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Nonlinear Control of Bioprocess Using Feedback Linearization, Backstepping, and Luenberger Observers

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Abstract

This paper addresses the analysis, design, and application of observer-based nonlinear controls by combining feedback linearization (FBL) and backstepping (BS) techniques with Luenberger observers. Complete development of observer-based controls is presented for a bioprocess. Controllers using input-output feedback linearization and backstepping techniques are designed first, assuming that all states are available for feedback. Next, the construction of observer in the transformed domain is presented based on input-output feedback linearization. This approach is then extended to observer design based on backstepping approach using the error equation resulted from the backstepping design procedure. Simulation results demonstrating the effectiveness of the techniques developed are presented and compared.

Keywords

Bioprocess, Feedback Linearization, Backstepping, Luenberger Observers, Observer-Based Control

1. Introduction

In process control, a major difficulty is to provide direct real-time measurements of the state variables required to implement advanced monitoring and control methods on bioreactors [1]-[5]. Dissolved oxygen concentration in bioreactors, temperature in non-isothermal reactors and gaseous flow rates, are available for on-line measurement while the values of concentration of products, reactants and/or biomass are often available only via on-line analysis [2]-[4], which means that these variables are not available for real-time feedback control. An alternative is to use state observers which, in conjunction with the process model and available measurements,

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can generate accurate estimates of the unmeasured and/or inaccessible states effectively. Exponential and asymptotic observers and their variants to estimate unmeasured states in bioprocess systems have appeared in [1]-[5]. In [6], Dochain and Perrier applied backstepping [7]-[9], techniques to the nonlinear control of microbial growth problem in a CSTR (continuously stirred tank reactor) and two controllers were proposed. The first one was a non-adaptive version, while the second one was an adaptive version in which the maximum specific growth rate was estimated on-line. However, backstepping-based observer design was not considered in [6].

In this paper, a complete development of observer-based control is presented that includes feedback linearization [7] [8] [10] [11], backstepping [7]-[9], Luenberger observer [12] with feedback linearization, and Luenberger observer with backstepping.

The paper is organized as follows. Section 2 presents the bioprocess model for control design. Theoretical foundation of input-output feedback linearization (FBL) and controller design are outlined in Section 3 with simulation results. Section 4 addresses the formulation and application of backstepping (BS) control with simulation results. In Section 5, simulation results are compared for both approaches, *i.e.*, FBL and BS. Section 6 addresses the design of Luenberger observers for FBL and BS controls with simulations. The conclusions are presented in Section 7.

2. Bioprocess Model

The model dynamics in a CSTR (continuous stirred tank reactor) with a simple microbial growth reaction, with one substrate S and biomass X, are given by the following equations [1]:

$$\frac{\mathrm{d}X}{\mathrm{d}t} = \mu X - DX , \tag{1}$$

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -k_1 \mu X + DS_{\text{in}} - DS , \tag{2}$$

where $k_1, \mu, D, S, S_{\text{in}}$, represent the yield coefficient, specific growth rate (h^{-1}), dilution rate (h^{-1}), and substrate concentration (grams/lit) in the influent and reactor, respectively.

The biomass concentration $X(t)$ (grams/lit) is the variable which is to be controlled. Defining the parameter θ_1 as $\theta_1 = D/k_o$ and expressing specific growth rate μ as $\mu = k_o r(S)$, the dynamical Equations (1) and (2) above can be written as [6]

$$\begin{cases} \dot{x}_1 = \theta_1^{-1} r(x_2) x_1 - x_1, \\ \dot{x}_2 = -k_1 \theta_1^{-1} r(x_2) x_1 - x_2 + u, \\ y = x_1, \end{cases} \tag{3}$$

where it is assumed that the biomass concentration $x_1(t) \triangleq X(t)$ can be measured with a sensor, *i.e.*, the output is given by $y = x_1$, while $x_2(t) \triangleq S(t)$ denotes the substrate concentration and $u(t) \triangleq S_{\text{in}}$ is the control input. The bioprocess model given by (3) can be written compactly in an alternate state-space form as:

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \dfrac{C_1 x_1 x_2}{K_s + x_2} - x_1 \\ \dfrac{C_2 x_1 x_2}{K_s + x_2} - x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \triangleq \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x}) u, \\ y = x_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} \boldsymbol{x} \triangleq \boldsymbol{C}\boldsymbol{x}, \end{cases} \tag{4}$$

where $C_1 \triangleq \frac{\mu_{\max}}{D}$ and $C_2 \triangleq -k_1 \frac{\mu_{\max}}{D}$. Note that in (4), $r(x_2)$ has been written using "Monod form" for reaction kinetics, which can expressed as

$$r(x_2) = \frac{\mu_{\max}}{k_o} \frac{x_2}{K_S + x_2} , \tag{5}$$

$$\Rightarrow \frac{\partial r}{\partial x_2} = \frac{\mu_{\max}}{k_o}\frac{K_S}{\left(K_S + x_2\right)^2}. \tag{6}$$

We will use (3) for back stepping control and observer design and (4) will be utilized for developing the control law and observer design using the feedback linearization approach. Typical values of the model parameters needed for the simulation studies are given in **Table 1** [6].

3. Feedback Linearization (FBL) Control Design

The main intent of this section is to investigate control design using the input-output feedback linearization (FBL) technique. Consider a general nonlinear control-affine SISO system described by [7] [8] [10] [11],

$$\dot{\boldsymbol{x}} = \boldsymbol{f}\left(\boldsymbol{x}\right) + \boldsymbol{g}\left(\boldsymbol{x}\right)u, \quad \boldsymbol{f}, \boldsymbol{g} : D \subset \mathbb{R}^n \to \mathbb{R}^n, \tag{7}$$

$$y = h\left(\boldsymbol{x}\right), \quad h : D \subset \mathbb{R}^n \to \mathbb{R}, \tag{8}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ is the state vector, $u, y \in \mathbb{R}$ are the control and output signals, respectively; h is a smooth function, and $\boldsymbol{f}$, $\boldsymbol{g}$ are smooth vector fields on *D*, where *D* is an open set. Given the nonlinear system (7) and the measurement (8), our goal is to find a *diffeomorphism* or nonlinear transformation of the form $\boldsymbol{z} = \boldsymbol{T}_{fbl}\left(\boldsymbol{x}\right)$, $\boldsymbol{z} \in \mathbb{R}^n$ with $\boldsymbol{T}_{fbl}\left(\boldsymbol{0}\right) = \boldsymbol{0}$ that transforms the nonlinear system in the x-*coordinates* to a linear system in the z-*coordinates*. Differentiating the output $y\left(t\right)$ with respect to *t* yields

$$\dot{y} = L_f h\left(\boldsymbol{x}\right) + L_g h\left(\boldsymbol{x}\right)u, \tag{9}$$

where $L_f h\left(\boldsymbol{x}\right) = \frac{\partial h}{\partial \boldsymbol{x}}\boldsymbol{f}$ and $L_g h\left(\boldsymbol{x}\right) = \frac{\partial h}{\partial \boldsymbol{x}}\boldsymbol{g}$ denote the Lie derivatives of $h\left(\boldsymbol{x}\right)$ with respect to $\boldsymbol{f}\left(\boldsymbol{x}\right)$ and $\boldsymbol{g}\left(\boldsymbol{x}\right)$, respectively. If $L_g h\left(\boldsymbol{x}\right) = 0$, then $\dot{y}\left(t\right)$ is independent of $u\left(t\right)$. Continuing successive differentiation ρ times until $u\left(t\right)$ appears explicitly, we obtain

$$y^{(\rho)} = L_f^{\rho} h\left(\boldsymbol{x}\right) + L_g L_f^{\rho-1} h\left(\boldsymbol{x}\right)u \triangleq \alpha\left(\boldsymbol{x}\right) + \beta\left(\boldsymbol{x}\right)u, \tag{10}$$

where $\alpha\left(\boldsymbol{x}\right) \triangleq L_f^{\rho} h\left(\boldsymbol{x}\right)$ is a *nonlinearity cancellation factor* and $\beta\left(\boldsymbol{x}\right) \triangleq L_g L_f^{\rho-1} h\left(\boldsymbol{x}\right)$ is a scalar function. The smallest integer ρ for which $u\left(t\right)$ appears is referred to as the *relative degree*, *i.e.*, when $L_g L_f^{\rho-1} h\left(\boldsymbol{x}\right) \neq 0$. The nonlinear system (7) - (8) is said to have a well-defined relative degree ρ in a region $D_0 \subset D$ if

$$\begin{cases} L_g L_f^{k} h\left(\boldsymbol{x}\right) = 0 \quad \forall k,\ 0 \le k < \rho - 1; \\ L_g L_f^{\rho-1} h\left(\boldsymbol{x}\right) \neq 0, \end{cases} \tag{11}$$

for all $\boldsymbol{x} \in D_0$. Note that $\rho \le n$. From (10), define

$$v \triangleq y^{(\rho)} = \alpha\left(\boldsymbol{x}\right) + \beta\left(\boldsymbol{x}\right)u, \tag{12}$$

where $v\left(t\right)$ is a one-dimensional transformed input created by the feedback linearization process. Equation (12) yields the linearizing feedback control law [7] [8] [10] [11]:

$$u = \beta^{-1}\left(\boldsymbol{x}\right)\left[-\alpha\left(\boldsymbol{x}\right) + v\right], \tag{13}$$

Table 1. Parameter data.

Symbol	Parameter	Numerical Value
k_1	Yield coefficient	2
$\mu_{\max}$	Specific growth rate	0.33 h^{-1}
D	Dilution rate	0.05 h^{-1}
K_s	Saturation constant	5 grams/lit

provided $\beta(\boldsymbol{x})$ is invertible. If $\rho = n$, then

$$\boldsymbol{z} = \boldsymbol{T}_{fbl}(\boldsymbol{x}) = \begin{bmatrix} h(\boldsymbol{x}) \\ L_f h(\boldsymbol{x}) \\ \vdots \\ L_f^{n-1} h(\boldsymbol{x}) \end{bmatrix}. \tag{14}$$

If $\rho < n$, the diffeomorphism $\boldsymbol{T}_{fbl}(\boldsymbol{x})$ comprises of both external and internal dynamics, *i.e.*, $\boldsymbol{T}_{fbl}(\boldsymbol{x}) = [\boldsymbol{\xi} \mid \boldsymbol{\eta}]^{\mathrm{T}}$, where $\boldsymbol{\xi} \in \mathbb{R}^{\rho}$ represents the external dynamics state vector and $\boldsymbol{\eta} \in \mathbb{R}^{n-\rho}$ the internal dynamics state vector, respectively; furthermore, the differential equation for $\boldsymbol{\xi}$ is linear, while that for $\boldsymbol{\eta}$ is typically nonlinear. For the bioprocess model given by (4), $\rho = n = 2$, so the system is fully linearizable. We obtain, from (4), (12) and (14),

$$\alpha(\boldsymbol{x}) = \frac{(C_1 x_2 - K_s - x_2)(C_1 x_1 x_2 - K_s x_1 - x_1 x_2)}{(K_s + x_2)^2} + \frac{C_1 C_2 K_S x_1^2 x_2}{(K_s + x_2)^3} - \frac{C_1 K_S x_1 x_2}{(K_s + x_2)^2}, \tag{15}$$

$$\beta(\boldsymbol{x}) = \frac{C_1 K_S x_1}{(K_s + x_2)^2}, \tag{16}$$

$$\boldsymbol{z} = \boldsymbol{T}_{fbl}(\boldsymbol{x}) = \begin{bmatrix} y \\ \dot{y} \end{bmatrix} = \begin{bmatrix} x_1 \\ \dfrac{C_1 x_1 x_2}{K_s + x_2} - x_1 \end{bmatrix}, \tag{17}$$

where $\boldsymbol{T}_{fbl}(\boldsymbol{x})$ is a local diffeomorphism for the system. Using (17) with (12) for $\rho = 2$, the original nonlinear system described by (3) and (4) is transformed into a linear system of the form

$$\begin{cases} \dot{\boldsymbol{z}} = \boldsymbol{A}_c \boldsymbol{z} + \boldsymbol{B}_c v, \quad \boldsymbol{z}(0) = \boldsymbol{z}_o, \\ y = \boldsymbol{C}_c \boldsymbol{z}, \end{cases} \tag{18}$$

where

$$\begin{cases} \boldsymbol{A}_c = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{B}_c = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \boldsymbol{C}_c = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\ y = \boldsymbol{C}_c \boldsymbol{z}, \end{cases} \tag{19}$$

and $[\boldsymbol{A}_c, \boldsymbol{B}_c]$ and $[\boldsymbol{A}_c, \boldsymbol{C}_c]$ are, respectively, controllable and observable pairs. A suitable tracking control law for the transformed input $v(t)$ in the linear system (18) for $\rho = 2$ can be formulated as, with (17),

$$v = -\boldsymbol{K}_{fbl} \boldsymbol{z} + \boldsymbol{K}_{fbl} \boldsymbol{Y}_r + \ddot{y}_r = -\boldsymbol{K}_{fbl} \boldsymbol{T}_{fbl}(\boldsymbol{x}) + \boldsymbol{K}_{fbl} \boldsymbol{Y}_r + \ddot{y}_r, \tag{20}$$

where $\boldsymbol{z} = \boldsymbol{T}_{fbl}(\boldsymbol{x})$, $y_r(t)$ is a bounded reference with bounded derivatives $\dot{y}_r(t)$ and $\ddot{y}_r(t)$, $\boldsymbol{Y}_r = [y_r \quad \dot{y}_r]^{\mathrm{T}}$, and the constant feedback gain matrix $\boldsymbol{K}_{fbl} = [K_{1fbl} \quad K_{2fbl}]$ is determined such that $(\boldsymbol{A}_c - \boldsymbol{B}_c \boldsymbol{K}_{fbl})$ is Hurwitz. Furthermore, the gain matrix $\boldsymbol{K}_{fbl}$ can be determined by various design methods, such as pole placement (PP) and linear quadratic regulator (LQR). We shall focus on the PP design in this paper. Substituting (20) into (18) yields the closed-loop system

$$\begin{cases} \dot{\boldsymbol{z}} = (\boldsymbol{A}_c - \boldsymbol{B}_c \boldsymbol{K}_{fbl}) \boldsymbol{z} + \boldsymbol{B}_c (\boldsymbol{K}_{fbl} \boldsymbol{Y}_r + \ddot{y}_r), \quad \boldsymbol{z}(0) = \boldsymbol{z}_o, \\ y = \boldsymbol{C}_c \boldsymbol{z}. \end{cases} \tag{21}$$

The linearizing feedback control law in the x-domain can be written by setting $u = u_{fbl}$ in (13) as

$$u_{fbl} = \beta^{-1}(\boldsymbol{x})[-\alpha(\boldsymbol{x}) + v] = \beta^{-1}(\boldsymbol{x})[-\alpha(\boldsymbol{x}) - \boldsymbol{K}_{fbl} \boldsymbol{T}_{fbl}(\boldsymbol{x}) + \boldsymbol{K}_{fbl} \boldsymbol{Y}_r + \ddot{y}_r], \tag{22}$$

which yields the closed-loop system

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x}) u_{fbl}, & \boldsymbol{x}(0) = \boldsymbol{x}_o, \\ y = x_1. \end{cases} \tag{23}$$

The design of a PP control law (20) for the 2[nd]-order system (18) is achieved by choosing a damping ratio $\xi = 1.0$ that prohibits overshoot, and an undamped natural frequency $\omega_n = 8.2$ (rad/sec). The resulting closed-loop poles are given by $\boldsymbol{p} = [-\xi\omega_n \pm j\omega_d] = [-8.2 \quad -8.2]$ where $\omega_d = \omega_n\sqrt{1-\xi^2} = 0$ and the resulting gain $\boldsymbol{K}_{fbl} = [67.24 \quad 16.4]$ is computed with Matlab's **ACKER** command. Simulation studies for the closed-loop system with FBL control were conducted using (23). The controller performance was evaluated for a square-wave set-point reference $y_r(t)$ that alternates every 20 hours between 3 grams/lit and 4 grams/lit as shown in **Figure 1** (dotted line). The initial conditions were chosen as $X(0) = x_1(0) = 2$ grams/lit and $S(0) = x_2(0) = 0.9$ [6]. The simulation results are depicted in **Figure 1** which shows that the responses are satisfactory.

4. Backstepping (BS) Control Design

We shall address the design of back stepping (BS) [7]-[9] control in this section, where the parameter θ_1 is assumed known. The objective here is to design a BS control law u_{bs} such that the output $y = x_1$ tracks the reference y_r. We will also compare the performance of the closed-loop bioprocess under FBL control u_{fbl} given by (22) and the BS control u_{bs} to be developed below. The formulation presented here considers a general bounded differentiable reference signal $y_r(t)$ instead of the constant set-point regulation in [6]. Consider the nonlinear system in the form of (3) reproduced below for ease of reference:

$$\begin{cases} \dot{x}_1 = \theta_1^{-1} r(x_2) x_1 - x_1, \\ \dot{x}_2 = -k_1\theta_1^{-1} r(x_2) x_1 - x_2 + u, \\ y = x_1, \end{cases} \tag{3}$$

where θ_1 is a known constant parameter. We treat $r(x_2)x_1$ as the virtual control of the first subsystem $\dot{x}_1 = \theta_1^{-1} r(x_2) x_1 - x_1$ in (3) and let α_1 be the stabilizing function such that $y = x_1$ racks y_r. Define the tracking errors as

$$q_1 = y - y_r = x_1 - y_r, \tag{24}$$

$$q_2 = \theta_1^{-1} r(x_2) x_1 - \alpha_1, \tag{25}$$

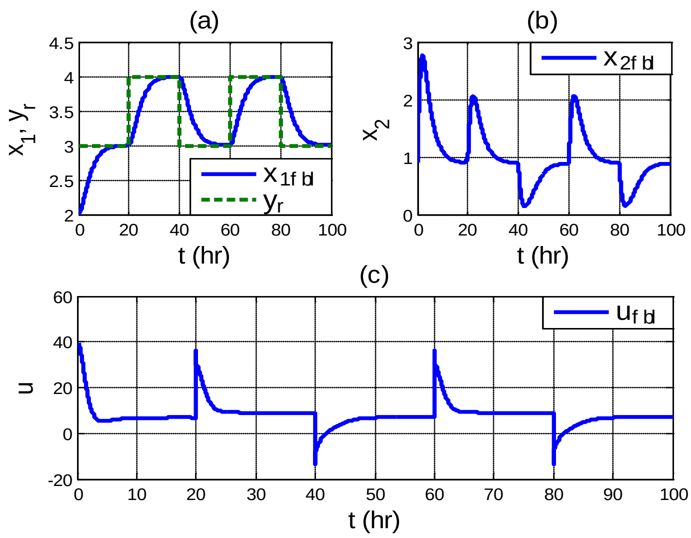

Figure 1. Responses of closed-loop bioprocess (23) under FBL control $u_{fbl}(t)$ with PP design.

where q_2 is the error between $\theta_1^{-1} r(x_2) x_1$ and α_1. Taking the derivative of q_1 yields, with (25),

$$\dot{q}_1 = \dot{x}_1 - \dot{y}_r = \theta_1^{-1} r(x_2) x_1 - x_1 - \dot{y}_r = q_2 + \alpha_1 - x_1 - \dot{y}_r . \tag{26}$$

Consider the Lyapunov function candidate

$$V_1 = \frac{1}{2} q_1^2 , \tag{27}$$

which yields the derivative, with (26),

$$\dot{V}_1 = q_1 q_2 + q_1 (\alpha_1 - x_1 - \dot{y}_r) . \tag{28}$$

Choosing the stabilizing function α_1 to make $\alpha_1 - x_1 - \dot{y}_r = -c_1 q_1$ in (28) yields

$$\alpha_1 = -c_1 q_1 + x_1 + \dot{y}_r , \quad c_1 > 0 . \tag{29}$$

Substituting (29) into (26) and (28) yields, respectively,

$$\dot{q}_1 = -c_1 q_1 + q_2 , \tag{30}$$

$$\dot{V}_1 = q_1 q_2 - c_1 q_1^2 , \quad \forall c_1 > 0. \tag{31}$$

From (31), if $q_2 = 0$, then $\dot{V}_1 = -c_1 q_1^2 < 0 \quad \forall q_1 \neq 0$ and the origin $q_1 = 0$ is globally asymptotically stable, whereby achieving global tracking with $y = x_1 \to y_r$. The term $q_1 q_2$ will be addressed in the next step.

The next step is to develop a BS control law for u. The derivative of $q_2 = \theta_1^{-1} r(x_2) x_1 - \alpha_1$ given by (25) satisfies, with (3) and (29),

$$\dot{q}_2 = \theta_1^{-1} r(x_2) \dot{x}_1 + \theta_1^{-1} x_1 \frac{\partial r}{\partial x_2} \dot{x}_2 - \dot{\alpha}_1 , \tag{32}$$

$$= \phi(\boldsymbol{x}) - \dot{\alpha}_1 + \theta_1^{-1} x_1 \frac{\partial r}{\partial x_2} u , \tag{33}$$

where

$$\phi(\boldsymbol{x}) \triangleq \theta_1^{-1} r (\theta_1^{-1} r - 1) x_1 + \theta_1^{-1} x_1 \frac{\partial r}{\partial x_2} (-k_1 \theta_1^{-1} r x_1 - x_2) , \tag{34}$$

and $\frac{\partial r}{\partial x_2}$ is given by (6).

To stabilize the (q_1, q_2)-system the Lyapunov function candidate as

$$V_2 = V_1 + \frac{1}{2} q_2^2 . \tag{35}$$

The derivative of V_2 is given by, with (31) and (33),

$$\dot{V}_2 = -c_1 q_1^2 + q_2 \left[q_1 + \phi(\boldsymbol{x}) - \dot{\alpha}_1 + \theta_1^{-1} x_1 \frac{\partial r}{\partial x_2} u \right] . \tag{36}$$

Defining $u_{bs} \triangleq u$ to be the BS control, and choosing u_{bs} to make the term $[\bullet] = -c_2 q_2$ in (36) yields

$$u_{bs} = \left(\theta_1^{-1} x_1 \frac{\partial r}{\partial x_2} \right)^{-1} \left[-c_2 q_2 - q_1 - \phi(\boldsymbol{x}) + \dot{\alpha}_1 \right] . \tag{37}$$

Substituting (37) into (33) and (36), we obtain,

$$\dot{q}_2 = -q_1 - c_2 q_2 , \tag{38}$$

$$\dot{V}_2 = -c_1 q_1^2 - c_2 q_2^2, \quad c_1 > 0, \ c_2 > 0. \tag{39}$$

Since $\dot{V}_2 < 0$ for all $c_1 > 0, c_2 > 0$, it follows that $(q_1, q_2) = (0, 0)$ is globally asymptotically stable. Additionally, the stability result can also be established by combing the error equations from (30) and (38) as

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} -c_1 & 1 \\ -1 & -c_2 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \Rightarrow \dot{\boldsymbol{q}} = \boldsymbol{A}_q \boldsymbol{q}. \tag{40}$$

Since $\boldsymbol{A}_q$ is a skew-symmetric Hurwitz matrix for all $c_1 > 0$ and $c_2 > 0$, it follows that the equilibrium $(q_1, q_2) = (0, 0)$ is globally asymptotically stable. Moreover, $[\boldsymbol{A}_q, \boldsymbol{C}_q]$ is an observable pair, where $\boldsymbol{C}_q = [1 \quad 0]$ (see (53)). Since (40) is in the form of a standard linear time-invariant (LTI) system, a Luenberger observer [12] for state estimation can be constructed for the system, and will be investigated in Section 6. Meanwhile, the closed-loop bioprocess under BS control is given by,

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x}) u_{bs}, \quad \boldsymbol{x}(0) = \boldsymbol{x}_o, \\ y = h(\boldsymbol{x}) = x_1, \end{cases} \tag{41}$$

where u_{bs} is given by (37).

Simulation studies were conducted using (41) with the backstepping gains $c_1 = c_2 = 8$. The reference signal $y_r(t)$ and the initial condition $\boldsymbol{x}(0) = \boldsymbol{x}_o$ were same as those used for the FBL control in Section 3. The simulation is depicted in **Figure 2** which shows that the responses were satisfactory.

5. Comparison of FBL and BS Designs

The simulation results for the FBL versus BS designs using the gains reported in Sections 3 and 4 are shown in **Figure 3** and **Figure 4** for comparison purposes.

It can be seen that both $x_{1fbl}(t) \to y_r(t)$ and $x_{1bs}(t) \to y_r(t)$ asymptotically with no overshoot. It can also be seen that the magnitudes of $u_{fbl}(t)$ are slightly larger than those of $u_{bs}(t)$. However, the reverse can also be obtained by tuning $\boldsymbol{K}_{fbl}$ and $\{c_1, c_2\}$.

6. Observer-Based FBL and BS Controls

As mentioned before that not all state variables are measured in the bioreactor systems; therefore, suitable observers are needed for realizing the full-state feedback control designs proposed in Sections 3 and 4. We shall

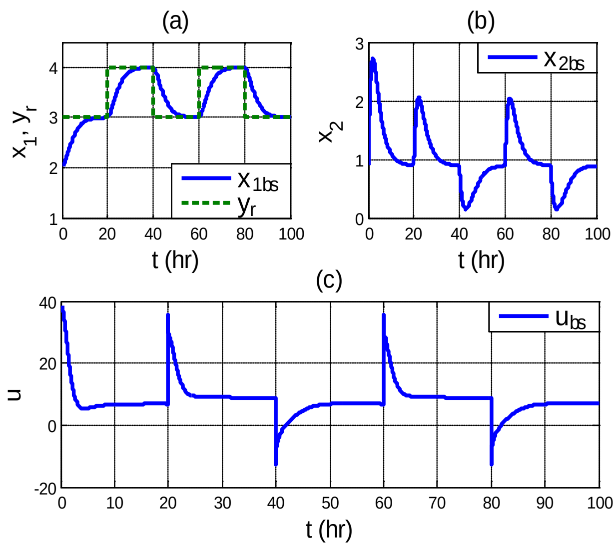

Figure 2. Responses of closed-loop bioprocess (41) under BS control $u_{bs}(t)$ for $c_1 = c_2 = 8$.

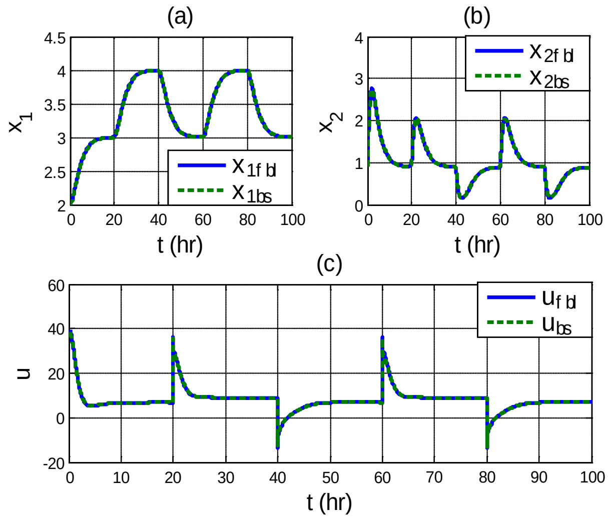

Figure 3. Comparing FBL responses in **Figure 1** and BS responses in **Figure 2**.

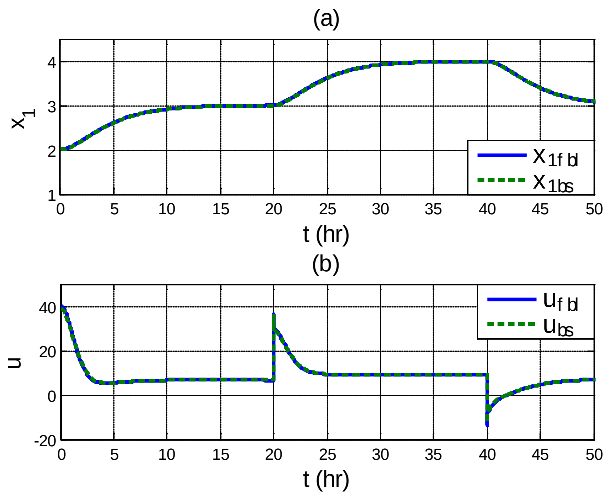

Figure 4. Zoomed-in view of $x_{1fbl}(t)$ and $x_{1bs}(t)$ and $u_{fbl}(t)$ and $u_{bs}(t)$ in **Figure 3**.

investigate the constructions of Luenberger observers for the FBL-based and BS-based control approaches in this section

6.1. Luenberger Observer for FBL Control

Since only $x_1(t)$ is measured in (4), a Luenberger observer [12] can be constructed for full-state estimation needed for full-state control of the bioprocess system. Using (21), a full-state observer can be constructed as

$$\begin{cases} \dot{\hat{\boldsymbol{z}}} = \left(\boldsymbol{A}_c - \boldsymbol{B}_c\boldsymbol{K}_{fbl}\right)\hat{\boldsymbol{z}} + \boldsymbol{B}_c\left(\boldsymbol{K}_{fbl}\boldsymbol{Y}_r + \ddot{y}_r\right) + \boldsymbol{L}\left(y - \hat{y}\right) = \boldsymbol{A}_o\hat{\boldsymbol{z}} + \boldsymbol{B}_c\left(\boldsymbol{K}_{fbl}\boldsymbol{Y}_r + \ddot{y}_r\right) + \boldsymbol{L}y, \\ y = \boldsymbol{C}_c\boldsymbol{z} = z_1 = x_1, \quad \hat{y} = \boldsymbol{C}_c\hat{\boldsymbol{z}} = \hat{z}_1 = \hat{x}_1, \end{cases} \tag{42}$$

where $\boldsymbol{A}_o \triangleq \boldsymbol{A}_c - \boldsymbol{B}_c\boldsymbol{K}_{fbl} - \boldsymbol{L}\boldsymbol{C}_c$ is the observer system matrix and $\boldsymbol{L} \in \mathbb{R}^{2\times 1}$ the observer gain matrix to be determined such that $\boldsymbol{A}_o$ is Hurwitz, provided that $\left[\left(\boldsymbol{A}_c - \boldsymbol{B}_c\boldsymbol{K}_{fbl}\right), \boldsymbol{C}_c\right]$ is an observable pair (which is the case in the present problem). The gain matrix $\boldsymbol{L}$ in (42) can be computed using a Luenberger observer [12] with pole placement (PP) and/or Kalman-Bucy filter [13] design techniques. We shall focus on the PP design method;

henceforth $\boldsymbol{L}$ and $\boldsymbol{A}_o = \boldsymbol{A}_c - \boldsymbol{B}_c\boldsymbol{K}_{fbl} - \boldsymbol{L}\boldsymbol{C}_c$ in (42) will be denoted by $\boldsymbol{L} = \boldsymbol{L}_{pp}$ and $\boldsymbol{A}_{opp} = \boldsymbol{A}_c - \boldsymbol{B}_c\boldsymbol{K}_{fbl} - \boldsymbol{L}_{pp}\boldsymbol{C}_c$, respectively. It should be noted that for a general LTI system characterized by $[\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}]$, where $[\boldsymbol{A}, \boldsymbol{C}]$ is an observable pair, the pair $\left[(\boldsymbol{A}-\boldsymbol{B}\boldsymbol{K}), \boldsymbol{C}\right]$ may not be observable, because full-state feedback can destroy observability; furthermore, $(\boldsymbol{A}-\boldsymbol{B}\boldsymbol{K}-\boldsymbol{L}\boldsymbol{C})$ may be unstable even though $(\boldsymbol{A}-\boldsymbol{L}\boldsymbol{C})$ is designed to be stable [14] [15].

Now using (21) and (42), it can readily be shown that the estimation error $\tilde{\boldsymbol{z}} = \boldsymbol{z} - \hat{\boldsymbol{z}}$ satisfies,

$$\dot{\tilde{\boldsymbol{z}}} = \boldsymbol{A}_{opp}\tilde{\boldsymbol{z}}, \quad \tilde{\boldsymbol{z}}(0) = \tilde{\boldsymbol{z}}_o, \tag{43}$$

where the initial condition $\tilde{\boldsymbol{z}}(0)$ is arbitrary. Since $\boldsymbol{A}_{opp}$ is Hurwitz, it follows that

$$\lim_{t\to\infty}\left[\tilde{\boldsymbol{z}}(t)\right] = \lim_{t\to\infty}\left[\boldsymbol{z}(t) - \hat{\boldsymbol{z}}(t)\right] = 0 \tag{44}$$

for all $\tilde{\boldsymbol{z}}(0) \neq 0$.

Using the transformation defined by (17), the observer described by (42) in the z-coordinates can be transformed back to the x-coordinates as

$$\dot{\hat{\boldsymbol{x}}} = \boldsymbol{Q}_{fbl}^{-1}(\hat{\boldsymbol{x}})\dot{\hat{\boldsymbol{z}}} = \boldsymbol{f}(\hat{\boldsymbol{x}}) + \boldsymbol{g}(\hat{\boldsymbol{x}})\hat{u}_{fbl} + \boldsymbol{Q}_{fbl}^{-1}(\hat{\boldsymbol{x}})\boldsymbol{L}_{pp}(y - \hat{x}_1), \tag{45}$$

where $\boldsymbol{Q}_{fbl}^{-1}(\hat{\boldsymbol{x}}) \triangleq \left(\dfrac{\partial \boldsymbol{T}_{fbl}}{\partial \boldsymbol{x}}(\hat{\boldsymbol{x}})\right)^{-1}$ and $\boldsymbol{Q}_{fbl}(\hat{\boldsymbol{x}})$ is the Jacobian matrix associated with (17) given by

$$\boldsymbol{Q}_{fbl}(\hat{\boldsymbol{x}}) = \begin{bmatrix} 1 & 0 \\ \dfrac{C_1\hat{x}_2}{K_s + \hat{x}_2} - 1 & \dfrac{C_1K_s\hat{x}_1}{(K_s + \hat{x}_2)^2} \end{bmatrix}. \tag{46}$$

In summary, the observer-based control system with feedback linearization for the bioprocess under consideration has the form

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\hat{u}_{fbl}, \quad \boldsymbol{x}(0) = \boldsymbol{x}_o, \tag{47}$$

$$\dot{\hat{\boldsymbol{x}}} = \boldsymbol{f}(\hat{\boldsymbol{x}}) + \boldsymbol{g}(\hat{\boldsymbol{x}})\hat{u}_{fbl} + \boldsymbol{Q}_{fbl}^{-1}(\hat{\boldsymbol{x}})\boldsymbol{L}_{pp}(y - \hat{x}_1), \quad \hat{\boldsymbol{x}}(0) = \hat{\boldsymbol{x}}_o, \tag{48}$$

where $\hat{\boldsymbol{x}}(0)$ is the initial estimate of $\boldsymbol{x}(0)$ and

$$\hat{u}_{fbl} = \beta^{-1}(\hat{\boldsymbol{x}})\left[-\alpha(\hat{\boldsymbol{x}}) + \hat{v}\right], \tag{49}$$

$$\hat{v} = -\boldsymbol{K}_{fbl}\boldsymbol{T}_{fbl}(\hat{\boldsymbol{x}}) + \boldsymbol{K}_{fbl}\boldsymbol{Y}_r + \ddot{y}_r, \tag{50}$$

$$\alpha(\hat{\boldsymbol{x}}) = \frac{(C_1\hat{x}_2 - K_s - \hat{x}_2)(C_1\hat{x}_1\hat{x}_2 - K_s\hat{x}_1 - \hat{x}_1\hat{x}_2)}{(K_s + \hat{x}_2)^2} + \frac{C_1C_2K_S\hat{x}_1^2\hat{x}_2}{(K_s + \hat{x}_2)^3} - \frac{C_1K_S\hat{x}_1\hat{x}_2}{(K_s + \hat{x}_2)^2}, \tag{51}$$

$$\beta(\boldsymbol{x}) = \frac{C_1K_S\hat{x}_1}{(K_s + \hat{x}_2)^2}. \tag{52}$$

6.2. Luenberger Observer for BS Control

In this section we pursue our final objective, *i.e.*, to design a Luenberger observer based on the BS formulation using the error Equation (40). To construct an observer for (40), we need an output equation which can be defined as,

$$\tilde{y} = q_1 = [1 \quad 0]\boldsymbol{q} \triangleq \boldsymbol{C}_q\boldsymbol{q}, \tag{53}$$

where $\tilde{y} = q_1 = x_1 - y_r$ is known and $\left[\boldsymbol{A}_q, \boldsymbol{C}_q\right]$ is an observable pair.

We present the following proposition.

Proposition 1

Consider the bioprocess system described by (3) and (4). A Luenberger observer for the associated error system (40) with measurement given by (53) can be constructed as

$$\dot{\hat{\boldsymbol{q}}} = \boldsymbol{A}_q\hat{\boldsymbol{q}} + \boldsymbol{L}_{bs}\left(\tilde{y} - \boldsymbol{C}_q\hat{\boldsymbol{q}}\right), \quad \hat{\boldsymbol{q}}(0) = \hat{\boldsymbol{q}}_o, \tag{54}$$

where $\tilde{y} - \boldsymbol{C}_q\hat{\boldsymbol{q}} = y - \hat{x}_1$ and $\boldsymbol{L}_{bs} \in \mathbb{R}^{2\times1}$ is the observer gain matrix to be determined by, for example poleplacement, such that the observer matrix $\boldsymbol{A}_{obs} = \boldsymbol{A}_q - \boldsymbol{L}_{bs}\boldsymbol{C}$ is Hurwitz. Since $\boldsymbol{A}_q$ is already Hurwitz and $\left[\boldsymbol{A}_q, \boldsymbol{C}_q\right]$ is an observable pair, $\boldsymbol{L}_{bs}$ can be determined such that the real parts of the eigenvalues of $\boldsymbol{A}_{obs} = \boldsymbol{A}_q - \boldsymbol{L}_{bs}\boldsymbol{C}_q$ lie on the left-side of those of $\boldsymbol{A}_q$ on the open left-half plane, if desirable. Furthermore, (54) can be expressed in the *x*-coordinates as

$$\dot{\hat{\boldsymbol{x}}} = \boldsymbol{f}(\hat{\boldsymbol{x}}) + \boldsymbol{g}(\hat{\boldsymbol{x}})\hat{u}_{bs} + \boldsymbol{Q}_{bs}^{-1}\boldsymbol{L}_{bs}\left(y - \hat{x}_1\right), \quad \hat{\boldsymbol{x}}(0) = \hat{\boldsymbol{x}}_o. \tag{55}$$

Proof: First, we need to show that the estimate $\hat{\boldsymbol{q}}$ converges to its true value $\boldsymbol{q}$. Define the estimation error as $\tilde{\boldsymbol{q}} = \boldsymbol{q} - \hat{\boldsymbol{q}}$. From (40) and (54), it follows that $\tilde{\boldsymbol{q}}$ satisfies,

$$\dot{\tilde{\boldsymbol{q}}} = \dot{\boldsymbol{q}} - \dot{\hat{\boldsymbol{q}}} = \left(\boldsymbol{A}_q - \boldsymbol{L}_{bs}\boldsymbol{C}_q\right)\left(\boldsymbol{q} - \hat{\boldsymbol{q}}\right) \triangleq \boldsymbol{A}_{obs}\tilde{\boldsymbol{q}}, \quad \tilde{\boldsymbol{q}}(0) = \tilde{\boldsymbol{q}}_o, \tag{56}$$

where $\tilde{\boldsymbol{q}}(0)$ is the initial condition. Since $\boldsymbol{A}_{obs}$ is Hurwitz, it follows that

$$\lim_{t\to\infty}\left[\tilde{\boldsymbol{q}}\right] = \lim_{t\to\infty}\left[\boldsymbol{q} - \hat{\boldsymbol{q}}\right] = 0, \tag{57}$$

for arbitrary $\tilde{\boldsymbol{q}}(0)$ Next, using, (24), (25) and (29), the coordinates transformation for the error-system can be obtained as

$$\boldsymbol{q} = \begin{bmatrix} x_1 - y_r \\ \theta_1^{-1}r(x_2)x_1 - \alpha_1 \end{bmatrix} \triangleq \boldsymbol{T}_{bs}\left(\boldsymbol{x}, y_r, \dot{y}_r\right) = \begin{bmatrix} x_1 - y_r \\ \theta_1^{-1}r(x_2)x_1 + (c_1 - 1)x_1 - c_1 y_r - \dot{y}_r \end{bmatrix}. \tag{58}$$

Equation (58) yields the Jacobian matrix

$$\boldsymbol{Q}_{bs}(\boldsymbol{x}) \triangleq \frac{\partial \boldsymbol{T}_{bs}}{\partial \boldsymbol{x}} = \begin{bmatrix} 1 & 0 \\ \theta_1^{-1}r(x_2) + (c_1 - 1) & \theta_1^{-1}\dfrac{\partial r}{\partial x_2}x_1 \end{bmatrix}, \tag{59}$$

where $\boldsymbol{Q}_{bs}$ is nonsingular for $\theta_1^{-1}\dfrac{\partial r}{\partial x_2}x_1 \neq 0$ so that $\boldsymbol{T}_{bs}\left(\boldsymbol{x}, y_r, \dot{y}_r\right)$ is a local diffeomorphism for (3). Equations (58) and (59) yield $\dot{\hat{\boldsymbol{q}}} = \boldsymbol{Q}_{bs}(\hat{\boldsymbol{x}})\dot{\hat{\boldsymbol{x}}}$ and, from (53) and (54),

$$\dot{\hat{\boldsymbol{x}}} = \boldsymbol{Q}_{bs}^{-1}(\hat{\boldsymbol{x}})\dot{\hat{\boldsymbol{q}}} = \boldsymbol{Q}_{bs}^{-1}(\hat{\boldsymbol{x}})\left[\boldsymbol{A}_q\hat{\boldsymbol{q}} + \boldsymbol{L}_{bs}\left(y - \hat{x}_1\right)\right] = \boldsymbol{f}(\hat{\boldsymbol{x}}) + \boldsymbol{g}(\hat{\boldsymbol{x}})\hat{u}_{bs} + \boldsymbol{Q}_{bs}^{-1}(\hat{\boldsymbol{x}})\boldsymbol{L}_{bs}\left(y - \hat{x}_1\right), \tag{60}$$

where $\boldsymbol{Q}_{bs}^{-1}(\hat{\boldsymbol{x}})$ is the inverse of $\boldsymbol{Q}_{bs}(\hat{\boldsymbol{x}})$, and

$$\hat{u}_{bs} = \left(\theta_1^{-1}\hat{x}_1\frac{\partial r}{\partial \hat{x}_2}\right)^{-1}\left[-c_2\hat{q}_2 - \hat{q}_1 - \phi(\hat{\boldsymbol{x}}) + \dot{\hat{\alpha}}_1\right], \tag{61}$$

$$\dot{\hat{\alpha}}_1 = -c_1\dot{\hat{q}}_1 + \dot{\hat{x}}_1 + \ddot{y}_r = -c_1\left(-c_1\hat{q}_1 + \hat{q}_2\right) + \theta_1^{-1}r(\hat{x}_2)\hat{x}_1 - \hat{x}_1 + \ddot{y}_r, \quad c_1 > 0, \tag{62}$$

which complete the proof. □

The observer design technique developed here is interesting and attractive and is different from the two-filter approach in [9]. The technique can be applied to a wide class of BS-based error systems.

In summary, the observer-based control system with the BS formulation for the bioprocess is described by

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\hat{u}_{bs}, \quad \boldsymbol{x}(0) = \boldsymbol{x}_o, \tag{63}$$

$$\dot{\hat{\boldsymbol{x}}} = \boldsymbol{f}(\hat{\boldsymbol{x}}) + \boldsymbol{g}(\hat{\boldsymbol{x}})\hat{u}_{bs} + \boldsymbol{Q}_{bs}^{-1}(\hat{\boldsymbol{x}})\boldsymbol{L}_{bs}(y - \hat{x}_1), \quad \hat{\boldsymbol{x}}(0) = \hat{\boldsymbol{x}}_o, \tag{64}$$

where $\hat{\boldsymbol{x}}(0)$ is the initial estimate of $\boldsymbol{x}(0)$.

Simulation studies for the proposed observer-based FBL and BS controls were conducted and compared. The initial conditions were chosen as $\boldsymbol{x}(0) = [2 \quad 0.9]^{\mathrm{T}}$ and $\hat{\boldsymbol{x}}(0) = [0.8 \quad 0.1]^{\mathrm{T}}$. The set-point reference $y_r(t)$ was the same as before. The model parameters were given in **Table 1** and **Table 2**.

In **Figure 5**, results for observer-based FBL control scheme described by (47) and (48) are shown. It can be seen that the estimates $\hat{x}_{1fbl}(t) \to x_1(t)$ and $\hat{x}_{2fbl}(t) \to x_2(t)$ converged to the true states around $t = 7$ h. In **Figure 6**, results for the observer-based BS control scheme are presented. Convergence of the estimated states to the actual states can also be seen from this figure, and are similar to those presented in **Figure 2**.

In **Figure 7**, the behavior of the error variables q_1 and q_2 defined by (24) and (25) which satisfy (40) in the backstepping scheme is shown. It is evident that $\hat{q}_1(t) \to q_1(t)$ and $\hat{q}_2(t) \to q_2(t)$ smoothly after the transients are over around $t = 8$ h.

7. Conclusion

Observers are critical to control system analysis and designs that employ full-state feedback, where not all the state variables are accessible for on-line, real-time measurements, and/or where the measurements are corrupted by noise. Indeed, the design of suitable linear or nonlinear observers or filters leading to observer-based control technology is an integral part of real world control system applications. In this paper, observer-based control strategies were developed for a nonlinear bioprocess system using feedback linearization and backstepping control techniques; in particular, a Luenberger observer for backstepping control was formulated using the error equation resulted from the backstepping design procedure. The observer design technique developed here is interesting and attractive and is different from the two-filter approach known in the literature. Simulation results with and without observers for both the FBL and BS schemes are presented and compared. The results were excellent and demonstrated the feasibility and effectiveness of the proposed approaches.

Table 2. Controllers and observer gains.

Observer-Based Control	Controller Gain/Coefficients	Observer Gain $\boldsymbol{L}$
FBL	$\boldsymbol{K}_{fbl} = [20 \quad 9]$	$\boldsymbol{L}_{fbl} = [19.4 \quad 92.13]^{\mathrm{T}}$
BS	$[c_1 \quad c_2] = [3.5 \quad 3.5]$	$\boldsymbol{L}_{bs} = [12.4 \quad 35.48]^{\mathrm{T}}$

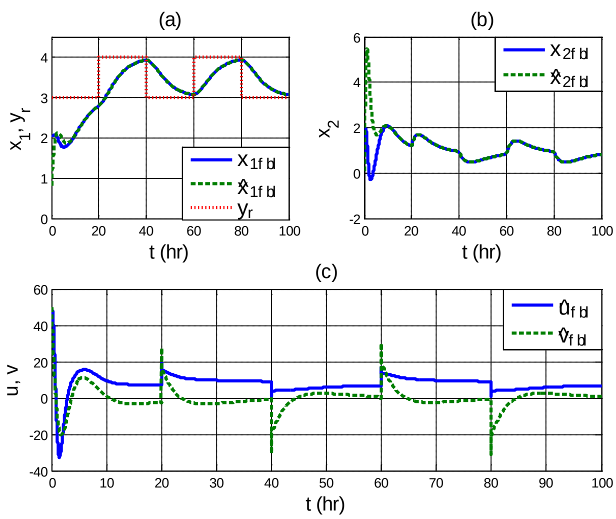

Figure 5. Responses of observer-based FBL control scheme (47) and (48): $\hat{x}_{1fbl}(t) \to x_1(t)$ and $\hat{x}_{2fbl}(t) \to x_2(t)$ smoothly.

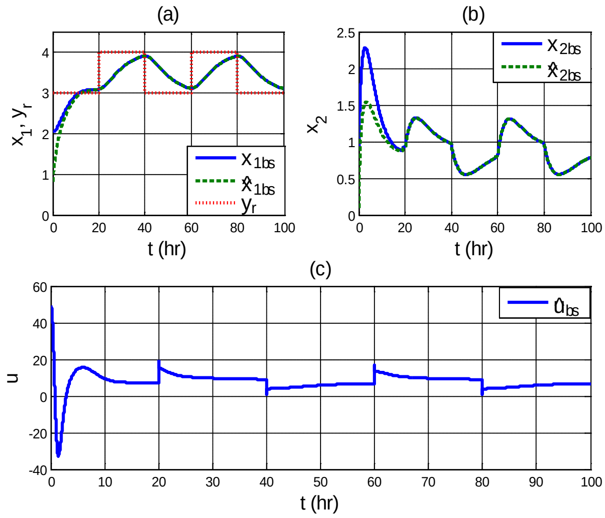

Figure 6. Responses of observer-based BS control scheme (63) and (64): $\hat{x}_{1bs}(t) \to x_1(t)$ and $\hat{x}_{2bs}(t) \to x_2(t)$ smoothly.

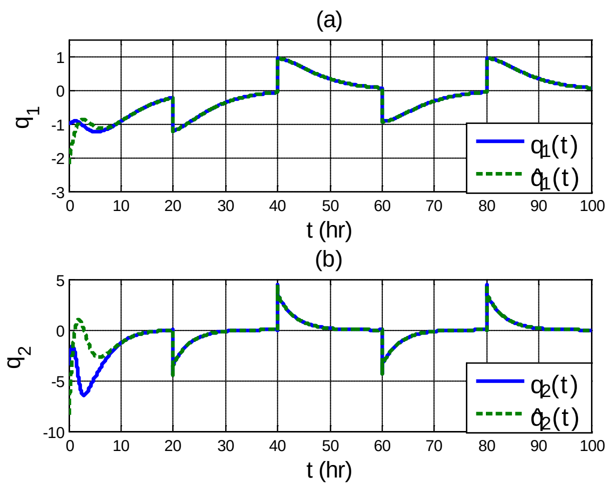

Figure 7. Evolution of the backstepping error variables: $\hat{q}_1(t) \to q_1(t)$ and $\hat{q}_2(t) \to q_2(t)$ smoothly.

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