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The Global Attractors for a Nonlinear Viscoelastic Wave Equation with Strong Damping and Linear Damping and Source Terms

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Abstract

In this paper, firstly, some priori estimates are obtained for the existence and uniqueness of solutions of a nonlinear viscoelastic wave equation with strong damping, linear damping and source terms. Then we study the global attractors of the equation.

Keywords

Global Attractors, Viscoelastic Equation, Priori Estimates

1. Introduction

We know that viscoelastic materials have memory effects. These properties are due to the mechanical response influenced by the history of the materials. As these materials have a wide application in the natural science, their dynamics are of great importance and interest. The memory effects can be modeled by a partial differential equation. In recent years, the behaviors of solutions for the PDE system have been studied extensively, and many achievements have been obtained. Many authors have focused on the problem of existence, decay and blow-up for the last two decades, see [1]-[5]. And the attractors are still important contents that are studied.

In [6], R.O. Araújo, T. Ma and Y.M. Qin studied the following equation

$$|u_t|^\rho u_{tt} - \Delta u - \Delta u_{tt} + \int_0^{+\infty} g(s)\Delta u(t-s)\mathrm{d}s + f(u) = h(x) \tag{1.1}$$

and they proved the global existence, uniqueness and exponential stability of solutions and existence of the

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global attractor.

In [7], Y.M. Qin, B.W. Feng and M. Zhang considered the following initial-boundary value problem:

$$\begin{cases} |u_t|^{\rho} u_{tt} - \Delta u - \Delta u_{tt} + \int_0^{+\infty} g(s)\Delta u(t-s)\,\mathrm{d}s + u_t = \sigma(x,t),\ x \in \Omega, t > \tau \\ u(x,t) = 0,\ x \in \partial\Omega,\ t \geq \tau \\ u(x,\tau) = u_0^{\tau}(x),\ u_t(x,\tau) = u_1^{\tau}(x),\ u(x,t) = u_{\tau}(x,t),\ x \in \Omega, \tau \in R^{+} \end{cases} \quad (1.2)$$

where Ω is a bounded domain of $R^n\ (n \geq 1)$ with a smooth boundary $\partial\Omega$, $u_{\tau}(x,t)$ (the past history of *u*) is a given datum which has to be known for all $t \leq \tau$, the function *g* represents the kernel of a memory, $\sigma = \sigma(x,t)$ is a non-autonomous term, called a symbol, and ρ is a real number such that $1 < \rho \leq \frac{2}{n-2}$ if $n \geq 3$; $\rho > 1$ if $n = 1, 2$. They proved the existence of uniform attractors for a non-autonomous viscoelastic equation with a past history. For more related results, we refer the reader to [8]-[14].

In this work, we intend to study the following initial-boundary problem:

$$\begin{cases} u_{tt} - \Delta u + \int_0^{+\infty} g(s)\Delta u(t-s)\,\mathrm{d}s - \varepsilon_1 \Delta u_t + \varepsilon_2 u_t + \varepsilon_3 |u|^{p-2} u = f(x), \quad x \in \Omega, t > 0 \\ u(x,t) = 0, \quad x \in \partial\Omega, t \geq 0 \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in \Omega \end{cases} \quad (1.3)$$

where $\varepsilon_1, \varepsilon_2, \varepsilon_3 \geq 0$, and $\Omega \subset R^n\ (n \geq 1)$ is a bounded domain with smooth boundary $\partial\Omega$, $2 < p < \min\left\{\frac{2n}{n-2}, \frac{2n+4}{n}\right\}$ if $n \geq 3$; $p > 2$ if $n = 1, 2$, for the problem (1.3), the memory term $\int_0^{+\infty} g(s)\Delta u(t-s)\,\mathrm{d}s$ replaces $\int_0^{t} g(t-s)\Delta u(s)\,\mathrm{d}s$, and we consider the strong damping term $-\varepsilon_1 \Delta u_t$, the linear damping term $\varepsilon_2 u_t$ and source terms $\varepsilon_3 |u|^{p-2} u$. We define

$$\eta = \eta(s) = \eta^t(x,s) = u(x,t) - u(x,t-s)$$

A direct computation yields

$$\eta_t(s) = -\eta_s(s) + u_t(t)$$

Thus, the original memory term can be written as

$$\int_0^{+\infty} g(s)\Delta u(t-s)\,\mathrm{d}s = \int_0^{+\infty} g(s)\,\mathrm{d}s \cdot \Delta u - \int_0^{+\infty} g(s)\Delta\eta(s)\,\mathrm{d}s$$

and we get a new system

$$u_{tt} - \left(1 - \int_0^{+\infty} g(s)\,\mathrm{d}s\right)\Delta u - \varepsilon_1 \Delta u_t - \int_0^{+\infty} g(s)\Delta\eta(s)\,\mathrm{d}s + \varepsilon_2 u_t + \varepsilon_3 |u|^{p-2} u = f(x) \quad (1.4)$$

$$\eta_t = -\eta_s + u_t \quad (1.5)$$

with the initial conditions

$$u(x,0) = u_0(x),\ u_t(x,0) = u_1(x),\ \eta(0) = \eta^t(x,0) = 0,\ x \in \Omega \quad (1.6)$$

and the boundary conditions

$$u(x,t) = 0,\ x \in \partial\Omega, t \geq 0 \quad (1.7)$$

The rest of this paper is organized as follows. In Section 2, we first obtain the priori estimates, then in Section 3, we prove the existence of the global attractors.

For convenience, we denote the norm and scalar product in $L^2(\Omega)$ by $\|\cdot\|$ and $(\cdot,\cdot)$, let $V=H^1(\Omega)$, $D(A)=H^2(\Omega)$.

2. The Priori Estimates of Solution of Equation

In this section, we present some materials needed in the proof of our results, state a global existence result, and prove our main result. For this reason, we assume that

(G1) $g:R^+\to R^+$ is a differentiable function satisfying $1-\int_0^{+\infty} g(s)\mathrm{d}s=l>0$;

(G2) $g(s)\ge 0, g'(s)\le 0, \forall s\in R^+$;

(G3) There exists a constant $\xi>0$ such that $g'(s)+\xi g(s)\le 0$, $\forall s\in R^+$;

Lemma 1. Assume (G1), (G2) and (G3) hold, let

$$\begin{cases} 2<p<\dfrac{2n}{n-2}, & n\ge 3 \\ p\ge 2, & n=1,2 \end{cases}$$

and $(u_0,u_1)\in H_0^1(\Omega)\times L^2(\Omega)$, $f\in L^2(\Omega)$, $v=u_t+\varepsilon u$, then the solution (u,v) of Equation (1.3) satisfies $(u,v)\in H_0^1(\Omega)\times L^2(\Omega)$ and

$$\|(u,v)\|^2_{H_0^1\times L^2}=\|\nabla u\|^2+\|v\|_2^2\le \frac{W(0)}{k}\mathrm{e}^{-\alpha_1 t}+\frac{C_1}{\alpha_1 k}\left(1-\mathrm{e}^{-\alpha_1 t}\right) \tag{2.1}$$

here $W(0)=\|v_0\|_2^2+(l-\varepsilon_1\varepsilon)\|\nabla u_0\|_2^2+\frac{2\varepsilon_3}{p}\|u_0\|_p^p$, thus there exists E_0 and $t_1=t_1(\Omega)>0$, such that

$$\|(u,v)\|^2_{H_0^1\times L^2}=\|\nabla u(t)\|_2^2+\|v(t)\|_2^2\le E_0\,(t>t_1) \tag{2.2}$$

Proof. We multiply $v=u_t+\varepsilon u$ with both sides of equation and obtain

$$\left(u_{tt}-\left(1-\int_0^{+\infty} g(s)\mathrm{d}s\right)\Delta u-\varepsilon_1\Delta u_t-\int_0^{+\infty} g(s)\Delta\eta(s)\mathrm{d}s+\varepsilon_2 u_t+\varepsilon_3|u|^{p-2}u,v\right)=(f,v)$$

By using Holder inequality, Young's inequality and Poincare inequality, we get

$$\begin{aligned}(u_{tt},v)&=(v_t-\varepsilon u_t,v)=\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|v\|_2^2-\varepsilon(u_t,v)=\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|v\|_2^2-\varepsilon(v-\varepsilon u,v)\\ &=\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|v\|_2^2-\varepsilon\|v\|_2^2+\varepsilon^2(u,v)\ge\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|v\|_2^2-\varepsilon\|v\|_2^2-\frac{\varepsilon^2}{2}\|u\|_2^2-\frac{\varepsilon^2}{2}\|v\|_2^2\\ &\ge\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|v\|_2^2-\varepsilon\|v\|_2^2-\frac{\varepsilon^2}{2\lambda_1}\|\nabla u\|_2^2-\frac{\varepsilon^2}{2}\|v\|_2^2,\end{aligned} \tag{2.3}$$

and

$$\left(-\left(1-\int_0^{+\infty} g(s)\mathrm{d}s\right)\Delta u,v\right)=-l(\Delta u,u_t+\varepsilon u)=\frac{l}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla u\|_2^2+l\varepsilon\|\nabla u\|_2^2 \tag{2.4}$$

and

$$\left(-\int_0^{+\infty} g(s)\Delta\eta(s)\mathrm{d}s,v\right)=\left(-\int_0^{+\infty} g(s)\Delta\eta(s)\mathrm{d}s,u_t\right)+\left(-\int_0^{+\infty} g(s)\Delta\eta(s)\mathrm{d}s,\varepsilon u\right) \tag{2.5}$$

For the first term on the right side (2.5), by using (G1), (G2) and (G3), we have

$$\begin{aligned}\int_0^{+\infty} g(s)\int_\Omega\nabla\eta(s)\cdot\nabla u_t\mathrm{d}x\mathrm{d}s&=\int_0^{+\infty} g(s)\int_\Omega\nabla\eta(s)\cdot(\nabla\eta_t+\nabla\eta_s)\mathrm{d}x\mathrm{d}s\\ &=\int_0^{+\infty} g(s)\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla\eta\|_2^2\mathrm{d}s+\int_0^{+\infty} g(s)\mathrm{d}\frac{1}{2}\|\nabla\eta\|_2^2\\ &\ge\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\eta\|_{g,V}^2+\frac{\xi}{2}\|\eta\|_{g,V}^2,\end{aligned} \tag{2.6}$$

where

$$\|\eta\|_{g,V}^2 = \int_0^{+\infty} g(s)\|\nabla\eta(s)\|_2^2\,\mathrm{d}s \tag{2.7}$$

For the second term on the right side (2.5), by using Holder inequality and Young's inequality, we get

$$\begin{aligned}\left(-\int_0^{+\infty} g(s)\Delta\eta(s)\mathrm{d}s, \varepsilon u\right) &= \varepsilon\int_0^{+\infty} g(s)\int_\Omega \nabla\eta(s)\nabla u\mathrm{d}x\mathrm{d}s \\ &\ge -\frac{\xi}{4}\|\eta\|_{g,V}^2 - \frac{\varepsilon^2}{\xi}\int_0^{+\infty} g(s)\mathrm{d}s\|\nabla u\|_2^2\end{aligned} \tag{2.8}$$

So, we have

$$\begin{aligned}\left(-\int_0^{+\infty} g(s)\Delta\eta(s)\mathrm{d}s, v\right) &= \left(-\int_0^{+\infty} g(s)\Delta\eta(s)\mathrm{d}s, u_t\right) + \left(-\int_0^{+\infty} g(s)\Delta\eta(s)\mathrm{d}s, \varepsilon u\right) \\ &\ge \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\eta\|_{g,V}^2 + \frac{\xi}{4}\|\eta\|_{g,V}^2 - \frac{\varepsilon^2}{\xi}\int_0^{+\infty} g(s)\mathrm{d}s\|\nabla u\|_2^2\end{aligned} \tag{2.9}$$

By using Poincare inequality, we obtain

$$\begin{aligned}(-\varepsilon_1\Delta u_t, v) &= \varepsilon_1(-\Delta v + \varepsilon\Delta u, v) = \varepsilon_1\|\nabla v\|_2^2 + \varepsilon_1\varepsilon(\Delta u, u_t + \varepsilon u) \\ &= \varepsilon_1\|\nabla v\|_2^2 - \frac{\varepsilon_1\varepsilon}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla u\|_2^2 - \varepsilon_1\varepsilon^2\|\nabla u\|_2^2 \\ &\ge \varepsilon_1\lambda_1\|v\|_2^2 - \frac{\varepsilon_1\varepsilon}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla u\|_2^2 - \varepsilon_1\varepsilon^2\|\nabla u\|_2^2\end{aligned} \tag{2.10}$$

and

$$\begin{aligned}(\varepsilon_2 u_t, v) &= \varepsilon_2(u_t, u_t + \varepsilon u) = \varepsilon_2\|u_t\|_2^2 - \varepsilon_2\varepsilon\|u\|_2\|u_t\|_2 \\ &\ge \varepsilon_2\|u_t\|_2^2 - \frac{\varepsilon_2\varepsilon}{2}\|u\|_2^2 - \frac{\varepsilon_2\varepsilon}{2}\|u_t\|_2^2 \\ &\ge \varepsilon_2\left(1 - \frac{\varepsilon}{2}\right)\|u_t\|_2^2 - \frac{\varepsilon_2\varepsilon}{2\lambda_1}\|\nabla u\|_2^2\end{aligned} \tag{2.11}$$

and

$$\left(\varepsilon_3|u|^{p-2}u, v\right) = \varepsilon_3\left(|u|^{p-2}u, u_t + \varepsilon u\right) = \frac{\varepsilon_3}{p}\frac{\mathrm{d}}{\mathrm{d}t}\|u\|_p^p + \varepsilon_3\varepsilon\|u\|_p^p \tag{2.12}$$

By using Holder inequality and Young's inequality, we obtain

$$(f(x), v) \le \|f\|\cdot\|v\| \le \frac{\lambda_1}{2}\|v\|_2^2 + \frac{1}{2\lambda_1}\|f\|_2^2 \tag{2.13}$$

Then, we have

$$\begin{aligned}&\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|v\|_2^2 - \varepsilon\|v\|_2^2 - \frac{\varepsilon^2}{2\lambda_1}\|\nabla u\|_2^2 - \frac{\varepsilon^2}{2}\|v\|_2^2 + \frac{l}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla u\|_2^2 + l\varepsilon\|\nabla u\|_2^2 + \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\eta\|_{g,V}^2 \\ &+\frac{\xi}{4}\|\eta\|_{g,V}^2 - \frac{\varepsilon^2}{\xi}\int_0^{+\infty} g(s)\mathrm{d}s\|\nabla u\|_2^2 + \varepsilon_1\lambda_1\|v\|_2^2 - \frac{\varepsilon_1\varepsilon}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla u\|_2^2 - \varepsilon_1\varepsilon^2\|\nabla u\|_2^2 \\ &+\varepsilon_2\left(1 - \frac{\varepsilon}{2}\right)\|u_t\|_2^2 - \frac{\varepsilon_2\varepsilon}{2\lambda_1}\|\nabla u\|_2^2 + \frac{\varepsilon_3}{p}\frac{\mathrm{d}}{\mathrm{d}t}\|u\|_p^p + \varepsilon_3\varepsilon\|u\|_p^p \\ &\le \frac{\lambda_1}{2}\|v\|_2^2 + \frac{1}{2\lambda_1}\|f\|_2^2\end{aligned} \tag{2.14}$$

That is

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\|v\|_2^2+(l-\varepsilon_1\varepsilon)\|\nabla u\|_2^2+\|\eta\|_{g,V}^2+\frac{2\varepsilon_3}{p}\|u\|_p^p\right]+\left(2\varepsilon_1\lambda_1-2\varepsilon-\varepsilon^2-\lambda_1\right)\|v\|_2^2$$
$$+2\varepsilon\left(l-\varepsilon_1\varepsilon-\frac{\varepsilon}{2\lambda_1}-\frac{\varepsilon}{\xi}\int_0^{+\infty}g(s)\mathrm{d}s-\frac{\varepsilon_2}{2\lambda_1}\right)\|\nabla u\|_2^2+\frac{\xi}{2}\|\eta\|_{g,V}^2+2\varepsilon_3\varepsilon\|u\|_p^p \tag{2.15}$$
$$+2\varepsilon_2\left[1-\frac{\varepsilon}{2}\right]\|u_t\|_2^2\le\frac{1}{\lambda_1}\|f\|_2^2$$

Next, we take proper $\varepsilon,\varepsilon_1,\varepsilon_2,\varepsilon_3$, such that

$$\begin{cases} a_1=2\varepsilon_1\lambda_1-2\varepsilon-\varepsilon^2-\lambda_1\ge 0\\ a_2=2\varepsilon\left(l-\varepsilon_1\varepsilon-\dfrac{\varepsilon}{2\lambda_1}-\dfrac{\varepsilon}{\xi}\displaystyle\int_0^{+\infty}g(s)\mathrm{d}s-\dfrac{\varepsilon_2}{2\lambda_1}\right)\ge 0\\ a_3=2\varepsilon_2\left(1-\dfrac{\varepsilon}{2}\right)\ge 0\end{cases} \tag{2.16}$$

Taking $\alpha_1=\min\left\{a_1,\frac{a_2}{l-\varepsilon_1\varepsilon},\frac{\xi}{2},p\varepsilon\right\}$, then

$$\frac{\mathrm{d}}{\mathrm{d}t}W(t)+\alpha_1 W(t)\le\frac{1}{\lambda_1}\|f\|_2^2:=C_1 \tag{2.17}$$

where $W(t)=\|v\|_2^2+(l-\varepsilon_1\varepsilon)\|\nabla u\|_2^2+\|\eta\|_{g,V}^2+\frac{2\varepsilon_3}{p}\|u\|_p^p$, by using Gronwall inequality, we obtain

$$W(t)\le W(0)\mathrm{e}^{-\alpha_1 t}+\frac{C_1}{\alpha_1}\left(1-\mathrm{e}^{-\alpha_1 t}\right) \tag{2.18}$$

From $2<p<\frac{2n}{n-2},n\ge 3$, according to Embedding Theorem then $H_0^1(\Omega)\subset L^p(\Omega)$, let $k=\min\{1,(l-\varepsilon_1\varepsilon)\}$, so we have

$$\|(u,v)\|_{H_0^1\times L^2}^2=\|\nabla u\|^2+\|v\|_2^2\le\frac{W(0)}{k}\mathrm{e}^{-\alpha_1 t}+\frac{C_1}{\alpha_1 k}\left(1-\mathrm{e}^{-\alpha_1 t}\right)$$

Then

$$\overline{\lim_{t\to\infty}}\|(u,v)\|_{H_0^1\times L^2}^2\le\frac{C_1}{\alpha_1 k}$$

So, there exists E_0 and $t_1=t_1(\Omega)>0$, such that

$$\|(u,v)\|_{H_0^1\times L^2}^2=\|\nabla u(t)\|_2^2+\|v\|_2^2\le E_0\ (t>t_1)$$

Lemma 2. Assume (G1), (G2) and (G3) hold, let

$$\begin{cases} 2<p<\dfrac{2n+4}{n}, & n\ge 3\\ p\ge 2, & n=1,2\end{cases}$$

and $(u_0,u_1)\in H^2(\Omega)\times H^1(\Omega)$, $f\in H^1(\Omega)$, $v=u_t+\varepsilon u$, then the solution (u,v) of Equation (1.3) satisfies $(u,v)\in H^2(\Omega)\times H^1(\Omega)$ and

$$\|(u,v)\|^2_{H_0^2\times H^1}=\|\Delta u\|_2^2+\|\nabla v\|_2^2\le\frac{W(0)}{k}\mathrm{e}^{-\alpha_2 t}+\frac{C_2}{\alpha_2 k}\left(1-\mathrm{e}^{-\alpha_2 t}\right)\tag{2.19}$$

Here $V(0)=\|\nabla u_1+\nabla u_0\|_2^2+(l-\varepsilon_1\varepsilon)\|\Delta u_0\|_2^2$, thus there exists E_1 and $t_2=t_2(\Omega)>0$, such that

$$\|(u,v)\|^2_{H^2\times H^1}=\|\Delta u(t)\|_2^2+\|\nabla v(t)\|_2^2\le E_1\quad(t>t_2)\tag{2.20}$$

Proof. We multiply $-\Delta v=-\Delta u_t-\varepsilon\Delta u$ with both sides of equation and obtain

$$\left(u_{tt}-\left(1-\int_0^{+\infty}g(s)\mathrm{d}s\right)\Delta u-\varepsilon_1\Delta u_t-\int_0^{+\infty}g(s)\Delta\eta(s)\mathrm{d}s+\varepsilon_2 u_t+\varepsilon_3|u|^{p-2}u,-\Delta v\right)=(f,-\Delta v)\tag{2.21}$$

By using Holder inequality, Young's inequality and Poincare inequality, we get

$$\begin{aligned}(u_{tt},-\Delta v)&=(v_t-\varepsilon u_t,-\Delta v)=\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla v\|_2^2-\varepsilon(u_t,-\Delta v)\\&=\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla v\|_2^2-\varepsilon(v-\varepsilon u,-\Delta v)\\&=\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla v\|_2^2-\varepsilon\|\nabla v\|_2^2+\varepsilon^2(\nabla u,\nabla v)\\&\ge\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla v\|_2^2-\varepsilon\|\nabla v\|_2^2-\frac{\varepsilon^2}{2}\|\nabla u\|_2^2-\frac{\varepsilon^2}{2}\|\nabla v\|_2^2\\&\ge\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla v\|_2^2-\varepsilon\|\nabla v\|_2^2-\frac{\varepsilon^2}{2\lambda_1}\|\Delta u\|_2^2-\frac{\varepsilon^2}{2}\|\nabla v\|_2^2\end{aligned}$$

and

$$\left(-\left(1-\int_0^{+\infty}g(s)\mathrm{d}s\right)\Delta u,-\Delta v\right)=-l(\Delta u,-\Delta u_t-\varepsilon\Delta u)=\frac{l}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\Delta u\|_2^2+l\varepsilon\|\Delta u\|_2^2\tag{2.22}$$

and

$$\left(-\int_0^{+\infty}g(s)\Delta\eta(s)\mathrm{d}s,-\Delta v\right)=\left(-\int_0^{+\infty}g(s)\Delta\eta(s)\mathrm{d}s,-\Delta u_t\right)+\left(-\int_0^{+\infty}g(s)\Delta\eta(s)\mathrm{d}s,-\varepsilon\Delta u\right)\tag{2.23}$$

For the first term on the right side (2.23), by using (G1), (G2) and (G3), we have

$$\begin{aligned}\int_0^{+\infty}g(s)\int_\Omega\Delta\eta(s)\cdot\Delta u_t\mathrm{d}x\mathrm{d}s&=\int_0^{+\infty}g(s)\int_\Omega\Delta\eta(s)\cdot(\Delta\eta_t+\Delta\eta_s)\mathrm{d}x\mathrm{d}s\\&=\int_0^{+\infty}g(s)\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\Delta\eta\|_2^2\mathrm{d}s+\int_0^{+\infty}g(s)\mathrm{d}\frac{1}{2}\|\Delta\eta\|_2^2\\&\ge\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\eta\|^2_{g,D(A)}+\frac{\xi}{2}\|\eta\|^2_{g,D(A)}\end{aligned}\tag{2.24}$$

where

$$\|\eta\|^2_{g,D(A)}=\int_0^{+\infty}g(s)\|\Delta\eta(s)\|_2^2\mathrm{d}s\tag{2.25}$$

For the second term on the right side (2.23), by using Holder inequality and Young's inequality, we get

$$\begin{aligned}\left(-\int_0^{+\infty}g(s)\Delta\eta(s)\mathrm{d}s,-\varepsilon\Delta u\right)&=\varepsilon\int_0^{+\infty}g(s)\int_\Omega\Delta\eta(s)\Delta u\mathrm{d}x\mathrm{d}s\\&\ge-\frac{\xi}{4}\|\eta\|^2_{g,D(A)}-\frac{\varepsilon^2}{\xi}\int_0^{+\infty}g(s)\mathrm{d}s\|\Delta u\|_2^2\end{aligned}\tag{2.26}$$

so, we have

$$\left(-\int_0^{+\infty} g(s)\Delta\eta(s)\mathrm{d}s, -\Delta v\right)$$
$$=\left(-\int_0^{+\infty} g(s)\Delta\eta(s)\mathrm{d}s, -\Delta u_t\right)+\left(-\int_0^{+\infty} g(s)\Delta\eta(s)\mathrm{d}s, -\varepsilon\Delta u\right)$$
$$=\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\eta\|_{g,D(A)}^2+\frac{\xi}{4}\|\eta\|_{g,D(A)}^2-\frac{\varepsilon^2}{\xi}\int_0^{+\infty} g(s)\mathrm{d}s\|\Delta u\|_2^2$$

By using Poincare inequality, we have

$$\begin{aligned}(-\varepsilon_1\Delta u_t, -\Delta v) &= \varepsilon_1(-\Delta v+\varepsilon\Delta u, -\Delta v)=\varepsilon_1\|\Delta v\|_2^2+\varepsilon_1\varepsilon(\Delta u, -\Delta u_t-\varepsilon\Delta u)\\ &= \varepsilon_1\|\Delta v\|_2^2-\frac{\varepsilon_1\varepsilon}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\Delta u\|_2^2-\varepsilon_1\varepsilon^2\|\Delta u\|_2^2,\end{aligned} \tag{2.27}$$

and

$$\begin{aligned}(\varepsilon_2 u_t, -\Delta v) &= \varepsilon_2(u_t, -\Delta u_t-\varepsilon\Delta u)=\varepsilon_2\|\nabla u_t\|_2^2-\varepsilon_2\varepsilon\|\nabla u\|_2\|\nabla u_t\|_2\\ &\geq \varepsilon_2\left(1-\frac{\varepsilon}{2}\right)\|\nabla u_t\|_2^2-\frac{\varepsilon_2\varepsilon}{2\lambda_1}\|\Delta u\|_2^2\end{aligned} \tag{2.28}$$

And using Interpolation Theorem, we have

$$\begin{aligned}\left(\varepsilon_3|u|^{p-2}u, -\Delta v\right) &\leq \varepsilon_3\|u\|_{2p-2}^{p-1}\|\Delta v\|_2 \leq \varepsilon_3 C_0(\|u\|_2)\|\Delta u\|_2^{\frac{n(p-2)}{4}}\|\Delta v\|_2\\ &\leq \frac{\varepsilon_1}{2}\|\Delta v\|_2^2+\frac{\varepsilon_3^2 C_0^2(\|u\|_2)}{2\varepsilon_1}\|\Delta u\|_2^{\frac{n(p-2)}{2}}\\ &\leq \frac{\varepsilon_1}{2}\|\Delta v\|_2^2+\frac{l\varepsilon}{2}\|\Delta u\|_2^2+C_0(\|u\|_2, \varepsilon_1, \varepsilon_3, l, \varepsilon).\end{aligned} \tag{2.29}$$

By using Holder inequality and Young's inequality, we have

$$(f(x), -\Delta v) \leq \|\nabla f\|\cdot\|\nabla v\| \leq \frac{\lambda_1\varepsilon_1}{4}\|\nabla v\|_2^2+\frac{1}{\lambda_1\varepsilon_1}\|\nabla f\|_2^2, \tag{2.30}$$

Then, we have

$$\begin{aligned}&\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla v\|_2^2-\varepsilon\|\nabla v\|_2^2-\frac{\varepsilon^2}{2\lambda_1}\|\Delta u\|_2^2-\frac{\varepsilon^2}{2}\|\nabla v\|_2^2+\frac{l}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\Delta u\|_2^2+l\varepsilon\|\Delta u\|_2^2\\ &+\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\eta\|_{g,D(A)}^2+\frac{\xi}{4}\|\eta\|_{g,D(A)}^2-\frac{\varepsilon^2}{\xi}\int_0^{+\infty} g(s)\mathrm{d}s\|\Delta u\|_2^2+\varepsilon_1\|\Delta v\|_2^2\\ &-\frac{\varepsilon_1\varepsilon}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\Delta u\|_2^2-\varepsilon_1\varepsilon^2\|\Delta u\|_2^2+\varepsilon_2\left(1-\frac{\varepsilon}{2}\right)\|\nabla u_t\|_2^2-\frac{\varepsilon_2\varepsilon}{2\lambda_1}\|\Delta u\|_2^2\\ &\leq \frac{\varepsilon_1}{2}\|\Delta v\|_2^2+\frac{l\varepsilon}{2}\|\Delta u\|_2^2+C_0(\|u\|_2, \varepsilon_1, \varepsilon_3, l, \varepsilon)+\frac{\lambda_1\varepsilon_1}{4}\|\nabla v\|_2^2+\frac{1}{\lambda_1\varepsilon_1}\|\nabla f\|_2^2\end{aligned}$$

That is

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\left[\|\nabla v\|_2^2+(l-\varepsilon_1\varepsilon)\|\Delta u\|_2^2+\|\eta\|_{g,D(A)}^2\right]+\left(\frac{\varepsilon_1\lambda_1}{2}-2\varepsilon-\varepsilon^2\right)\|\nabla v\|_2^2\\ &+2\varepsilon\left(\frac{l}{2}-\varepsilon_1\varepsilon-\frac{\varepsilon}{2\lambda_1}-\frac{\varepsilon}{\xi}\int_0^{+\infty} g(s)\mathrm{d}s-\frac{\varepsilon_2}{2\lambda_1}\right)\|\Delta u\|_2^2+\frac{\xi}{2}\|\eta\|_{g,D(A)}^2\\ &+2\varepsilon_2\left[1-\frac{\varepsilon}{2}\right]\|\nabla u_t\|_2^2 \leq \frac{2}{\lambda_1\varepsilon_1}\|\nabla f\|_2^2+2C_0(\|u\|_2, \varepsilon_1, \varepsilon_3, l, \varepsilon).\end{aligned} \tag{2.31}$$

Next, we take proper $\varepsilon,\varepsilon_1,\varepsilon_2,\varepsilon_3$, such that

$$\begin{cases} b_1=\dfrac{\varepsilon_1\lambda_1}{2}-2\varepsilon-\varepsilon^2\ge 0 \\ b_2=2\varepsilon\left(\dfrac{l}{2}-\varepsilon_1\varepsilon-\dfrac{\varepsilon}{2\lambda_1}-\dfrac{\varepsilon}{\xi}\int_0^{+\infty}g(s)\mathrm{d}s-\dfrac{\varepsilon_2}{2\lambda_1}\right)\ge 0 \\ b_3=2\varepsilon_2\left(1-\dfrac{\varepsilon}{2}\right)\ge 0 \end{cases} \tag{2.32}$$

Taking $\alpha_2=\min\left\{b_1,\dfrac{b_2}{l-\varepsilon_1\varepsilon},\dfrac{\xi}{2}\right\}$, then

$$\frac{\mathrm{d}}{\mathrm{d}t}V(t)+\alpha_2V(t)\le\frac{2}{\lambda_1\varepsilon_1}\|\nabla f\|_2^2+2C_0\left(\|u\|_2,\varepsilon_1,\varepsilon_3,l,\varepsilon\right):=C_2, \tag{2.33}$$

where $V(t)=\|\nabla v\|_2^2+(l-\varepsilon_1\varepsilon)\|\Delta u\|_2^2+\|\eta\|_{g,D(A)}^2$, by Gronwall inequality, we have

$$V(t)\le V(0)\mathrm{e}^{-\alpha_2 t}+\frac{C_2}{\alpha_2}\left(1-\mathrm{e}^{-\alpha_2 t}\right) \tag{2.34}$$

From $2<p\le\dfrac{2n}{n-2}$, according to Embedding Theorem, then $H^2(\Omega)\subset W^{1,p}(\Omega)$, let $k=\min\{1,(l-\varepsilon_1\varepsilon)\}$, so, we have

$$\|(u,v)\|_{H^2\times H^1}^2=\|\Delta u\|_2^2+\|\nabla v\|_2^2\le\frac{V(0)}{k}\mathrm{e}^{-\alpha_2 t}+\frac{C_2}{\alpha_2 k}\left(1-\mathrm{e}^{-\alpha_2 t}\right)$$

then

$$\overline{\lim_{t\to\infty}}\|(u,v)\|_{H^2\times H^1}^2\le\frac{C_2}{\alpha_2 k}$$

So, there exists $E_1>0$ and $t_2=t_2(\Omega)>0$, such that

$$\|(u,v)\|_{H^2\times H^1}^2=\|\Delta u(t)\|_2^2+\|\nabla v(t)\|_2^2\le E_1\quad(t>t_2).$$

3. Global Attractors

Theorem 1. Assume (G1), (G2) and (G3) hold, let

$$\begin{cases} 2<p<\min\left\{\dfrac{2n}{n-2},\dfrac{2n+4}{n}\right\}, & n\ge 3 \\ p\ge 2, & n=1,2 \end{cases}$$

and $(u_0,u_1)\in H^2(\Omega)\times H^1(\Omega)$, $f\in H^1(\Omega)$, $v=u_t+\varepsilon u$, so Equation (1.3) exists a unique smooth solution

$$(u,v)\in L^\infty\left([0,+\infty);H^2(\Omega)\times H^1(\Omega)\right)$$

Proof. By the method of Galerkin and **Lemma 1** and **Lemma 2**, we can easily obtain the existence of solutions. Next, we prove the uniqueness of solutions in detail.

Assume that u,v are two solutions of equation, let $w=u-v$, then, the two equations subtract and obtain

$$\begin{aligned} & w''-\left(1-\int_0^{+\infty}g(s)\mathrm{d}s\right)\Delta w-\int_0^{+\infty}g(s)(\Delta\eta_1-\Delta\eta_2)\mathrm{d}s-\varepsilon_1\Delta w'+\varepsilon_2 w' \\ & =\varepsilon_3\left(|v|^{p-2}v-|u|^{p-2}u\right) \end{aligned} \tag{3.1}$$

where

$$\eta_1 = u(x,t) - u(x,t-s),\ \ \eta_2 = v(x,t) - v(x,t-s) \tag{3.2}$$

By multiplying the equation by w' and integrating over Ω, we get

$$\begin{aligned}&\left(w'' - \left(1 - \int_0^{+\infty} g(s)\,\mathrm{d}s\right)\Delta w - \int_0^{+\infty} g(s)(\Delta\eta_1 - \Delta\eta_2)\,\mathrm{d}s - \varepsilon_1 \Delta w' + \varepsilon_2 w', w'\right)\\ &= \left(\varepsilon_3\left(|v|^{p-2} v - |u|^{p-2} u\right), w'\right)\end{aligned} \tag{3.3}$$

here

$$(w'', w') = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|w'\|_2^2 \tag{3.4}$$

and

$$\left(-\left(1 - \int_0^{+\infty} g(s)\,\mathrm{d}s\right)\Delta w, w'\right) = \frac{l}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla w\|_2^2 \tag{3.5}$$

by using (G1), (G2) and (G3), we have

$$\begin{aligned}\left(-\int_0^{+\infty} g(s)(\Delta\eta_1 - \Delta\eta_2)\,\mathrm{d}s, w'\right) &= \left(-\int_0^{+\infty} g(s)(\Delta\eta_1 - \Delta\eta_2)\,\mathrm{d}s, (\eta_1 - \eta_2)_t + (\eta_1 - \eta_2)_s\right)\\ &\geq \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\eta_1 - \eta_2\|_{g,V}^2 + \frac{\xi}{2}\|\eta_1 - \eta_2\|_{g,V}^2\end{aligned} \tag{3.6}$$

By using Poincare inequality, we have

$$(-\varepsilon_1 \Delta w', w') = \varepsilon_1 \|\nabla w'\|_2^2 \geq \varepsilon_1 \lambda_1 \|w'\|_2^2 \tag{3.7}$$

and

$$(\varepsilon_2 w', w') = \varepsilon_2 \|w'\|_2^2 \tag{3.8}$$

By using Holder inequality, Young's inequality and Poincare inequality, we have

$$\begin{aligned}&\varepsilon_3\left(|u|^{p-2} u - |v|^{p-2} v, w'\right) = \varepsilon_2 \int\left(|u|^{p-2} u - |v|^{p-2} v\right) w'\,\mathrm{d}x\\ &\leq \varepsilon_2 p \int\left(|u|^{p-1} + |v|^{p-1}\right)|w||w'|\,\mathrm{d}x \leq \varepsilon_3 C_0 \|w\|\|w'\|\\ &\leq 2\varepsilon_3 \lambda_1 \|w'\|_2^2 + \frac{\varepsilon_3 C_0^2}{8\lambda_1}\|w\|_2^2 \leq 2\varepsilon_3 \lambda_1 \|w'\|_2^2 + \frac{\varepsilon_3 C_0^2}{8\lambda_1^2}\|\nabla w\|_2^2\end{aligned} \tag{3.9}$$

then, we have

$$\begin{aligned}&\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|w'\|_2^2 + \frac{l}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla w\|_2^2 + \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\eta_1 - \eta_2\|_{g,V}^2 + \frac{\xi}{2}\|\eta_1 - \eta_2\|_{g,V}^2 + \varepsilon_1 \lambda_1 \|w'\|_2^2 + \varepsilon_2 \|w'\|_2^2\\ &\leq 2\varepsilon_3 \lambda_1 \|w'\|_2^2 + \frac{\varepsilon_3 C_0^2}{8\lambda_1^2}\|\nabla w\|_2^2\end{aligned} \tag{3.10}$$

That is

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\|w'\|_2^2 + l\|\nabla w\|_2^2 + \|\eta_1 - \eta_2\|_{g,V}^2\right] \leq 4\varepsilon_3 \lambda_1 \|w'\|_2^2 + \frac{\varepsilon_3 C_0^2}{4\lambda_1^2}\|\nabla w\|_2^2 + \|\eta_1 - \eta_2\|_{g,V}^2 \tag{3.11}$$

Taking $m = \max\left\{4\varepsilon_3 \lambda_1, \frac{\varepsilon_3 C_0^2}{4\lambda_1^2 l}, 1\right\}$, then

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\|w'\|_2^2+l\|\nabla w\|_2^2+\|\eta_1-\eta_2\|_{g,V}^2\right]\le m\left(\|w'\|_2^2+l\|\nabla w\|_2^2+\|\eta_1-\eta_2\|_{g,V}^2\right) \tag{3.12}$$

By using Gronwall inequality, we have

$$\|w'\|_2^2+l\|\nabla w\|_2^2+\|\eta_1-\eta_2\|_{g,V}^2\le\left[\|w'(0)\|_2^2+l\|\nabla w(0)\|_2^2+\|\eta_1(0)-\eta_2(0)\|_{g,V}^2\right]\mathrm{e}^{mt} \tag{3.13}$$

So we get $w(t)\equiv 0$, the uniqueness is proved.

Theorem 2. Let *X* be a Banach space, and $\{S(t)\}(t\ge 0)$ are the semigroup operator on *X*. $S(t):X\to X$, $S(t)S(\tau)=S(t+\tau)$, $S(0)=I$, here *I* is a unit operator. Set $S(t)$ satisfy the follow conditions.

1) $S(t)$ is bounded, namely $\forall R>0$, $\|u\|_X\le R$, it exists a constant $C(R)$, so that

$$\|S(t)u\|_X\le C(R)\ \ (t\in[0,+\infty))$$

2) It exists a bounded absorbing set $B_0\subset X$, namely, $\forall B\subset B_0$, it exists a constant t_0, so that

$$S(t)B\subset B_0\ \ (t\ge t_0)$$

3) When $t>0$, $S(t)$ is a completely continuous operator *A*.

Therefore, the semigroup operators $S(t)$ exist a compact global attractor.

Theorem 3. Under the assume of **Theorem 1**, equations have global attractor

$$A=\omega(B_0)=\bigcap_{s\ge 0}\overline{\bigcup_{t\ge s}S(t)B_0}$$

where $B_0=\left\{(u,v)\in H_0^2\times H^1:\|(u,v)\|_{H_0^2\times H^1}^2=\|u\|_{H_0^2}^2+\|v\|_{H^1}^2\le E_0+E_1\right\}$, B_0 is the bounded absorbing set of $H^2(\Omega)\times H_0^1(\Omega)$ and satisfies

1) $S(t)A=A$, $t>0$;

2) $\lim\limits_{t\to\infty}dist\left(S(t)B_0,A\right)=0$, here $B\subset H_0^2(\Omega)\times H^1(\Omega)$ and it is a bounded set,

$$dist(X,Y)=\sup_{x\in X}\inf_{y\in Y}\|x-y\|_{H^2\times H^1}$$

Proof. Under the conditions of **Theorem 1**, it exists the solution semigroup $S(t)$, here $X=H_0^2(\Omega)\times H^1(\Omega)$, $S(t):H^2\times H^1\to H^2\times H^1$.

1) From **Lemma 1** to **Lemma 2**, we can get that $\forall B\subset H_0^2(\Omega)\times H^1(\Omega)$ is a bounded set that includes in the ball $\left\{\|(u,v)\|_{H_0^2\times H^1}\le R\right\}$,

$$\|S(t)(u_0,v_0)\|_{H^2\times H_0^1}^2=\|u\|_{H^2}^2+\|v\|_{H_0^1}^2\le\|u_0\|_{H^2}^2+\|v_0\|_{H_0^1}^2+C\le R^2+C,\quad (t\ge 0,(u_0,v_0)\in B)$$

This shows that $S(t)(t\ge 0)$ is uniformly bounded in $H^2(\Omega)\times H_0^1(\Omega)$.

2) Furthermore, for any $(u_0,v_0)\in H^2(\Omega)\times H^1(\Omega)$, when $t\ge\max\{t_1,t_2\}$, we have

$$\|S(t)(u_0,v_0)\|_{H_0^2\times H^1}^2=\|u\|_{H_0^2}^2+\|v\|_{H^1}^2\le E_0+E_1$$

So, we get B_0 is the bounded absorbing set.

3) Since $H^2(\Omega)\times H_0^1(\Omega)\to H_0^1(\Omega)\times L^2(\Omega)$ is tightly embedded, which means that the bounded set in $H^2(\Omega)\times H_0^1(\Omega)$ is the tight set in $H_0^1(\Omega)\times L^2(\Omega)$, so the semigroup operator $S(t)$ is completely continuous.

So, the semigroup operators $S(t)$ exist a compact global attractor *A*. The proof is completed.

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