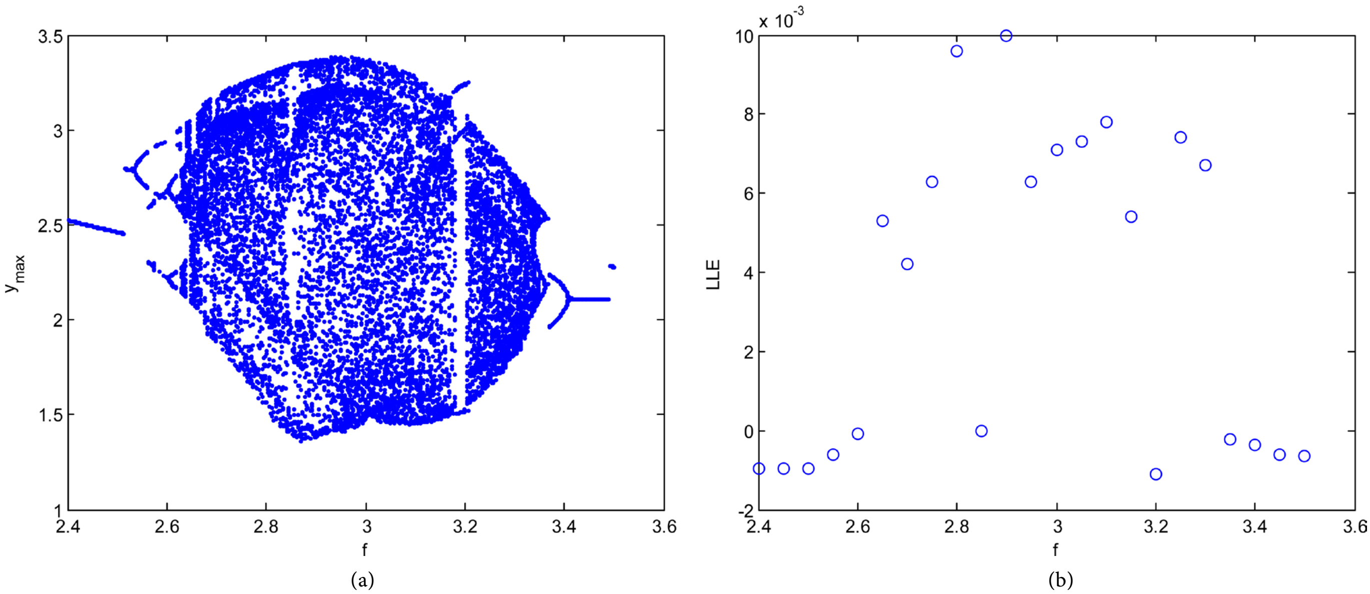

Figure 1. Bifurcation diagram and the LLE versus f for $q = 0.95$: (a) Bifurcation diagram; (b) The LLE.

algorithm [25] and plotted in Figure 1(b). The FOSMIB power system is chaotic over most of the range $f \in [2.65, 3.3]$, where the LLEs are positive. The phase portraits for different values of *f* are plotted in Figure 2. With the increase of *f* from 2.4, period-1, period-2 and period-4 orbits are obtained at $f = 2.5$, $f = 2.55$ and $f = 2.61$, respectively. After a cascade of period-doubling bifurcations, the system loses its stability and enters chaos at $f = 2.65$. As *f* increases further, the system becomes stable again via inverse period-doubling bifurcations.

Now, let $c = 0.5$, $\beta = 1$, $\omega = 1$, $f = 2.8$ and vary q from 0.87 to 1. The resulting bifurcation diagram is plotted in Figure 3(a), which indicates period-doubling bifurcations and chaos. The fractional-order SMIB power system is chaotic over most of the range $q \in [0.92, 1]$, where the LLEs are positive as shown in Figure 3(b). The phase portraits for different values of q are plotted in Figure 4. With the increase of q from 0.87, period-1, period-2 and period-4 orbits are obtained at $q = 0.88$, $q = 0.893$ and $q = 0.913$, respectively. As q increases further, after a cascade of period-doubling bifurcations, a chaotic attractor is obtained at $q = 0.92$.

5. Adaptive Backstepping Control of Chaos

In this section, an active controller is designed using fractional-order backstepping method to suppress chaos in the FOSMIB power system and stabilize it to the unstable equilibrium point $E(\pi, 0)$.

5.1. Controller Design

Consider the controlled FOSMIB power system

$$\begin{cases} D_t^q x = y, \\ D_t^q y = -cy - \beta \sin x + f \sin \omega t + u, \end{cases} \tag{16}$$

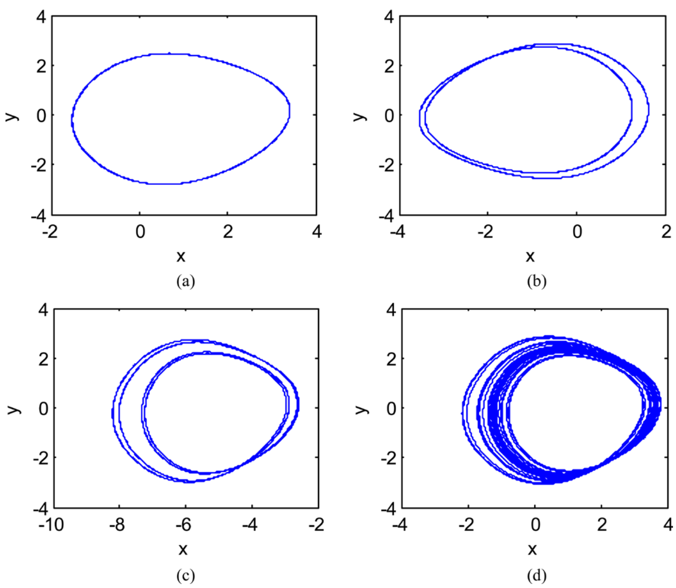

Figure 2. Phase portraits for different values of *f*: (a) *f* = 2.5; (b) *f* = 2.55; (c) *f* = 2.61; (d) *f* = 2.65.

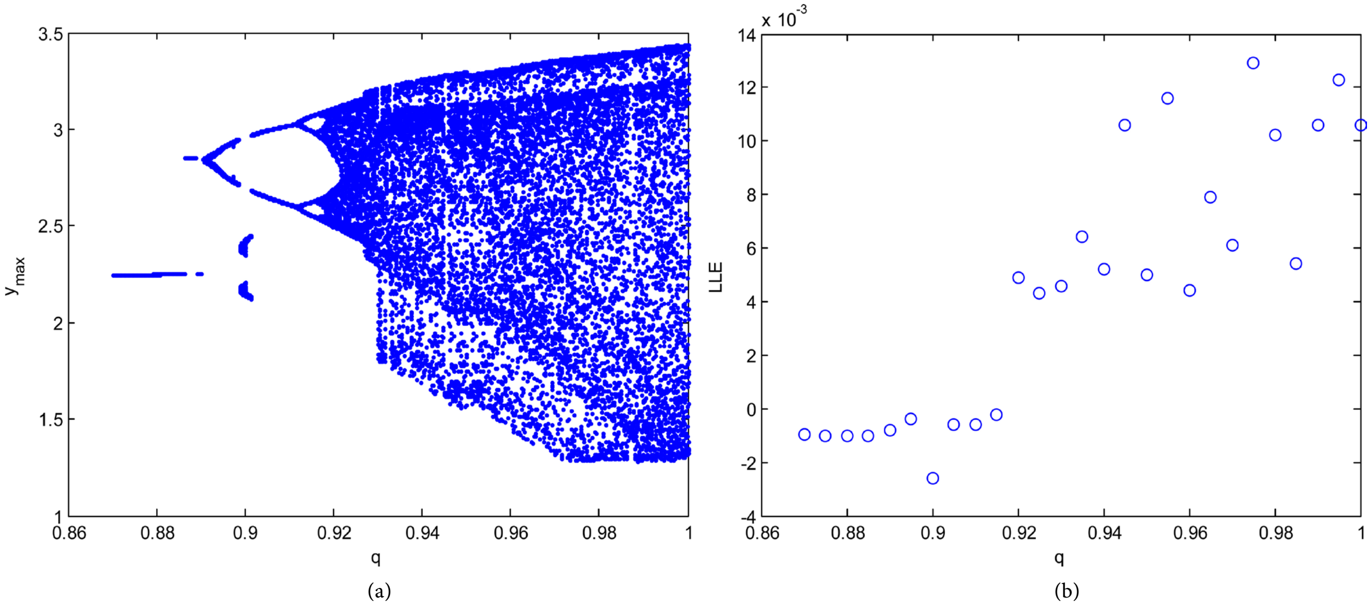

Figure 3. Bifurcation diagram and the LLE versus q for $f = 2.8$: (a) Bifurcation diagram; (b) The LLE.

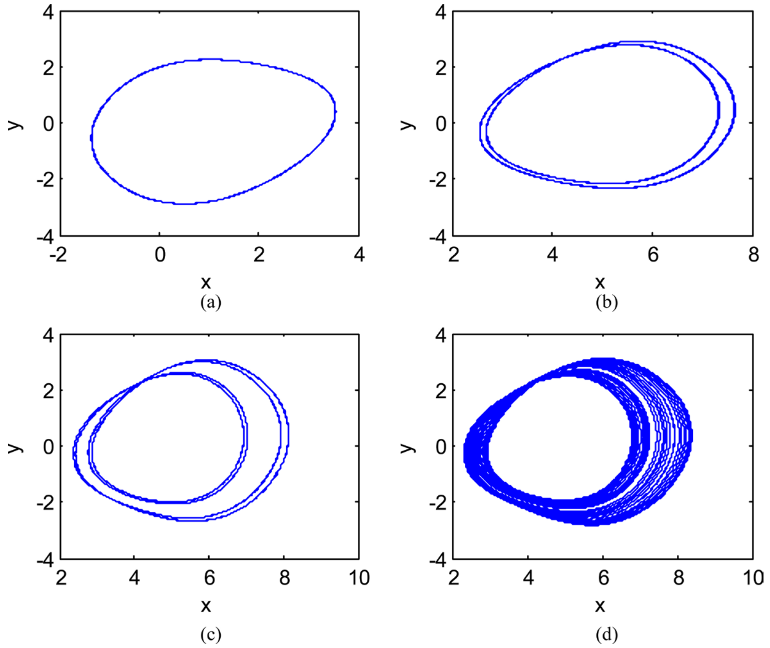

Figure 4. Phase portraits for different values of q: (a) $q = 0.88$; (b) $q = 0.893$; (c) $q = 0.913$; (d) $q = 0.92$.

where $q \in (0,1)$ and the parameter f is unknown. The backstepping design procedure consists of two steps.

Step 1. Define $e_1 = x - \pi$. Its derivative is given by

$$D_t^q e_1 = D_t^q x = e_2 + \alpha_1, \tag{17}$$

where $e_2 = y - \alpha_1$, α_1 is the virtual control to be defined later.

Select the candidate Lyapunov function as

$$V_1 = \frac{1}{2}e_1^2, \tag{18}$$

Now, applying Lemma 2, it can be found that

$$D_t^q V_1 \le e_1\left(e_2 + \alpha_1\right). \tag{19}$$

Define the virtual control α_1 as

$$\alpha_1 = -c_1 e_1, \tag{20}$$

where c_1 is a positive constant, which leads to $e_2 = y + c_1 e_1$. Substituting Equation (20) into Equation (17) and inequality (19), we have

$$D_t^q e_1 = -c_1 e_1 + e_2, \tag{21}$$

$$D_t^q V_1 \le -c_1 e_1^2 + e_1 e_2. \tag{22}$$

Step 2. The derivative of e_2 is expressed as

$$D_t^q e_2 = D_t^q y + c_1 D_t^q e_1 = -c_1^2 e_1 + c_1 e_2 - cy - \beta \sin x + \left(f - \hat{f}\right)\sin \omega t + \hat{f} \sin \omega t + u, \tag{23}$$

where $\hat{f}$ is the estimate of f. Choose the candidate Lyapunov function as

$$V_2 = V_1 + \frac{1}{2}e_2^2 + \frac{1}{2k}\left(f - \hat{f}\right)^2, \tag{24}$$

where k is a positive constant, which can adjust the speed of the adaptive law. Using Lemma 2, it can be found that

$$\begin{aligned} D_t^q V_2 &\le D_t^q V_1 + e_2 D_t^q e_2 - \frac{1}{k}\left(f - \hat{f}\right) D_t^q \hat{f} \\ &\le -c_1 e_1^2 + e_2\left[\left(1 - c_1^2\right)e_1 + c_1 e_2 - cy - \beta \sin x + \hat{f} \sin \omega t + u\right] \\ &\quad + \left(f - \hat{f}\right)\left(e_2 \sin \omega t - \frac{1}{k} D_t^q \hat{f}\right). \end{aligned} \tag{25}$$

Choose the control input and the adaptive law as

$$u = -\left(1 - c_1^2\right)e_1 - \left(c_1 + c_2\right)e_2 + cy + \beta \sin x - \hat{f} \sin \omega t, \tag{26}$$

$$D_t^q \hat{f} = k e_2 \sin \omega t, \tag{27}$$

where c_2 is a positive constant. Substituting Equation (26) and Equation (27) into Equation (23) and inequality (25), we have

$$D_t^q e_2 = -e_1 - c_2 e_2 + \left(f - \hat{f}\right)\sin \omega t, \tag{28}$$

$$D_t^q V_2 \le -c_1 e_1^2 - c_2^2 e_2. \tag{29}$$

According to Lemma 3, the closed-loop error system is asymptotically stable at the origin $(0,0)$. It means that, with the proposed controller and adaptive law, the FOSMIB power system is asymptotically stable at the equilibrium point $E_2(\pi, 0)$.

5.2. Simulation Results

In the simulation, the fractional order q is equal to 0.95. The parameters of system (16) are taken as $c = 0.5$, $\beta = 1$, $\omega = 1$ and $f = 2.66$. The parameters of the controller

(26) and the adaptive law (27) are chosen as $c_1 = c_2 = 0.5$ and $k = 2$. The initial conditions are taken as $x(0) = 1$, $y(0) = -0.3$. The initial parameter estimate is given by $\hat{f}(0) = 0.5$. The closed-loop system consisted of Equations ((16), (26) and (27)) is solved by using the predictor-corrector algorithm. The simulation results are shown in Figure 5.

The time-domain waveforms the states of the controlled system (16) are shown in Figure 5(a) and Figure 5(b). The FOSMIB power system has experienced chaotic behavior before the controller is put into effect. By activating the controller u at $t = 20$ s, the chaotic behavior is suppressed and the controlled system converges to the equilibrium point $E_2(\pi, 0)$ quickly. The parameter estimate $\hat{f}$ is converged to f as shown in Figure 5(c) and the controller u is bounded as shown in Figure 5(d). From Figure 5, it can be seen that the proposed controller is feasible for suppressing chaos in the FOSMIB power system.

6. Conclusion

In this paper, we have numerically investigated the FOSMIB power system. The parameter f and the fractional order q are selected as bifurcation parameters respectively. Complex dynamical behaviors, such as periodic orbits, period-doubling bifurcations and chaotic attractors, are observed in the FOSMIB power system. The LLE is calculated using Wolf algorithm to confirm the existence of chaos. Furthermore, by exploiting the fractional-order backstepping method, we propose an adaptive controller to suppress chaos in the FOSMIB power system. The effectiveness of the presented controller is verified by numerical simulation results.

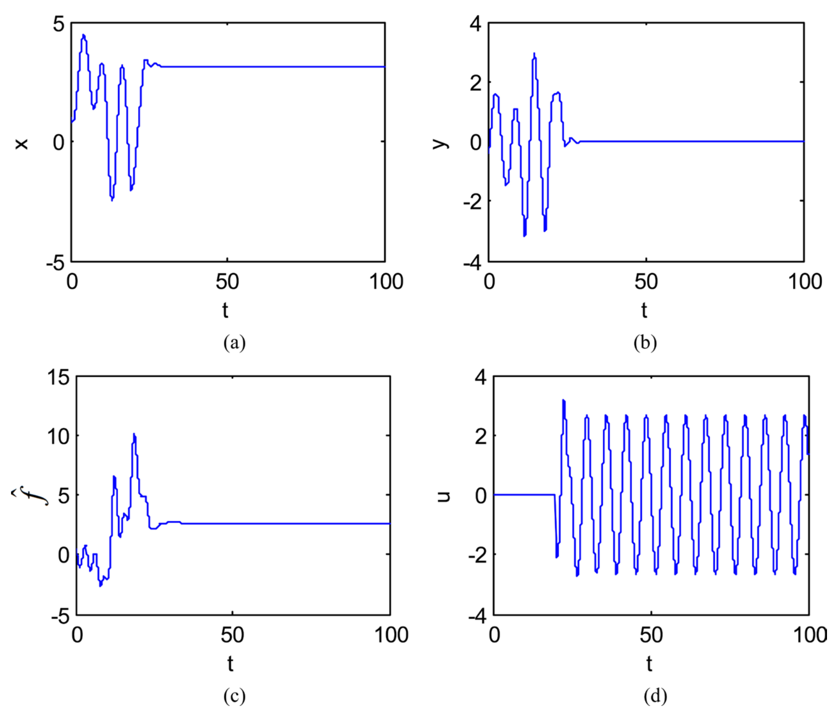

Figure 5. The time-domain waveforms of the controlled system (16).

Acknowledgements

The work was supported by the Natural Science Foundation of Henan Province, China (Grant No. 14A120005) and Excellent Young Scientist Development Foundation of Zhengzhou University, China (Grant No. 1421319086).

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