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The Global Attractors of the Solution for 2D Maxwell-Navier-Stokes with Extra Force Equations

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Abstract

In this paper, we studied the solution existence and uniqueness and the attractors of the 2D Maxwell-Navier-Stokes with extra force equations.

Keywords

Maxwell-Navier-Stokes Equations, Existence, Uniqueness, Attractor

1. Introduction

In recent years, the Maxwell-Navier-Stokes equations have been studied extensively, and the studies have obtained many achievements [1] [2]. The Maxwell-Navier-Stokes equations are a coupled system of equations consisting of the Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism. The coupling comes from the Lorentz force in the fluid equation and the electric current in the Maxwell equations. In [1], the authors studied the non-resistive limit of the 2D Maxwell-Navier-Stokes equations and established the convergence rate of the non-resistive limit for vanishing resistance by using the Fourier localization technique. In [2], the author has proved the existence and uniqueness of global strong solutions to the non-resistive of the 2D Maxwell-Navier-Stokes equations for initial data $(v_0, E_0, B_0) \in \left(L^2\left(H^2\right)\right) \times \left(H^s\left(R^2\right)\right)^2$ with $s > 0$. The long time behaviors of the solutions of nonlinear partial differential equations also are seen in [3]-[10].

In this paper,we will study the 2D Maxwell-Navier-Stokes equations with extra force and dissipation in a bounded area under homogeneous Dirichlet boundary condition problems:

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$$\begin{cases} \frac{dv}{dt} + v\nabla v - \gamma\Delta v = j\times B + f(x), \Omega\times(0,T), \\ \frac{dE}{dt} - curlB - \varepsilon\Delta E = -j + g(x), \Omega\times(0,T), \\ \frac{dB}{dt} + curlE - \eta\Delta B = h(x), \Omega\times(0,T), \\ j = E + v\times B, \Omega\times(0,T), \\ divv = divB = 0, \Omega\times(0,T), \\ v(0,x) = v_0(x); E(0,x) |= E_0(x); B(0,x) |= B_0(x), \\ v(x,t)|_{\partial\Omega} = E(x,t)|_{\partial\Omega} = B(x,t)|_{\partial\Omega} = 0, \end{cases} \tag{1.1}$$

here $\Omega \subset R^2$ is bounded set, $\partial\Omega$ is the bound of Ω, v is the velocity of the fluid, γ is the viscosity, ε and η are resistive constants, j is the electric current which is given by Ohm's law, E is the electric field, B is the magnetic field and $j\times B$ is the Lorentz force.

Let $\|\cdot\| = \|\cdot\|_{L^2(\Omega)}$ and $\|f\|^2 = (f,f) = \int_\Omega f^2(x)\mathrm{d}x$.

2. The Priori Estimate of Solution of Questions (1.1)

Lemma 1. Assume $f,g,h \in L^2(\Omega); \|v_0\| \le \rho_0, \|E_0\| \le \rho_0, \|B_0\| \le \rho_0$, so the solution (v,E,B) of the Dirichlet bound questions (1.1) satisfies

$$\|v\|^2 + \|E\|^2 + \|B\|^2 \le \begin{cases} \left(\|v_0\|^2 + \|E_0\|^2 + \|B_0\|^2\right)e^{-\alpha t} + \frac{\|f\|^2 + \|g\|^2 + \|h\|^2}{\alpha^2}, & (t \ge 0), \\ \frac{2\left(\|f\|^2 + \|g\|^2 + \|h\|^2\right)}{\alpha^2}, & (t \ge t_0), \end{cases}$$

here $t_0 = \max\left\{0, \frac{1}{\alpha}\ln\left[\frac{\rho_0\alpha^2}{\|f\|^2 + \|g\|^2 + \|h\|^2}\right]\right\}$.

Proof. For the system (1.1) multiply the first equation by v with both sides and obtain

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|v\|^2 + \gamma\|\nabla v\|^2 = (j\times B, v) + (f(x), v). \tag{2.1}$$

For the system (1.1) multiply the second equation by E with both sides and obtain

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|E\|^2 + (-curlB, E) + \varepsilon\|\nabla E\|^2 = (-j, E) + (g(x), E). \tag{2.2}$$

For the system (1.1) multiply the third equation by B with both sides and obtain

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|B\|^2 + (curlE, B) + \eta\|\nabla B\|^2 = (h(x), B). \tag{2.3}$$

Because $(curlE, B) = (curlB, E)$, so $(2.1)+(2.2)+(2.3)$ is

$$\begin{aligned} &\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\|v\|^2 + \|E\|^2 + \|B\|^2\right) + \gamma\|\nabla v\|^2 + \varepsilon\|\nabla E\|^2 + \eta\|\nabla B\|^2 \\ &= (j\times B, v) + (f(x), v) + (-j, E) + (g(x), E) + (h(x), B). \end{aligned} \tag{2.4}$$

According to Poincare's inequality, we obtain

$$\gamma\|\nabla v\|^2 \ge \gamma C_0\|v\|^2, \ \varepsilon\|\nabla E\|^2 \ge \varepsilon C_1\|E\|^2, \ \eta\|\nabla B\|^2 \ge \eta C_2\|B\|^2. \tag{2.5}$$

According to $j = E + v\times B$, we obtain

$$(j\times B,v)+(-j,E)=(j,B\times v)-(j,E)=-(j,v\times B)-(j,E)=-(j,E+v\times B)=-\|j\|^2. \tag{2.6}$$

According to Young's inequality, we obtain

$$(f(x),v)\le\|f(x)\|\|v\|\le\frac{\gamma C_0}{2}\|v\|^2+\frac{1}{2\gamma C_0}\|f\|^2, \tag{2.7}$$

$$(g(x),E)\le\|g(x)\|\|E\|\le\frac{\varepsilon C_1}{2}\|E\|^2+\frac{1}{2\varepsilon C_1}\|g\|^2, \tag{2.8}$$

$$(h(x),B)\le\|h(x)\|\|B\|\le\frac{\eta C_2}{2}\|B\|^2+\frac{1}{2\eta C_2}\|h\|^2. \tag{2.9}$$

From (2.4) (2.5) (2.6) (2.7) (2.8) (2.9), we obtain

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\|v\|^2+\|E\|^2+\|B\|^2\right)+\frac{\gamma C_0}{2}\|v\|^2+\frac{\varepsilon C_1}{2}\|E\|^2+\frac{\eta C_2}{2}\|B\|^2+\|j\|^2$$
$$\le\frac{1}{2\gamma C_0}\|f\|^2+\frac{1}{2\varepsilon C_1}\|g\|^2+\frac{1}{2\eta C_2}\|h\|^2,$$

so

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\|v\|^2+\|E\|^2+\|B\|^2\right)+\frac{\gamma C_0}{2}\|v\|^2+\frac{\varepsilon C_1}{2}\|E\|^2+\frac{\eta C_2}{2}\|B\|^2$$
$$\le\frac{1}{2\gamma C_0}\|f\|^2+\frac{1}{2\varepsilon C_1}\|g\|^2+\frac{1}{2\eta C_2}\|h\|^2.$$

Let $\alpha=\min\{\gamma C_0,\varepsilon C_1,\eta C_2\}$, according that we obtain

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\|v\|^2+\|E\|^2+\|B\|^2\right)+\frac{\alpha}{2}\left(\|v\|^2+\|E\|^2+\|B\|^2\right)\le\frac{1}{2\alpha}\left(\|f\|^2+\|g\|^2+\|h\|^2\right),$$

so

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\|v\|^2+\|E\|^2+\|B\|^2\right)+\alpha\left(\|v\|^2+\|E\|^2+\|B\|^2\right)\le\frac{1}{\alpha}\left(\|f\|^2+\|g\|^2+\|h\|^2\right).$$

Using the Gronwall's inequality, the Lemma 1 is proved.

Lemma 2. Under the condition of Lemma 1, and $f,g,h\in H^1(\Omega)$; $v_0,E_0,B_0\in H^1(\Omega)$; $\gamma\ge\frac{9C_6}{4},\eta\ge\frac{3C_7}{2}$, so the solution (v,E,B) of the Dirichlet bound questions (1.1) satisfies

$$\|\nabla v\|^2+\|\nabla E\|^2+\|\nabla B\|^2\le\begin{cases}\left(\|\nabla v_0\|^2+\|\nabla E_0\|^2+\|\nabla B_0\|^2\right)\mathrm{e}^{-kt}+\frac{k}{C}, & (t\ge 0),\\ \frac{k}{C}, & (t\ge t_0),\end{cases}$$

here $t_0=\max\left\{0,\ln\left[\frac{\rho_0 C}{k}\right]\right\}$.

Proof. For the system (1.1) multiply the first equation by $-\Delta v$ with both sides and obtain

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla v\|^2+(v\nabla v,-\Delta v)+\gamma\|\Delta v\|^2=(j\times B,-\Delta v)+(f(x),-\Delta v). \tag{2.10}$$

For the system (1.1) multiply the second equation by $-\Delta E$ with both sides and obtain

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla E\|^2-(curlB,-\Delta E)+\varepsilon\|\Delta E\|^2=(-j,-\Delta E)+(g(x),-\Delta E). \tag{2.11}$$

For the system (1.1) multiply the third equation by $-\Delta B$ with both sides and obtain

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla B\|^2+(curlE,-\Delta B)+\eta\|\Delta B\|^2=(h(x),-\Delta B). \tag{2.12}$$

According $(curlB,-\Delta E)=-(curlB,\Delta E)=-(curlE,\Delta B)$ and (2.10) (2.11) (2.12) we obtain

$$\begin{aligned}&\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\|\nabla v\|^2+\|\nabla E\|^2+\|\nabla B\|^2\right)-(v\nabla v,\Delta v)+\gamma\|\Delta v\|^2+\varepsilon\|\Delta E\|^2+\eta\|\Delta B\|^2\\&=(j\times B,-\Delta v)+(f(x),-\Delta v)+(-j,-\Delta E)+(g(x),-\Delta E)+(h(x),-\Delta B),\end{aligned} \tag{2.13}$$

here

$$(v\nabla v,\Delta v)=-\left((\nabla v)^2+v\Delta v,\nabla v\right)=-\left((\nabla v)^2,\nabla v\right)-(v\nabla v,\Delta v),$$

so

$$(v\nabla v,\Delta v)=-\frac{1}{2}\left((\nabla v)^2,\nabla v\right)\le\frac{1}{2}\|\nabla v\|_{L^3}^3.$$

According to the Sobolev's interpolation inequalities

$$\frac{1}{2}\|\nabla v\|_{L^3}^3\le C_3\|\Delta v\|^{\frac{2}{3}}\|v\|^{\frac{1}{3}}\le\frac{\gamma}{6}\|\Delta v\|^2+\frac{2C_3}{3\left(\frac{\gamma}{2C_3}\right)^{\frac{1}{3}}}\|v\|^{\frac{1}{2}},$$

so

$$(v\nabla v,\Delta v)\le\frac{\gamma}{6}\|\Delta v\|^2+\frac{2C_3}{3\left(\frac{\gamma}{2C_3}\right)^{\frac{1}{3}}}\|v\|^{\frac{1}{2}}; \tag{2.14}$$

$$\begin{aligned}|(j\times B,-\Delta v)|&=|(E\times B+(v\times B)\times B,-\Delta v)|\\&\le|(E\times B,-\Delta v)|+|((v\times B)\times B,-\Delta v)|\le\|\Delta v\|\|E\|_{L^4}\|B\|_{L^4}+\left|\left((Bv)\cdot B-B^2\cdot v,-\Delta v\right)\right|.\end{aligned} \tag{2.15}$$

According to the Sobolev's interpolation inequalities and Young's inequalities

$$\begin{aligned}&\|\Delta v\|\|E\|_{L^4}\|B\|_{L^4}\le\|\Delta v\|\|\Delta E\|^{\frac{1}{4}}\|E\|^{\frac{3}{4}}\|\Delta B\|^{\frac{1}{4}}\|B\|^{\frac{3}{4}}\le C_4\|\Delta v\|\|\Delta E\|^{\frac{1}{4}}\|\Delta B\|^{\frac{1}{4}}\le\frac{\gamma}{3}\|\Delta v\|^2+\frac{3C_4}{4\gamma}\|\Delta E\|^{\frac{1}{2}}\|\Delta B\|^{\frac{1}{2}}\\&\le\frac{\gamma}{3}\|\Delta v\|^2+\frac{\varepsilon}{3}\|\Delta E\|^2+\frac{3}{4\left(\frac{4\varepsilon}{3}\right)^{\frac{1}{3}}}\left(\frac{3C_4^2}{4\gamma}\right)^{\frac{4}{3}}\|\Delta B\|^{\frac{2}{3}}\\&\le\frac{\gamma}{3}\|\Delta v\|^2+\frac{\varepsilon}{3}\|\Delta E\|^2+\frac{\eta}{3}\|\Delta B\|^2+\frac{2}{3\eta^{\frac{1}{2}}}\left(\frac{3}{4\left(\frac{4\varepsilon}{3}\right)^{\frac{1}{3}}}\right)^{\frac{3}{2}}\left(\frac{3C_4^2}{4\gamma}\right)^2.\end{aligned} \tag{2.16}$$

According to the Holder's inequalities and inequalities

$$\begin{aligned}\left|\left((Bv)\cdot B-B^2\cdot v,-\Delta v\right)\right|&\le2\int_\Omega|B|^2|v||\Delta v|\mathrm{d}x\le2\|v\|_\infty\int_\Omega|B|^2|\Delta v|\mathrm{d}x\le2\|v\|_\infty\|\Delta v\|\|B\|_{L^4}^2\\&\le2C_5\|v\|^{\frac{1}{2}}\|\Delta v\|^{\frac{3}{2}}\|B\|^{\frac{3}{2}}\|\Delta B\|^{\frac{1}{2}}\le\frac{3}{4}\|\Delta v\|^2\|B\|^2+\frac{C_5^2}{2}\|\Delta B\|^2\|v\|^2\\&\le\frac{3C_6}{4}\|\Delta v\|^2+\frac{C_7}{2}\|\Delta B\|^2\le\frac{\gamma}{3}\|\Delta v\|^2+\frac{\eta}{3}\|\Delta B\|^2,\end{aligned} \tag{2.17}$$

and

$$\begin{aligned}|(-j,-\Delta E)| &= |(E,\Delta E)+(v\times B,\Delta E)| \le \|\nabla E\|^2 + \int_\Omega |v||B||\Delta E|\,\mathrm{d}x \\ &\le \|\nabla E\|^2 + \|B\|_\infty \|\Delta E\|\|v\| \le \|\nabla E\|^2 + C_8\|B\|^{\frac{1}{2}}\|\Delta B\|^{\frac{1}{2}}\|\Delta E\|\|v\| \\ &\le \|\nabla E\|^2 + \frac{\varepsilon}{6}\|\Delta E\|^2 + \frac{3C_8^2}{2\varepsilon}\|B\|\|\Delta B\|\|v\|^2 \\ &\le \|\nabla E\|^2 + \frac{\varepsilon}{6}\|\Delta E\|^2 + \frac{\eta}{6}\|\Delta B\|^2 + \frac{27C_9^4}{8\eta\varepsilon^2}\|B\|^2\|v\|^4.\end{aligned} \tag{2.18}$$

According to the (2.13) (2.14) (2.15) (2.16) (2.17) (2.18), we obtain

$$\begin{aligned}&\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\|\nabla v\|^2 + \|\nabla E\|^2 + \|\nabla B\|^2\right) + \frac{\gamma}{3}\|\Delta v\|^2 + \frac{\varepsilon}{3}\|\Delta E\|^2 + \frac{\eta}{3}\|\Delta B\|^2 \\ &\le (f(x),-\Delta v) + (g(x),-\Delta E) + (h(x),-\Delta B) + \|\nabla E\|^2 + C_{10},\end{aligned} \tag{2.19}$$

here

$$C_{10} \ge \frac{2}{3\eta^{\frac{1}{2}}}\left(\frac{3}{4\left(\frac{4\varepsilon}{3}\right)^{\frac{1}{3}}}\right)^{\frac{3}{2}}\left(\frac{3C_4^2}{4\gamma}\right)^2 + \frac{27C_9^4}{8\eta\varepsilon^2}\|B\|^2\|v\|^4 - \frac{2C_3}{3\left(\frac{\gamma}{2C_3}\right)^{\frac{1}{3}}}\|v\|^{\frac{1}{2}}.$$

According to the Poincare's inequalities

$$\frac{\gamma}{3}\|\Delta v\|^2 \ge \frac{\gamma}{2}C_{11}\|\nabla v\|^2;\ \frac{\varepsilon}{3}\|\Delta E\|^2 \ge \frac{\varepsilon}{2}C_{12}\|\nabla E\|^2;\ \frac{\eta}{3}\|\Delta B\|^2 \ge \frac{\eta}{2}C_{13}\|\eta B\|^2. \tag{2.20}$$

According to the Young's inequalities

$$(f(x),-\Delta v) = (\nabla f(x),\nabla v) \le \|\nabla f(x)\|\|\nabla v\| \le \frac{\gamma}{2}\|\nabla v\|^2 + \frac{1}{2\gamma}\|\nabla f(x)\|^2. \tag{2.21}$$

In a similar way,we can obtain

$$(g(x),-\Delta E) \le \frac{\varepsilon}{2}\|\nabla E\|^2 + \frac{1}{2\varepsilon}\|\nabla g(x)\|^2, \tag{2.22}$$

$$(h(x),-\Delta B) \le \frac{\eta}{2}\|\nabla B\|^2 + \frac{1}{2\eta}\|\nabla h(x)\|^2. \tag{2.23}$$

From (2.19)-(2.23), we have

$$\begin{aligned}&\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\|\nabla v\|^2 + \|\nabla E\|^2 + \|\nabla B\|^2\right) + \frac{\gamma}{2}(C_{11}-1)\|\nabla v\|^2 + \frac{\varepsilon}{2}\left(C_{12}-1-\frac{2}{\varepsilon}\right)\|\nabla E\|^2 + \frac{\eta}{2}(C_{13}-1)\|\nabla B\|^2 \\ &\le \frac{1}{2\gamma}\|\nabla f(x)\|^2 + \frac{1}{2\varepsilon}\|\nabla g(x)\|^2 + \frac{1}{2\eta}\|\nabla h(x)\|^2 + C_{10}.\end{aligned}$$

Let $k = \min\left\{\gamma(C_{11}-1),\varepsilon\left(C_{12}-1-\frac{2}{\varepsilon}\right),\eta(C_{13}-1)\right\}$, because $f,g,h \in H^1(\Omega)$, so existing C satisfied

$$\frac{1}{2}C \ge \frac{1}{2\gamma}\|\nabla f(x)\|^2 + \frac{1}{2\varepsilon}\|\nabla g(x)\|^2 + \frac{1}{2\eta}\|\nabla h(x)\|^2 + C_{10}.$$

So

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\|\nabla v\|^2 + \|\nabla E\|^2 + \|\nabla B\|^2\right) + k\left(\|\nabla v\|^2 + \|\nabla E\|^2 + \|\nabla B\|^2\right) \le C.$$

According to the Gronwall's inequality,we can get the Lemma 2.

3. Solution's Existence and Uniqueness and Attractor of Questions (1.1)

Theorem 1. Assume that $f, g, h \in L^2\left(R^+; H^1(\Omega)\right)$, and $v_0, E_0 B_0 \in H^1(\Omega)$, so questions (1.1) exist a unique solution $w(v, E, B) \in L^\infty\left(R^+; H^1(\Omega)\right)$.

Proof. By the method of Galerkin and Lemma 1 - Lemma 2, we can easily obtain the existence of solutions. Next, we prove the uniqueness of solutions in detail.

Assume $w_1(v_1, E_1, B_1), w_2(v_2, E_2, B_2)$ are two solutions of questions (1.1), let $w(v, E, B) = w_1(v_1, E_1, B_1) - w_2(v_2, E_2, B_2)$. Here $v = v_1 - v_2, E = E_1 - E_2, B = B_1 - B_2, j = j_1 - j_2$, so the difference of the two solution satisfies

$$\begin{cases} \frac{\mathrm{d}v_1}{\mathrm{d}t} + v_1\nabla v_1 - \gamma\Delta v_1 = j_1 \times B_1 + f(x), \Omega\times(0,T). \\ \frac{\mathrm{d}E_1}{\mathrm{d}t} - curlB_1 - \varepsilon\Delta E_1 = -j_1 + g(x), \Omega\times(0,T). \\ \frac{\mathrm{d}B_1}{\mathrm{d}t} + curlE_1 - \eta\Delta B_1 = h(x), \Omega\times(0,T). \\ j_1 = E_1 + v_1 \times B_1, \Omega\times(0,T). \\ divv_1 = divB_1 = 0, \Omega\times(0,T). \\ v_1(0,x) = v_{10}(x); E_1(0,x)|_{t=0} = E_{10}(x); B_1(0,x) |= B_{10}(x). \\ v_1(x,t)|_{\partial\Omega} = E_1(x,t)|_{\partial\Omega} = B_1(x,t)|_{\partial\Omega} = 0. \end{cases}$$

$$\begin{cases} \frac{\mathrm{d}v_2}{\mathrm{d}t} + v_2\nabla v_2 - \gamma\Delta v_2 = j_2 \times B_2 + f(x), \Omega\times(0,T). \\ \frac{\mathrm{d}E_2}{\mathrm{d}t} - curlB_2 - \varepsilon\Delta E_2 = -j_2 + g(x), \Omega\times(0,T). \\ \frac{\mathrm{d}B_2}{\mathrm{d}t} + curlE_2 - \eta\Delta B_2 = h(x), \Omega\times(0,T). \\ j_2 = E_2 + v_2 \times B_2, \Omega\times(0,T). \\ divv_2 = divB_2 = 0, \Omega\times(0,T). \\ v_2(0,x) = v_{20}(x); E_2(0,x) = E_{20}(x); B_2(0,x) = B_{20}(x). \\ v_2(x,t)|_{\partial\Omega} = E_2(x,t)|_{\partial\Omega} = B_2(x,t)|_{\partial\Omega} = 0. \end{cases}$$

The two above formulae subtract and obtain

$$\begin{cases} \frac{\mathrm{d}v}{\mathrm{d}t} + v_2\nabla v + v\nabla v_1 - \gamma\Delta v = j_1 \times B + j \times B_2, \Omega\times(0,T). \\ \frac{\mathrm{d}E}{\mathrm{d}t} - curlB - \varepsilon\Delta E = -j, \Omega\times(0,T). \\ \frac{\mathrm{d}B}{\mathrm{d}t} + curlE - \eta\Delta B = 0, \Omega\times(0,T). \\ j = E + v_1 \times B + v \times B_2, \Omega\times(0,T). \\ divv = divB = 0, \Omega\times(0,T). \\ v(0,x) = v_0(x); E(0,x) = E_0(x); B(0,x) = B_0(x). \\ v(x,t)|_{\partial\Omega} = E(x,t)|_{\partial\Omega} = B(x,t)|_{\partial\Omega} = 0. \end{cases} \tag{3.1}$$

For the system (3.1) multiply the first equation by v with both sides and obtain

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|v\|^2+\left(v_2\nabla v+v\nabla v_1,v\right)+\gamma\|\nabla v\|^2=\left(j_1\times B+j\times B_2,v\right). \tag{3.2}$$

For the system (3.1) multiply the second equation by E with both sides and obtain

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|E\|^2+\left(-curlB,E\right)+\varepsilon\|\nabla E\|^2=\left(-j,E\right). \tag{3.3}$$

For the system (3.1) multiply the third equation by B with both sides and obtain

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|B\|^2+\left(curlE,B\right)+\eta\|\nabla B\|^2=0. \tag{3.4}$$

According to (3.2) + (3.3) + (3.4), we obtain

$$\begin{aligned}&\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\|v\|^2+\|E\|^2+\|B\|^2\right)+\left(v_2\nabla v+v\nabla v_1,v\right)+\gamma\|\nabla v\|^2+\varepsilon\|\nabla E\|^2+\eta\|\nabla B\|^2\\&=\left(j_1\times B+j\times B_2,v\right)+\left(-j,E\right),\end{aligned} \tag{3.5}$$

here $\left(v_2\nabla v+v\nabla v_1,v\right)=\left(v_2\nabla v,v\right)+\left(v\nabla v_1,v\right)$, and $\left(v_2\nabla v,v\right)=-\left(\nabla v_2,v^2\right)-\left(\nabla v_2,v^2\right)$, so $\left(v_2\nabla v,v\right)=-\frac{1}{2}\left(\nabla v_2,v^2\right)$. From that, we have

$$\begin{aligned}\left|\left(v_2\nabla v+v\nabla v_1,v\right)\right|&\le\left|\left(v\nabla v_1,v\right)\right|+\left|-\frac{1}{2}\left(\nabla v_2,v^2\right)\right|\le\|\nabla v_1\|\|v\|_{L^4}^2+\frac{1}{2}\|\nabla v_2\|\|v\|_{L^4}^2\\&\le\left(\|\nabla v_1\|+\frac{1}{2}\|\nabla v_2\|\right)C_{14}\|v\|\|\nabla v\|\le\frac{\gamma}{2}\|\nabla v\|^2+\frac{1}{2\gamma}\left(\|\nabla v_1\|+\frac{1}{2}\|\nabla v_2\|\right)^2C_{14}^2\|v\|^2\\&\le\frac{\gamma}{2}\|\nabla v\|^2+\frac{1}{2\gamma}C_{15}\|v\|^2.\end{aligned} \tag{3.6}$$

$$\begin{aligned}&\left(j_1\times B+j\times B_2,v\right)+\left(-j,E\right)\\&=\left(j_1,B\times v\right)+\left(j,B_2\times v\right)+\left(-j,E\right)\\&=\left(j_1,B\times v\right)-\left(j,v\times B_2\right)-\left(j,v_1\times B\right)-\left(j,E\right)+\left(j,v_1\times B\right)\\&=\left(j_1,B\times v\right)+\left(j,v_1\times B\right)-\|j\|^2\\&\le\|j_1\|_{L^4}\|B\|_{L^4}\|v\|+\|j\|\|B\|_{L^4}\|v_1\|_{L^4}-\|j\|^2\\&\le C_{16}\|j_1\|^{\frac{1}{2}}\|B\|^{\frac{1}{2}}\|\nabla j_1\|^{\frac{1}{2}}\|\nabla B\|^{\frac{1}{2}}\|v\|+C_{17}\|j\|\|B\|^{\frac{1}{2}}\|\nabla B\|^{\frac{1}{2}}\|v_1\|^{\frac{1}{2}}\|\nabla v_1\|^{\frac{1}{2}}-\|j\|^2\\&\le\frac{1}{2}\|v\|^2+\frac{1}{2}C_{16}^2\|j_1\|\|B\|\|\nabla j_1\|\|\nabla B\|+\frac{1}{2}\|j\|^2+\frac{1}{2}C_{17}^2\|B\|\|\nabla B\|\|v_1\|\|\nabla v_1\|-\|j\|^2\\&\le\frac{1}{2}\|v\|^2+\frac{1}{2}\left(\frac{\eta}{2}\|\nabla B\|^2+\frac{1}{2\eta}C_{16}^4\|j_1\|^2\|B\|^2\|\nabla j_1\|^2\right)\\&\quad+\frac{1}{2}\left(\frac{\eta}{2}\|\nabla B\|^2+\frac{1}{2\eta}C_{17}^4\|B\|^2\|v_1\|^2\|\nabla v_1\|^2\right)-\frac{1}{2}\|j\|^2\\&\le\frac{1}{2}\|v\|^2+\frac{\eta}{2}\|\nabla B\|^2+\frac{1}{2\eta}C_{18}\|B\|^2-\frac{1}{2}\|j\|^2.\end{aligned} \tag{3.7}$$

Notice that

$$\gamma\|\nabla v\|^2\ge\gamma C_0\|v\|^2,\ \ \varepsilon\|\nabla E\|^2\ge\varepsilon C_1\|E\|^2,\ \ \eta\|\nabla B\|^2\ge\eta C_2\|B\|^2. \tag{3.8}$$

From the (3.5), (3.6), (3.7) and (3.8), we can obtain

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\|v\|^2+\|E\|^2+\|B\|^2\right)+\left(\frac{\gamma C_0}{2}-\frac{1}{2}\right)\|v\|^2+\varepsilon C_1\|E\|^2+\left(\frac{\eta C_2}{2}-\frac{C_{18}}{2\eta}\right)\|B\|^2\le-\frac{1}{2}\|j\|^2.$$

Let

$$-\frac{m}{2}=\max\left\{\frac{\gamma C_0}{2}-\frac{1}{2},\varepsilon C_1,\frac{\eta C_2}{2}-\frac{C_{18}}{2\eta}\right\},$$

so, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\|v\|^2+\|E\|^2+\|B\|^2\right)\le m\left(\|v\|^2+\|E\|^2+\|B\|^2\right).$$

According to the consistent Gronwall inequality, the uniqueness is proved.

Theorem 2. [8] Let X be a Banach space, and $\{S(t)\}(t\ge 0)$ are the semigroup operators on X. $S(t):X\to X$, $S(t)\cdot S(\tau)=S(t+\tau)$, $S(0)=I$, here I is a unit operator. Set $S(t)$ satisfy the follow conditions.

1) $S(t)$ is bounded. Namely $\forall R>0,\|u\|_X\le R$, it exists a constant $C(R)$, so that $\|S(t)u\|_X\le C(R)\left(t\in[0,+\infty)\right)$;

2) It exists a bounded absorbing set $M_0\subset X$, namely $\forall M\subset X$, it exists a constant t_0, so that $S(t)M\subset M_0\ (t>t_0)$;

3) When $t>0, S(t)$ is a completely continuous operator A.

Therefor, the semigroup operators $S(t)$ exist a compact global attractor.

Theorem 3. Under the assume of Theorem 1, questions (1.1) have global attractor $A=w(M_0)=\bigcap_{s\ge 0}\overline{\bigcup_{t\ge s}S(t)M_0}$, M_0 is the bounded absorbing set of $H^1(\Omega)$ and satisfies

1) $S(t)A=A, t>0$;

2) $\lim_{t\to\infty} dist\left(S(t)M,A\right)=0$, here $\forall M\subset L^2(\Omega)$ and it is a bounded set, $dist(X,Y)=\sup_{x\in X}\inf_{y\in Y}\|x-y\|_{L^2(\Omega)}$.

Proof. Under the conditions of Theorem 1 and Theorem 2, it exists the solution semigroup $S(t)$ of questions (1.1), $S(t):H(\Omega)\to H(\Omega)$.

From Lemma 1 - Lemma 2, to $\forall M\subset H^1(\Omega)$ is a bounded set that includes in the ball $\left\{\|v\|_{H^1}+\|E\|_{H^1}+\|B\|_{H^1}\le R\right\}$,

$$\begin{aligned}&\|S(t)v_0\|_{H^1}^2+\|S(t)E_0\|_{H^1}^2+\|S(t)B_0\|_{H^1}^2=\|v\|_{H^1}^2+\|E\|_{H^1}^2+\|B\|_{H^1}^2\\&\le\|v_0\|_{H^1}^2+\|E_0\|_{H^1}^2+\|B_0\|_{H^1}^2+C_1\left(\|f\|^2+\|g\|^2+\|h\|^2\right)+C_2\\&\le R^2+C',\ \left(t\ge 0,v_0,E_0,B_0\in M\right).\end{aligned}$$

This shows $\{S(t)\}(t\ge 0)$ is uniformly bounded in $H^1(\Omega)$.

Furthermore, when $t\ge t_0=t_0\left(R,\|f\|+\|g\|+\|h\|\right)$, there is

$$\|S(t)v_0\|_{H^1}^2+\|S(t)E_0\|_{H^1}^2+\|S(t)B_0\|_{H^1}^2=\|v\|_{H^1}^2+\|E\|_{H^1}^2+\|B\|_{H^1}^2\le 2\left(\frac{\|f\|^2+\|g\|^2+\|h\|^2}{\alpha^2}+\frac{k}{C}\right),$$

therefore,

$$\left\{v,E,B\in H^1(\Omega),\|v\|_{H^1}+\|E\|_{H^1}+\|B\|_{H^1}\le\sqrt{2\left(\frac{\|f\|^2+\|g\|^2+\|h\|^2}{\alpha^2}+\frac{k}{C}\right)}\right\}\subset M_0,$$

is the bounded absorbing set of semigroup $S(t)$.

Since $H^1(\Omega)\to H(\Omega)$ is tightly embedded, which is that the bounded set in $H(\Omega)$ is the tight set in

$H^1(\Omega)$, so the semigroup operator $S(t): H(\Omega) \to H(\Omega)$ to $t > 0$ is completely continuous.

4. Discussion

If we want to estimate the Hausdorff and fractal dimension of the attractor A of question (1.1), we need proof of the solution of question (1.1) that is differentiable. We are studying the solution's differentiability hardly and positively. Over a time, we will get some results.

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