

Exact Traveling Wave Solutions for the (1 + 1)-Dimensional Compound KdVB Equation via the Novel (G'/G) -Expansion Method

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Abstract

In this work, while applying a new and novel (G'/G) -expansion version technique, we identify four families of the traveling wave solutions to the (1 + 1)-dimensional compound KdVB equation. The exact solutions are derived, in terms of hyperbolic, trigonometric and rational functions, involving various parameters. When the parameters are tuned to special values, both solitary, and periodic wave models are distinguished. State of the art symbolic algebra graphical representations and dynamical interpretations of the obtained solutions physics are provided and discussed. This in turn ends up revealing salient solutions features and demonstrating the used method efficiency.

Keywords

Novel (G'/G) -Expansion Method, The (1 + 1)-Dimensional Compound KdVB Equation, Traveling Wave Solutions, Solitary Wave Solutions, Solitons

1. Introduction

Nonlinear Evolution Equations (NLEEs) are encountered in various fields of engineering, and many theoretical and applied sciences physics, such as applied mathematics, chemistry, biology and many applications. Exact analytical solutions of NLEEs have come to play a significant role in understanding of qualitative nature of

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many phenomena, and the suitable modeling of corresponding processes, in different areas of applied science. Graphical representations of solutions of the NLEEs equations permit the unscrambling of mechanisms pertaining to compound nonlinear phenomena. This includes for instance spatial localization of transfer processes, multiplicity or non-appearance steady states under different conditions, and existence of peaking regimes. Even special exact solutions data that may seem not to have a clear physical meaning, can often be used as test problems to verify processes reliability, and help estimate errors of various numerical, asymptotic, and approximate analytical methods.

Exact solutions can also serve as a basis for perfecting and testing computer algebra software packages, designed for solving NLEEs. Furthermore, exact solutions allow researchers to design and run experiments, by creating appropriate natural conditions, to determine these parameters or functions. Therefore, investigations of exact traveling wave solutions are becoming increasingly attractive in nonlinear phenomena investigations and analyses. On the other hand, not all equations posed by the advent of NLEEs models are readily solvable. As a result, many original techniques have been successfully urbanized by various groups of researchers, such as the Cole-Hopf transformation method [1], the Miura transformation method [2], the Hirota's bilinear method [3], the $\exp(-\Phi(\eta))$ -expansion method [4]-[6], the Sumudu transform method [7]-[14], the Fan sub-equation method [15] [16], the spectral-homotopy analysis method [17] [18], the least-squares finite element scheme [19], the (G'/G) -expansion method [20]-[23], the improved (G'/G) -expansion method [24], the trial function method [25], the nonlinear transform method [26], the extended Tanh-function method [27] [28], and the novel (G'/G) -expansion method [29]-[34], homotopy analysis method [35], to name a few. The latter sequence of papers really constituted a ladder honed in the current wealth of repeated experimental and theoretical successes that sprang us to the work at hand, that we hope will greatly benefit the readership, towards the further understanding of NLEEs dynamics and solutions, and mechanisms for recognizing and classifying them.

The aim of this article is to demonstrate the efficiency of the novel (G'/G) -expansion method to exhibit exact solutions for NLEEs in mathematical physics via the $(1 + 1)$ -dimensional compound KdVB equation [36]. This equation can be thought of as a generalization of KdV-mKdV and Burgers equations, involving nonlinear dispersion and dissipation effects (see for instance [37]). Below, twenty-five solutions, stratified into four separate families are stratified for the $(1 + 1)$ -dimensional compound KdVB equation.

The article is arranged as follows: the novel (G'/G) -expansion method is discussed in Section 2, and directly applied in Section 3 to nonlinear evolution equations elaborated upon previously. This is ensued by a discussion of obtained solutions and physical interpretations revealing the dynamics modeled into the equations and sampled by graphical representations and comparisons with published literature results, [38], culminating into the final conclusions relegated to Section 4. A rich list of references is availed for the convenience of the paper flow and the benefit of the readers, and consequently any reflective feedback fostering further developments would be highly welcome.

2. The Novel (G'/G) -Expansion Method

Suppose the nonlinear evolution equation,

$$P(u, u_t, u_x, u_{tt}, u_{tx}, u_{xx}, \dots) = 0, \quad (1)$$

where, P , is a polynomial in the function, $u(x, t)$, and its partial derivatives. The main steps of the novel (G'/G) -expansion method are:

Step 1: Combining the real variables x and t by a complex variable ξ , we suppose that

$$u(x, t) = u(\xi), \text{ and } \xi = x \pm Vt \quad (2)$$

where V is the speed of the traveling wave. Equation (2) is then used to transform Equation (1) into an ODE for $u = u(\xi)$:

$$Q(u, u', u'', u''', \dots) = 0, \quad (3)$$

where Q is a function of $u(\xi)$ and its derivatives wherein prime stands for derivative with respect to ξ .

Step 2: Assuming that the solution of Equation (3) can be expressed in terms of powers in $\psi(\xi)$,

$$u(\xi) = \sum_{j=-N}^N \alpha_j (\psi(\xi))^j \quad (4)$$

where,

$$\psi(\xi) = (d + \Phi(\xi)) \quad (5)$$

with, $\Phi(\xi) = \frac{G'(\xi)}{G(\xi)}$.

Herein α_{-N} or α_N may be zero, but not simultaneously, the, α_j ($j=0, \pm 1, \pm 2, \dots, \pm N$) and d are constants to be determined later. The function, $G = G(\xi)$ satisfies the second order nonlinear ODE:

$$GG'' = \lambda GG' + \mu G^2 + \nu (G')^2, \quad (6)$$

where prime denotes differentiation with respect ξ .

Using the Hopf-Cole transformation, $\Phi(\xi) = \ln(G(\xi))_\xi = \frac{G'(\xi)}{G(\xi)}$, reduces the Equation (6) into Riccati equation,

$$\Phi'(\xi) = \mu + \lambda \Phi(\xi) + (\nu - 1) \Phi^2(\xi). \quad (7)$$

We like to recall that Equation (7) can exhibit a plethora of solutions in the number of twenty five as in (Zhu [39]).

Step 3: The value of the positive integer N can be determined by balancing the highest order linear terms with the nonlinear terms of the highest order come out in Equation (3). If the degree of $u(\xi)$ is $D[u(\xi)] = n$, then the degree of the other expressions will be as follows:

$$D\left[\frac{d^p u(\xi)}{d\xi^p}\right] = n + p, \quad D\left[u^p \left(\frac{d^q u(\xi)}{d\xi^q}\right)^s\right] = np + s(n + q).$$

Step 4: Substitute Equation (4) including Equations. (5) and (6) into Equation (3), we obtain polynomials in $\left(d + \frac{G'(\xi)}{G(\xi)}\right)^j$ and $\left(d + \frac{G'(\xi)}{G(\xi)}\right)^{-j}$, ($j=0, 1, 2, \dots, N$). Next we collect each coefficient of the resulted polynomials to zero, yields an over-determined set of algebraic equations for α_j , d and V .

Step 5: Suppose that the value of the constants can be obtained by solving the algebraic equations obtained in Step 4, then substituting the values of the constants together with the solutions of Equation (6), will yield new and comprehensive exact traveling wave solutions of the nonlinear evolution equation (1).

3. Application of the Novel (G'/G)-Expansion Method

Let us consider the (1 + 1)-dimensional compound KdVB equation,

$$u_t + \alpha u u_x + \beta u^2 u_x + \gamma u_{xx} - \delta u_{xxx} = 0. \quad (8)$$

Using the traveling wave transformation $\xi = x - Vt$, Equation (8) is converted into the following ODE:

$$-Vu' + \alpha uu' + \beta u^2 u' + \gamma u'' - \delta u''' = 0. \quad (9)$$

Integrating Equation (9), we obtain

$$C - Vu + \frac{1}{2} \alpha u^2 + \frac{1}{3} \beta u^3 + \gamma u' - \delta u'' = 0 \quad (10)$$

where C is a constant of integration. Inserting Equation (4) into Equation (10) and balancing the highest order derivative u'' with the nonlinear term of the highest order u^3 , we obtain $M = 1$.

Therefore, the solution of Equation (10) takes the form,

$$u(\xi) = \alpha_{-1} (\psi(\xi))^{-1} + \alpha_0 + \alpha_1 (\psi(\xi)). \quad (11)$$

Substituting Equation (11) into Equation (10), the left hand side is transformed into polynomials of $\left(d + \frac{G'(\xi)}{G(\xi)}\right)^j$ and $\left(d + \frac{G'(\xi)}{G(\xi)}\right)^{-j}$, ($j=0, 1, 2, \dots, M$). Equating the coefficients of like power of these polynomials to zero, we obtain an over-determine set of algebraic equations (for simplicity we leave out to display the equations) for α_0 , α_1 , α_{-1} , d , C and V . Solving the over-determined set of algebraic equations by using the symbolic computation software, such as Maple 13, we obtain

$$C = -\frac{\alpha\gamma^2}{12\delta\beta} - \frac{\alpha\delta\lambda^2}{4\beta} - \frac{\alpha\delta\mu}{\beta} + \frac{\alpha^3}{24\beta^2} + \frac{\delta\mu\nu}{\beta} \pm \frac{1}{9\delta\sqrt{6\delta\beta}} (36\delta^2\mu\gamma(1-\nu) - \gamma^3 + 9\delta^2\lambda^2\gamma) \quad (12)$$

$$\alpha_0 = -\frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\delta\beta}} (12\delta d(1-\nu) + 6\delta\lambda - 2\gamma), \quad V = 2\delta\mu(1-\nu) + \frac{\gamma^2}{6\delta} - \frac{\alpha^2}{4\beta} + \frac{\delta\lambda^2}{2},$$

$$\alpha_{-1} = 0, \quad d = d, \quad \alpha_1 = \pm \sqrt{\frac{6\delta}{\beta}} (\nu - 1).$$

Substituting Equation (12) into Equation (11), with $\xi = x - Vt$, d , λ , μ , δ , γ and ν , being arbitrary constants. We obtain

$$u(\xi) = -\frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\delta\beta}} (12\delta d(1-\nu) + 6\delta\lambda - 2\gamma) + \left\{ \pm \sqrt{\frac{6\delta}{\beta}} (\nu - 1) \right\} \times (d + (G'/G)). \quad (13)$$

Substituting the solutions $G(\xi)$ of the Equation (6) into Equation (13) and simplifying, we obtain the following solutions:

When $\Omega = \lambda^2 - 4\mu\nu + 4\mu > 0$ and $\lambda(\nu - 1) \neq 0$ (or $\mu(\nu - 1) \neq 0$),

$$u_1(x, t) = \left\{ \pm \sqrt{\frac{6\delta}{\beta}} (\nu - 1) \right\} \times \left\{ d - \frac{1}{2(\nu - 1)} \left(\lambda + \sqrt{\Omega} \tanh\left(\frac{1}{2}\sqrt{\Omega}\xi\right) \right) \right\} - \frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\delta\beta}} (12\delta d(1-\nu) + 6\delta\lambda - 2\gamma). \quad (14)$$

where $\xi = x - \left\{ 2\delta\mu(1-\nu) + \frac{\gamma^2}{6\delta} - \frac{\alpha^2}{4\beta} + \frac{\delta\lambda^2}{2} \right\} t$, and d , λ , μ , δ , γ and ν are arbitrary constants.

With A & B real constants, when occurring in expressions, the next solutions, are given by,

$$u_2(x, t) = \left\{ \pm \sqrt{\frac{6\delta}{\beta}} (\nu - 1) \right\} \times \left\{ d - \frac{1}{2(\nu - 1)} \left(\lambda + \sqrt{\Omega} \coth\left(\frac{1}{2}\sqrt{\Omega}\xi\right) \right) \right\} - \frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\delta\beta}} (12\delta d(1-\nu) + 6\delta\lambda - 2\gamma). \quad (15)$$

$$u_3(x, t) = \left\{ \pm \sqrt{\frac{6\delta}{\beta}} (\nu - 1) \right\} \times \left[d - \frac{1}{2(\nu - 1)} \left\{ \lambda + \sqrt{\Omega} \left(\tanh(\sqrt{\Omega}\xi) \pm i \operatorname{sech}(\sqrt{\Omega}\xi) \right) \right\} \right] - \frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\delta\beta}} (12\delta d(1-\nu) + 6\delta\lambda - 2\gamma). \quad (16)$$

$$u_4(x,t) = \left\{ \pm \sqrt{\frac{6\delta}{\beta}} (\nu-1) \right\} \times \left[d - \frac{1}{2(\nu-1)} \left\{ \lambda + \sqrt{\Omega} \left(\coth(\sqrt{\Omega} \xi) \pm \operatorname{csc} h(\sqrt{\Omega} \xi) \right) \right\} \right] - \frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\beta\delta}} (12\delta d(1-\nu) + 6\delta\lambda - 2\gamma). \quad (17)$$

$$u_5(x,t) = \left\{ \pm \sqrt{\frac{6\delta}{\beta}} (\nu-1) \right\} \times \left[d - \frac{1}{4(\nu-1)} \left\{ 2\lambda + \sqrt{\Omega} \left(\tanh\left(\frac{1}{4}\sqrt{\Omega} \xi\right) + \coth\left(\frac{1}{4}\sqrt{\Omega} \xi\right) \right) \right\} \right] - \frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\beta\delta}} (12\delta d(1-\nu) + 6\delta\lambda - 2\gamma). \quad (18)$$

$$u_6(x,t) = \left\{ \pm \sqrt{\frac{6\delta}{\beta}} (\nu-1) \right\} \times \left[d + \frac{1}{2(\nu-1)} \left\{ -\lambda + \frac{\pm\sqrt{\Omega(A^2+B^2)} - A\sqrt{\Omega} \cosh(\sqrt{\Omega} \xi)}{A \sinh(\sqrt{\Omega} \xi) + B} \right\} \right] - \frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\beta\delta}} (12\delta d(1-\nu) + 6\delta\lambda - 2\gamma). \quad (19)$$

$$u_7(x,t) = \left\{ \pm \sqrt{\frac{6\delta}{\beta}} (\nu-1) \right\} \times \left[d + \frac{1}{2(\nu-1)} \left\{ -\lambda + \frac{\pm\sqrt{\Omega(A^2+B^2)} + A\sqrt{\Omega} \cosh(\sqrt{\Omega} \xi)}{A \sinh(\sqrt{\Omega} \xi) + B} \right\} \right] - \frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\beta\delta}} (12\delta d(1-\nu) + 6\delta\lambda - 2\gamma). \quad (20)$$

$$u_8(x,t) = \left\{ \pm \sqrt{\frac{6\delta}{\beta}} (\nu-1) \right\} \times \left[d + \frac{2\mu \cosh\left(\frac{1}{2}\sqrt{\Omega} \xi\right)}{\sqrt{\Omega} \sinh\left(\frac{1}{2}\sqrt{\Omega} \xi\right) - \lambda \cosh\left(\frac{1}{2}\sqrt{\Omega} \xi\right)} \right] - \frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\beta\delta}} (12\delta d(1-\nu) + 6\delta\lambda - 2\gamma). \quad (21)$$

$$u_9(x,t) = \left\{ \pm \sqrt{\frac{6\delta}{\beta}} (\nu-1) \right\} \times \left[d + \frac{2\mu \sinh\left(\frac{1}{2}\sqrt{\Omega} \xi\right)}{\sqrt{\Omega} \cosh\left(\frac{1}{2}\sqrt{\Omega} \xi\right) - \lambda \sinh\left(\frac{1}{2}\sqrt{\Omega} \xi\right)} \right] - \frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\beta\delta}} (12\delta d(1-\nu) + 6\delta\lambda - 2\gamma). \quad (22)$$

$$u_{10}(x,t) = \left\{ \pm \sqrt{\frac{6\delta}{\beta}} (\nu-1) \right\} \times \left[d + \frac{2\mu \cosh(\sqrt{\Omega} \xi)}{\sqrt{\Omega} \sinh(\sqrt{\Omega} \xi) - \lambda \cosh(\sqrt{\Omega} \xi) \pm i\sqrt{\Omega}} \right] - \frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\beta\delta}} (12\delta d(1-\nu) + 6\delta\lambda - 2\gamma). \quad (23)$$

$$u_{11}(x,t) = \left\{ \pm \sqrt{\frac{6\delta}{\beta}} (\nu-1) \right\} \times \left[d + \frac{2\mu \sinh(\sqrt{\Omega} \xi)}{\sqrt{\Omega} \cosh(\sqrt{\Omega} \xi) - \lambda \sinh(\sqrt{\Omega} \xi) \pm \sqrt{\Omega}} \right] - \frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\beta\delta}} (12\delta d(1-\nu) + 6\delta\lambda - 2\gamma). \quad (24)$$

When $\Omega = \lambda^2 - 4\mu\nu + 4\mu < 0$ and $\lambda(\nu-1) \neq 0$ (or $\mu(\nu-1) \neq 0$),

$$u_{12}(x,t) = \left\{ \pm \sqrt{\frac{6\delta}{\beta}}(\nu-1) \right\} \times \left\{ d + \frac{1}{2(\nu-1)} \left(-\lambda + \sqrt{-\Omega} \tan\left(\frac{1}{2}\sqrt{-\Omega}\xi\right) \right) \right\} - \frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\beta\delta}}(12\delta d(1-\nu) + 6\delta\lambda - 2\gamma). \quad (25)$$

$$u_{13}(x,t) = \left\{ \pm \sqrt{\frac{6\delta}{\beta}}(\nu-1) \right\} \times \left\{ d - \frac{1}{2(\nu-1)} \left(\lambda + \sqrt{-\Omega} \cot\left(\frac{1}{2}\sqrt{-\Omega}\xi\right) \right) \right\} - \frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\beta\delta}}(12\delta d(1-\nu) + 6\delta\lambda - 2\gamma). \quad (26)$$

$$u_{14}(x,t) = \left\{ \pm \sqrt{\frac{6\delta}{\beta}}(\nu-1) \right\} \times \left[d + \frac{1}{2(\nu-1)} \left\{ -\lambda + \sqrt{-\Omega} \left(\tan(\sqrt{-\Omega}\xi) \pm \sec(\sqrt{-\Omega}\xi) \right) \right\} \right] - \frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\beta\delta}}(12\delta d(1-\nu) + 6\delta\lambda - 2\gamma). \quad (27)$$

$$u_{15}(x,t) = \left\{ \pm \sqrt{\frac{6\delta}{\beta}}(\nu-1) \right\} \times \left[d - \frac{1}{2(\nu-1)} \left\{ \lambda + \sqrt{-\Omega} \left(\cot(\sqrt{-\Omega}\xi) \pm \csc(\sqrt{-\Omega}\xi) \right) \right\} \right] - \frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\beta\delta}}(12\delta d(1-\nu) + 6\delta\lambda - 2\gamma). \quad (28)$$

$$u_{16}(x,t) = \left\{ \pm \sqrt{\frac{6\delta}{\beta}}(\nu-1) \right\} \times \left[d + \frac{1}{4(\nu-1)} \left\{ -2\lambda + \sqrt{-\Omega} \left(\tan\left(\frac{1}{4}\sqrt{-\Omega}\xi\right) - \cot\left(\frac{1}{4}\sqrt{-\Omega}\xi\right) \right) \right\} \right] - \frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\beta\delta}}(12\delta d(1-\nu) + 6\delta\lambda - 2\gamma). \quad (29)$$

$$u_{17}(x,t) = \left\{ \pm \sqrt{\frac{6\delta}{\beta}}(\nu-1) \right\} \times \left[d + \frac{1}{2(\nu-1)} \left\{ -\lambda + \frac{\pm\sqrt{-\Omega}(A^2 - B^2) - A\sqrt{-\Omega} \cos(\sqrt{-\Omega}\xi)}{A \sin(\sqrt{-\Omega}\xi) + B} \right\} \right] - \frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\beta\delta}}(12\delta d(1-\nu) + 6\delta\lambda - 2\gamma). \quad (30)$$

$$u_{18}(x,t) = \left\{ \pm \sqrt{\frac{6\delta}{\beta}}(\nu-1) \right\} \times \left[d + \frac{1}{2(\nu-1)} \left\{ -\lambda + \frac{\pm\sqrt{-\Omega}(A^2 - B^2) + A\sqrt{-\Omega} \cos(\sqrt{-\Omega}\xi)}{A \sin(\sqrt{-\Omega}\xi) + B} \right\} \right] - \frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\beta\delta}}(12\delta d(1-\nu) + 6\delta\lambda - 2\gamma). \quad (31)$$

where A and B are arbitrary constants such that $A^2 - B^2 > 0$.

$$u_{19}(x,t) = \left\{ \pm \sqrt{\frac{6\delta}{\beta}}(\nu-1) \right\} \times \left\{ d - \frac{2\mu \cos\left(\frac{1}{2}\sqrt{-\Omega}\xi\right)}{\sqrt{-\Omega} \sin\left(\frac{1}{2}\sqrt{-\Omega}\xi\right) + \lambda \cos\left(\frac{1}{2}\sqrt{-\Omega}\xi\right)} \right\} - \frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\beta\delta}}(12\delta d(1-\nu) + 6\delta\lambda - 2\gamma). \quad (32)$$

$$u_{20}(x,t) = \left\{ \pm \sqrt{\frac{6\delta}{\beta}} (\nu-1) \right\} \times \left\{ d + \frac{2\mu \sin\left(\frac{1}{2}\sqrt{-\Omega} \xi\right)}{\sqrt{-\Omega} \cos\left(\frac{1}{2}\sqrt{-\Omega} \xi\right) - \lambda \sin\left(\frac{1}{2}\sqrt{-\Omega} \xi\right)} \right\} - \frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\beta\delta}} (12\delta d(1-\nu) + 6\delta\lambda - 2\gamma). \quad (33)$$

$$u_{21}(x,t) = \left\{ \pm \sqrt{\frac{6\delta}{\beta}} (\nu-1) \right\} \times \left\{ d - \frac{2\mu \cos(\sqrt{-\Omega} \xi)}{\sqrt{-\Omega} \sin(\sqrt{-\Omega} \xi) + \lambda \cos(\sqrt{-\Omega} \xi) \pm \sqrt{-\Omega}} \right\} - \frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\beta\delta}} (12\delta d(1-\nu) + 6\delta\lambda - 2\gamma). \quad (34)$$

$$u_{22}(x,t) = \left\{ \pm \sqrt{\frac{6\delta}{\beta}} (\nu-1) \right\} \times \left\{ d + \frac{2\mu \sin\left(\frac{1}{2}\sqrt{-\Omega} \xi\right)}{\sqrt{-\Omega} \cos\left(\frac{1}{2}\sqrt{-\Omega} \xi\right) - \lambda \sin\left(\frac{1}{2}\sqrt{-\Omega} \xi\right) \pm \sqrt{-\Omega}} \right\} - \frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\beta\delta}} (12\delta d(1-\nu) + 6\delta\lambda - 2\gamma). \quad (35)$$

When $\mu = 0$ and $\lambda(\nu-1) \neq 0$, where k is an arbitrary constant.

$$u_{23}(x,t) = \left\{ \pm \sqrt{\frac{6\delta}{\beta}} (\nu-1) \right\} \times \left\{ d - \frac{\lambda k}{(\nu-1)\{k + \cosh(\lambda\xi) - \sinh(\lambda\xi)\}} \right\} - \frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\beta\delta}} (12\delta d(1-\nu) + 6\delta\lambda - 2\gamma). \quad (36)$$

$$u_{24}(x,t) = \left\{ \pm \sqrt{\frac{6\delta}{\beta}} (\nu-1) \right\} \times \left\{ d - \frac{\lambda \{\cosh(\lambda\xi) + \sinh(\lambda\xi)\}}{(\nu-1)\{k + \cosh(\lambda\xi) + \sinh(\lambda\xi)\}} \right\} - \frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\beta\delta}} (12\delta d(1-\nu) + 6\delta\lambda - 2\gamma) \quad (37)$$

where k is an arbitrary constant.

When $(\nu-1) \neq 0$ and $\lambda = \mu = 0$, the solution of Equation (8) is,

$$u_{25}(x,t) = \left\{ \pm \sqrt{\frac{6\delta}{\beta}} (\nu-1) \right\} \times \left\{ d - \frac{1}{(\nu-1)\xi + c_1} \right\} - \frac{\alpha}{2\beta} \pm \frac{1}{2\sqrt{6\beta\delta}} (12\delta d(1-\nu) + 6\delta\lambda - 2\gamma) \quad (38)$$

where c_1 is an arbitrary constant.

4. Discussions and Conclusions

Here, we first discuss physical interpretation, and graphical representation of four families of solutions determined above. The introduction of dispersion without introducing nonlinearity destroys the solitary wave as different Fourier harmonics start propagating at different group velocities. On the other hand, introducing nonlinearity without dispersion also prevents the formation of solitary waves, because the pulse energy is frequently pumped into higher frequency modes. However, if both dispersion and nonlinearity are present, solitary waves can be sustained. Similarly to dispersion, dissipation can also give rise to solitary waves when combined with

nonlinearity. Hence, it is interesting to point out the delicate balance between the nonlinearity effect of uu_x and u^2u_x , and the dissipative effect of u_{xx} . The u_{xxx} gives rise to solitons that after a fully interaction with others the solitons come back retaining their identities with the same speed and shape.

The $(1 + 1)$ -dimensional compound KdVB equation has solitary wave solutions that have exponentially decaying wings. If two solitons of the $(1 + 1)$ -dimensional compound KdVB equation collide, the solitons just pass through each other and emerge unchanged. For special values of the parameters solitary wave solutions are originated from the obtained exact solutions.

Figure 1: Kink solution, shape of Equation (14) when $\delta = 2$, $\gamma = 1$, $\alpha = 1$, $\beta = 1$, $\lambda = 1$, $\mu = -1$, $\nu = 2$, $d = 1$ and $-10 \leq x, t \leq 10$. Solution of Equations (14), (16), (19), (22), (23), (36) and (37) represents kink. Kink waves are traveling waves which arise from one asymptotic state to another. The kink solutions are approach to a constant at infinity. Other figures are omitted for convenience.

Figure 2: Single soliton, shape of Equation (15) when $\delta = 2$, $\gamma = 1$, $\alpha = 1$, $\beta = 1$, $\lambda = 1$, $\mu = -1$, $\nu = 2$, $d = 1$ and $-10 \leq x, t \leq 10$. Solution of Equations (15), (17), (18), (26), (28) and (29) represents single soliton. Solitons are special kinds of solitary waves. The soliton solution is a specially localized solution, hence $u'(\xi)$, $u''(\xi)$, $u'''(\xi) \rightarrow 0$ as $\xi \rightarrow \pm\infty$, $\xi = x - ct$. Solitons have a remarkable property that it keeps its identity upon interacting with other solitons. Other figures are omitted for convenience.

Figure 3: Modulus plot of periodic wave solutions, shape of Equation (25) when $\delta = 2$, $\gamma = 1$, $\alpha = 1$, $\beta = 2$, $\lambda = 1$, $\mu = 1$, $\nu = 2$, $d = 1$ and $-1 \leq x, t \leq 1$. Solution of Equations (25), (27), (30)-(35) represents periodic wave solutions. Periodic solutions are traveling wave solutions that are periodic such as $\cos(x-t)$. Other figures are omitted for convenience.

Figure 4: Singular kink solution, shape of Equation (38) when $\delta = 2$, $\gamma = 1$, $\alpha = 1$, $\beta = 2$, $\lambda = 1$, $\mu = 1$, $\nu = 2$, $d = 1$ and $-10 \leq x, t \leq 10$. Solution of Equations (20), (21), (24) and (38) represents singular kink solutions. Other figures are omitted for convenience.

The graphical illustrations of the solutions are depicted in the **Figures 1-4** with the aid of commercial software Maple.

Comparison between Zayed [38] solutions and our solutions: Zayed [38] considered solutions of the $(1 + 1)$ -dimensional compound KdVB equation using the basic (G'/G) -expansion method combined with the Riccati equation. The solutions of the $(1 + 1)$ -dimensional compound KdVB equation obtained by the novel (G'/G) -expansion method are different from those of the basic (G'/G) -expansion method combined with the Riccati equation. Moreover, Zayed [38] investigated the well-established $(1 + 1)$ -dimensional compound KdVB equation to obtain exact solutions via the basic (G'/G) -expansion method and achieved only five solutions (see Appendix). Furthermore, twenty-five solutions of the well-known $(1 + 1)$ -dimensional compound KdVB equation are constructed by applying the novel (G'/G) -expansion method. On the other hand, the auxiliary equation used in this paper is different, so obtained solutions are also different. Similarly for any nonlinear evolution equation it can be shown that the novel (G'/G) -expansion method is much easier than other methods.

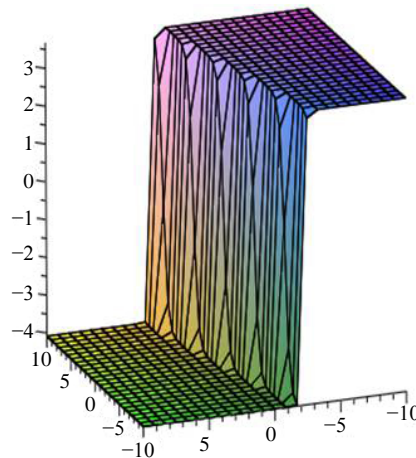


Figure 1. Kink solution, shape of Equation (14) when $\delta = 2$, $\gamma = 1$, $\alpha = 1$, $\beta = 1$, $\lambda = 1$, $\mu = -1$, $\nu = 2$, $d = 1$ and $-10 \leq x, t \leq 10$.

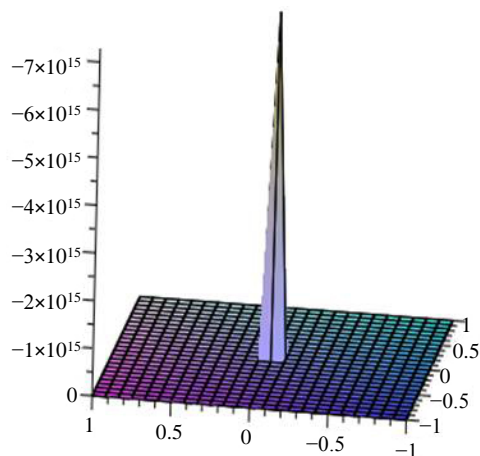


Figure 2. Single soliton, shape of Equation (15) when $\delta = 2, \gamma = 1, \alpha = 1, \beta = 1, \lambda = 1, \mu = -1, \nu = 2, d = 1$ and $-10 \leq x, t \leq 10$.

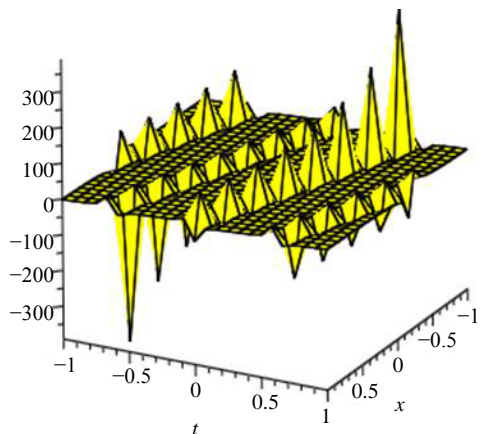


Figure 3. Modulus plot of periodic wave solutions, shape of Equation (25) when $\delta = 2, \gamma = 1, \alpha = 1, \beta = 2, \lambda = 1, \mu = -1$, and $-1 \leq x, t \leq 1$.

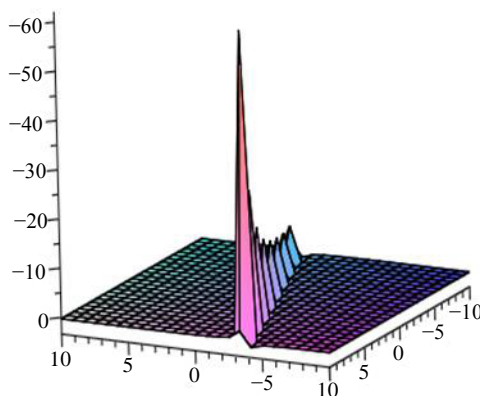


Figure 4. Singular kink solution, shape of Equation (38) when $\delta = 2, \gamma = 1, \alpha = 1, \beta = 2, \lambda = 1, \mu = 1, \nu = 2, d = 1$ and $-10 \leq x, t \leq 10$.

In this paper, the novel (G'/G) -expansion method has been successfully applied to find the traveling wave solutions for the $(1 + 1)$ -dimensional compound KdVB equation. The method has been used to find new exact solutions. As a result, hyperbolic function solutions, rational function solutions and trigonometric function solutions with several free parameters have been obtained. The solutions gotten with tuned parameters may be used to detect and explain complex physical phenomena. Results in this paper show that the devised algorithm is highly effective, and can be used for many other NLEEs in mathematical physics. Thus, we can say that the novel (G'/G) -expansion method can be extended to solve the problems of nonlinear evolution equations arising in the theory of solitons, and other nonlinear sciences. Consequently, thus far not only were we successful at solving the KdVB equation, but also, we contributed to establishing the novel (G'/G) -expansion method as a versatile and efficient tool that can be applied to NLEEs.

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Appendix. Zayed Solutions [38]

Zayed [38] examined the exact solutions of the nonlinear $(1 + 1)$ -dimensional compound KdVB equation by using the (G'/G) -expansion method. They found the following five solutions of the form,

$$u(\xi) = -\frac{\alpha}{2\beta} - \sqrt{\frac{6\delta}{\beta}} i \operatorname{sech} \xi. \quad (\text{A.1})$$

$$u(\xi) = -\frac{\alpha}{2\beta} + \sqrt{\frac{6\delta}{\beta}} \sec \xi. \quad (\text{A.2})$$

$$u(\xi) = -\frac{\alpha}{2\beta} \pm \sqrt{\frac{6\delta}{\beta}} (\coth \xi - \tanh \xi). \quad (\text{A.3})$$

$$u(\xi) = -\frac{\alpha}{2\beta} \pm \sqrt{\frac{6\delta}{\beta}} (\cot \xi + \tan \xi). \quad (\text{A.4})$$

$$u(\xi) = -\frac{\alpha}{2\beta} \pm \sqrt{\frac{6\delta}{\beta}} \left(\frac{B}{B\xi + c_1} \right). \quad (\text{A.5})$$