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Pulse Soliton Solutions of the Modified KdV and Born-Infeld Equations

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ABSTRACT

In this work, we use the Bogning-Djeumen Tchaho-Kofané method to look for all solutions of shape $Sech^n$- of the modified KdV and Born-Infeld Equations. n being a real number, we obtain the soliton solutions when n is positive and the non soliton solutions when n is negative.

Keywords: KdV Equation; Born-Infeld Equation; Soliton Solution; BDKm; Modified KdV Equation

1. Introduction

Dynamics of physical phenomena are more often described analytically by nonlinear partial differential equations (NPDEs). These equations are varied and are met in various branches of the Physics, notably in Classical Mechanics, Quantum Mechanics, Fluid Mechanics, Electrodynamics, and so on. These NPDEs are not always easy to solve. Knowing that all analytic solutions of a NPDE always bring supplementary information, many researchers develop every day the means to solve these NPDEs or to improve some existing solutions. It is in this optics that several methods and techniques facilitating the resolution of NPDEs have been proposed [1-3]. From all these equations, those which contain the scattering terms and nonlinear terms create a particular interest, because they generally admit some soliton solutions; the soliton being a futuristic concept which imposes itself progressively in the world of Physics and especially as alternative in the modern telecommunications. This propensity is translated besides by all recent works that have been published [4-7].

Among these numerous equations, two have drawn our attention in this work. It is precisely the equation of Born-Infeld developed by Born and Infeld in the years 1930 and admits numerous applications in physics [8-15]. The second is the equation of KdV. This equation which is the first that describes the dynamics of the navy waves has been also the object of several investigations and modifications [16-22].

Abdul-Majid Wazwaz in one of his recent works dem-onstrated that the modified KdV equation admitted several types of solutions notably the solitons, the peakons and the cuspons [23]. In this work, we follow the same logic to look for all solutions of the shape $Sech^n$- of the modified KdV equation. We are also going to determine all solutions $Sech^n$- of the equation of Born-Infeld. The problem we want to solve has been motivated in the following manner: We know that the equation of KdV under its initial shape admits a solution in $Sech^n$- (for $n = 2$) but we don't know what happens precisely when $n \neq 2$. Thus, we at first look for the equation of KdV under its initial shape then admit a solution of shape $Sech^n$- with $n \neq 2$. But it is not the first objective of this work, we want to test the efficiency of the method rightly by using on a simplest case. The goal of work is to look for all pulse solutions of the shape $Sech^n$- in the modified KdV and Born-Infeld equations by the Bogning-Djeumen Tchaho-Kofané method (BDKm) [24-28].

This work is organized as follows: in Section 2, we verify if the solution in $Sech^n$- of the initial KdV equation is only obtained for $n = 2$. Section 3 is devoted in research of all solutions in $Sech^n$- of the modified KdV equation. The same analysis is made in Section 4 for equation of Born-Infeld. Finally, in Section 5, we conclude our work.

2. $Sech^n$- Soliton Solution of the Initial KdV Equation

The KdV equation in its initial form is given by

$$\partial_t \phi + \partial_x^3 \phi + \eta \phi \partial_x \phi = 0, \qquad (1)$$

where ϕ represents the amplitude of navy wave, t is the time and x the spatial variable. The problem here is to construct the solution of Equation (1) in the form

$$\phi(x,t) = a\,\text{sech}^n\alpha(x-V_g t), \qquad (2)$$

where a is a constant and n a real number which is different of two. When we introduce the Equation (2) in the Equation (1) we obtain an equation of which the use of the different transformations linked to BDKm [24-28] permits to write it under the form

$$\sum_{i,j,n} F(a_{ij})\Big/\cosh^n\alpha x + \sum_{i,j,m} G(a_{ij})\sinh\alpha x\Big/\cosh^m\alpha x$$

$$+\sum_{i,j,k} H(a_{ij})\cosh^k\alpha x + \sum_{i,j,l} T(a_{ij})\cosh^l\alpha x\sinh\alpha x \qquad (3)$$

$$+\sum_{i,j} W(a_{ij}) = 0.$$

called equation of ranges where $F(a_{ij})$, $G(a_{ij})$, $H(a_{ij})$, $T(a_{ij})$ and $W(a_{ij})$ are functions of the coefficients a_{ij} to determine and i,j,n,m,k,l are positive whole integers.

These transformations concern the terms obtain when we introduce Equation (2) in Equation (1) and they are given by the following equations

$$(\tanh\alpha x)^{2k} = \sum_{i=0}^{k}(-1)^i C_k^i(\text{sech}\alpha x)^{2i}, \qquad (4)$$

$$(\tanh\alpha x)^{2k+1} = \sum_{i=0}^{k}(-1)^i C_k^i(\text{sech}\alpha x)^{2i+1}\sinh\alpha x, \qquad (5)$$

$$(\text{sech}\alpha x)^{2k+2}(\sinh\alpha x)^{2k+1}$$
$$= \sum_{i=0}^{k}(-1)^i C_k^i(\text{sech}\alpha x)^{2i+2}\sinh\alpha x, \qquad (6)$$

$$(\text{sech}\alpha x)^{2k+2n}(\sinh\alpha x)^{2k+1}$$
$$= \sum_{i=0}^{k}(-1)^i C_k^i(\text{sech}\alpha x)^{2i+2n}\sinh\alpha x, \qquad (7)$$

$$(\text{sech}\alpha x)^{2k-1}(\sinh\alpha x)^{2k} = \sum_{i=0}^{k}(-1)^i C_k^i(\text{sech}\alpha x)^{2i-1}, \qquad (8)$$

$$(\text{sech}\alpha x)^{2k+1}(\sinh\alpha x)^{4k+2}$$
$$= \sum_{i=0}^{2k+1}(-1)^i C_{2k+1}^i(\text{sech}\alpha x)^{2i-2k-1}. \qquad (9)$$

$$(\text{sech}\alpha x)^{2k}(\sinh\alpha x)^{2k+1}$$
$$= \sum_{i=0}^{k}(-1)^i C_k^i(\text{sech}\alpha x)^{2i}\sinh\alpha x, \qquad (10)$$

$$(\text{sech}\alpha x)^{4k+2}(\sinh\alpha x)^{2k+1}$$
$$= \sum_{i=0}^{k}(-1)^i C_k^i(\text{sech}\alpha x)^{2(i+k+1)}, \qquad (11)$$

$$(\text{sech}\alpha x)^{2k-1}(\sinh\alpha x)^{2k+1} = \sum_{i=0}^{k}(-1)^i C_k^i(\text{sech}\alpha x)^{2i-1}, (12)$$

$$(\text{sech}\alpha x)^{2k+1}(\sinh\alpha x)^{2k} = \sum_{i=0}^{k}(-1)^i C_k^i(\text{sech}\alpha x)^{2i+1}, \quad (13)$$

where $i,k\in\Box$, $\alpha\in\Box$ and n are fixed integers such that $C_k^i = \dfrac{k!}{i!(k-i)!}$. Thus, substituting Equation (2) in Equation (1), we obtain with the help of the preceding transformations the general equation

$$(na\alpha V_g - \alpha^3 n^3 a)\frac{\sinh\alpha(x-V_g t)}{\cosh^{n+1}\alpha(x-V_g t)}$$

$$-\eta na^2\alpha\frac{\sinh\alpha(x-V_g t)}{\cosh^{2n+1}\alpha(x-V_g t)} \qquad (14)$$

$$+n(n+1)(n+2)a\alpha^3\frac{\sinh\alpha(x-V_g t)}{\cosh^{n+3}\alpha(x-V_g t)} = 0.$$

After haven gotten the Equation (13), the principle of analysis is as follows: before passing to the identification of coefficients of the terms in

$$\sinh\alpha(x-V_g t)\Big/\cosh^i\alpha(x-V_g t) \quad (i = n+1, 2n+1, n+3)$$

of Equation (14), we verify if there exist some values of n for which some terms of the Equation (14) have the same factor $\sinh\alpha(x-V_g t)\Big/\cosh^i\alpha(x-V_g t)$? So for the Equation (14), the factors

$$\sinh\alpha(x-V_g t)\Big/\cosh^{2n+1}\alpha(x-V_g t)$$

and

$$\sinh\alpha(x-V_g t)\Big/\cosh^{n+3}\alpha(x-V_g t)$$

are identical for $n = 2$ (it is sufficient to solve the equation $2n+1 = n+3$). We also notice that the factors

$$\sinh\alpha(x-V_g t)\Big/\cosh^{n+1}\alpha(x-V_g t)$$

and

$$\sinh\alpha(x-V_g t)\Big/\cosh^{2n+1}\alpha(x-V_g t)$$

are identical for $n = 0$. But for $n = 0$, the Equation (14) leads to a trivial solution of the type $\phi(x,t) = a$ (constant) that is not interesting. For $n = 2$, Equation (14) is written as

$$(2a\alpha V_g - 8\alpha^3 a)\frac{\sinh\alpha(x-V_g t)}{\cosh^3\alpha(x-V_g t)}$$

$$+(24a\alpha^3 - 2\eta a^2\alpha)\frac{\sinh\alpha(x-V_g t)}{\cosh^5\alpha(x-V_g t)} = 0. \qquad (15)$$

From Equation (15), we obtain the following ranges of equations

$$12\alpha^2 - \eta a = 0 , \tag{16}$$

and

$$V_g - 4\alpha^2 = 0 . \tag{17}$$

The resolution of Equation (16) and Equation (17) permits to have $a = 12\alpha^2/\eta$ and $\alpha = \pm\sqrt{V_g}/2$. While reporting these values obtained in the expression (2), we at first find a solution considered like particular solution of the Equation (1) and given by

$$\phi(x,t) = \frac{3V_g}{\eta}\text{sech}^2 \pm \frac{\sqrt{V_g}}{2}(x - V_g t), \tag{18}$$

with $V_g \geq 0$ and $\eta \neq 0$. The particular cases $n = 0$ and $n = 2$ being studied, we now look at the case where $n \neq 0$ and $n \neq 2$. So for $n \neq 0$ and $n \neq 2$, the Equation (14) doesn't present more terms that can merge themselves. At this level, the only acceptable solution is the trivial solution $U(x,t) = 0$ which doesn't have importance. In conclusion, the equation of KdV as considered above admits non trivial solutions only for $n = 2$ and Equation (18) also gives the general solution of the KdV equation considered under its initial shape given by Equation (1).

3. Sech^n- Soliton Solution of the Modified KdV Equation

The modified KdV equation that we are going to use here is in the form [23]

$$\begin{aligned} UU_{xxt} - U_x U_{xt} - 4U^3 U_t + 4UU_{xxx} \\ -4U_x U_{xx} - 16U^3 U_x = 0. \end{aligned} \tag{19}$$

We look for the solutions of Equation (19) under the shape

$$U_n(x,t) = a\text{sech}^n(\alpha x - \alpha_0 t), \tag{20}$$

where a and α are constants to be determined, α_0 a known constant and n a real number to determine. Thus, taking into account Equation (20) into Equation (19) gives with the help of transformations (4), (5),···, (13) the following ranges equation

$$\begin{aligned} &\left[-a\alpha^2\alpha_0(n+1)(n+2) + a^2\alpha^2\alpha_0 n^2(n+1) \right. \\ &\left. +4a^2\alpha^3 n(n+1)(n+2) - 4a^2\alpha^3 n^2(n+1) \right] \\ &\cdot \frac{\sinh(\alpha x - \alpha_0 t)}{\cosh^{2n+3}(\alpha x - \alpha_0 t)} \\ &+ \left(16a^4\alpha n - 4a^4 n\alpha_0\right)\frac{\sinh(\alpha x - \alpha_0 t)}{\cosh^{4n+1}(\alpha x - \alpha_0 t)} = 0. \end{aligned} \tag{21}$$

The two terms of the Equation (21) merge themselves for $n = 1$ (it is sufficient to solve the equation $2n + 3 = 4n + 1$). So from the Equation (21), we obtain

$$\left(8\alpha - 2\alpha_0\right)a^3 + \left(\alpha^2\alpha_0 + 8\alpha^3\right)a - 3\alpha^2\alpha_0 = 0, \tag{22}$$

with $a \neq 0$. Equation (22) can also be written as

$$a^3 + \left(\frac{\alpha^2\alpha_0 + 8\alpha^3}{8\alpha - 2\alpha_0}\right)a - \frac{3\alpha^2\alpha_0}{8\alpha - 2\alpha_0} = 0, \tag{23}$$

with $\dfrac{\alpha}{\alpha_0} \neq \dfrac{1}{4}$. By setting $P = \dfrac{\alpha^2\alpha_0 + 8\alpha^3}{8\alpha - 2\alpha_0}$ and $q = \dfrac{-3\alpha^2\alpha_0}{8\alpha - 2\alpha_0}$, Equation (23) becomes

$$a^3 + Pa + q = 0 . \tag{24}$$

By using the Cardano's method, we set $a = u + v$ leads to the relations

$$u^3 + v^3 = -q , \tag{25}$$

and

$$u^3 v^3 = \frac{-P^3}{27} . \tag{26}$$

The resolution of the coupled system (25) and (26) gives

$$a = \sqrt[3]{\frac{-q + \sqrt{\dfrac{27q^2 + 4P^3}{27}}}{2}} + \sqrt[3]{\frac{-q - \sqrt{\dfrac{27q^2 + 4P^3}{27}}}{2}} . \tag{27}$$

Therefore the solution obtained for $n = 1$ is given by

$$U_1(x,t) = \left[\sqrt[3]{\frac{-q + \sqrt{\dfrac{27q^2 + 4P^3}{27}}}{2}} \right. \\ \left. + \sqrt[3]{\frac{-q - \sqrt{\dfrac{27q^2 + 4P^3}{27}}}{2}} \right] \text{sech}(\alpha x - \alpha_0 t). \tag{28}$$

For $n \neq 1$, Equation (21) leads to the following relations:

Term in $\dfrac{\sinh(\alpha x - \alpha_0 t)}{\cosh^{4n+1}(\alpha x - \alpha_0 t)}$,

$$\frac{\alpha}{\alpha_0} = \frac{1}{4} . \tag{29}$$

Term in $\dfrac{\sinh(\alpha x - \alpha_0 t)}{\cosh^{2n+3}(\alpha x - \alpha_0 t)}$,

$$a = \frac{1}{n} \quad \text{with } n \neq 0, n \neq 1 . \tag{30}$$

Then for $n \neq 0$ and $n \neq 1$, the solution of the modi-

fied KdV equation is given by

$$U_n(x,t) = \frac{1}{n}\operatorname{sech}^n \frac{\alpha_0}{4}(x-4t). \qquad (31)$$

In the following section, we are going to make an analogous survey for the equation of Born-Infeld.

4. Sech^n- Solution of the Born-Infeld's Equation

The Born-Infeld's equation that we want to analyze in this section is given by

$$\left(1-U_t^2\right)U_{xx} + 2U_x U_t U_{xt} - \left(1+U_x^2\right)U_{tt} = 0. \qquad (32)$$

Like in the preceeding section, we look for the solution of Equation (31) in the form of Equation (20). With the help of the transformations (4), (5),$\cdots$, (13), we obtain

$$\frac{a\left(\alpha^2-\alpha_0^2\right)n^2}{\cosh^n(\alpha x-\alpha_0 t)} + \frac{a\left(\alpha_0^2-\alpha^2\right)n(n+1)}{\cosh^{n+2}(\alpha x-\alpha_0 t)}$$
$$+\frac{2\alpha^2\alpha_0^2\left(a^2-a^3\right)n^4}{\cosh^{3n}(\alpha x-\alpha_0 t)}$$
$$+\frac{4\alpha^2\alpha_0^2\left(a^3-a^2\right)n^2+2\alpha^2\alpha_0^2\left(a^3-a^2\right)n^3}{\cosh^{3n+2}(\alpha x-\alpha_0 t)} \qquad (33)$$
$$+\frac{2\alpha^2\alpha_0^2\left(a^2-a^3\right)n^3(n+1)}{\cosh^{3n+4}(\alpha x-\alpha_0 t)} = 0.$$

As in the previous cases, we first look for the values of n for which the terms which constituted Equation (33) merge themselves. Thus, the terms in $1/\cosh^{n+2}(\alpha x-\alpha_0 t)$ and $1/\cosh^{3n+2}(\alpha x-\alpha_0 t)$ merge for $n=0$ as well as the terms in $1/\cosh^n(\alpha x-\alpha_0 t)$ and $1/\cosh^{3n}(\alpha x-\alpha_0 t)$. The terms in $1/\cosh^{n+2}(\alpha x-\alpha_0 t)$ and $1/\cosh^{3n+4}(\alpha x-\alpha_0 t)$ as well as the terms in $1/\cosh^{3n+2}(\alpha x-\alpha_0 t)$ and $1/\cosh^n(\alpha x-\alpha_0 t)$ merge for $n=-1$. The terms in $1/\cosh^n(\alpha x-\alpha_0 t)$ and $1/\cosh^{3n+4}(\alpha x-\alpha_0 t)$ merge for $n=-2$. The terms in $1/\cosh^{n+2}(\alpha x-\alpha_0 t)$ and $1/\cosh^{3n}(\alpha x-\alpha_0 t)$ merge for $n=1$. Then, for $n \in \{-2,-1,0,1\}$, the Equation (33) contains the terms which merge. In the continuation we are going to study in detail the Equation (33) when n is equal to these found values.

- For $n=-2$, the resolution of Equation (33) gives $\alpha=\pm\alpha_0$ and $a=1$. The solution of Equation (32) in this case is given by

$$U_{-2}(x,t) = \operatorname{sech}^{-2}\left[\pm\alpha_0(x-t)\right]$$
$$= \cosh^2\left[\pm\alpha_0(x-t)\right]. \qquad (34)$$

- For $n=-1$, the resolution of Equation (33) gives $\alpha=\pm\alpha_0$ and $a=1$. The solution of Equation (32) in this case is given by

$$U_{-1}(x,t) = \operatorname{sech}^{-1}\left[\pm\alpha_0(x-t)\right]$$
$$= \cosh\left[\pm\alpha_0(x-t)\right]. \qquad (35)$$

- For $n=0$, we obtain $U_0(x,t)=a$ which is a constant.
- For $n=1$, the Equation (33) becomes

$$\frac{4a\left(\alpha^2-\alpha_0^2\right)}{\cosh(\alpha x-\alpha_0 t)} + \frac{2a\left(\alpha^2-\alpha_0^2\right)+2\alpha^2\alpha_0^2\left(a^2-a^3\right)}{\cosh^3(\alpha x-\alpha_0 t)}$$
$$+\frac{6\alpha^2\alpha_0^2\left(a^3-a^2\right)}{\cosh^5(\alpha x-\alpha_0 t)} + \frac{4\alpha^2\alpha_0^2\left(a^2-a^3\right)}{\cosh^7(\alpha x-\alpha_0 t)} = 0. \qquad (36)$$

While equating the terms of the Equation (36) to be zero we obtain $\alpha=\pm\alpha_0$; $a=1$ and the solution of the Equation (32) is

$$U_1(x,t) = \operatorname{sech}\left[\pm\alpha_0(x-t)\right]. \qquad (37)$$

In the case where $n \neq \{-2,-1,0,1\}$, we identify the terms of Equation (32) to zero and look for the non trivial solutions $(a \neq 0)$ which give $\alpha=\pm\alpha_0$ and $a=1$. So we can affirm without hesitation that the general solution of the equation of Born-Infeld is

$$U_n(x,t) = \operatorname{sech}^n\left[\pm\alpha_0(x-t)\right],$$
$$\forall n \in \square. \qquad (38)$$

5. Conclusion

In this work, we constructed with success the solutions of shape Sech^n- of the KdV and Born-Infeld equations. Our survey has been guided by the fact that, we already know that by a direct integration, the initial KdV equation admits a solution in Sech^n- $(n=2)$ and we don't know what happens precisely when $n \neq 2$. To this effect, we supposed that its solution is under the shape Sech^n- to arrive at the conclusion $n=2$. The survey has been extended to the case of the modified KdV and Born-Infeld equations. The solutions of shape

$$U_n(x,t) = \frac{1}{n}\operatorname{sech}^n \frac{\alpha_0}{4}(x-4t),$$

for $n \neq 0$, $n \neq 1$ and $U_n(x,t) = \operatorname{sech}^n \pm\alpha_0(x-t)$,

$\forall n \in \square$, have been obtained respectively. The success of this survey is due to the BDKm whose mastery permits to push the analysis as far as possible in the equations that present an elevated nonlinearity as seen in the two equations studied scrupulously.

6. Acknowledgements

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