

# Size Exclusion Mechanism, Suspension Flow through Porous Medium

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## ABSTRACT

A lot of investigations have been done in order to understand the mechanisms of the transport of particulate suspension flow through porous medium. In general, Deep Bed Filtration studies have been conducted to analyse the mechanism involved in the processes of capturing and retaining particles occurs throughout the entire depth of the filter and not just on the filter surface. In this study, the deep bed filtration mechanism and the several mechanisms for the capture of suspended particles are explained then the size exclusion mechanism has been focused (particle capture from the suspension by the rock by the size exclusion). The effects of particle flux reduction and pore space inaccessibility due to selective flow of different size particles will be included in the model for deep bed filtration. The equations for particle and pore size distributions have been derived. The model proposed is a generalization of stochastic Sharma-Yortsos equations. Analytical solution for low concentration is obtained for any particle and pore size distributions. As we will see, the averaged macro scale solutions significantly differ from the classical deep bed filtration model.

**Keywords:** Transport; Porous Medium; Particulate Suspension Flow; Size Exclusion

## 1. Introduction

The following model predicts that the particle breakthrough happens after injection of one pore volume. Nevertheless, several cases where the break through time significantly differs from one pore volume injected, have been reported in the literature for particulate and polymer suspensions [1].

That model does not distinguish between different mechanisms of formation damage so it can not be used for diagnostic purposes. Several attempts to correlate the formation damage with sizes of particles and pores were unsuccessful [2] (A model for average concentrations is not general enough or may be size exclusion mechanisms never dominate).

Sharma and Yortsos [3] derived basic population balance equations for the transport of particulate suspensions in porous media. It is assumed that an overall pore space is accessible for particles and the particle population moves with the averaged flow velocity of the carrier water. In the case of a porous medium with the uniform pores size distribution, this assumption results in independent deep bed filtration of different particle size populations. Nevertheless, as we will see, if we consider size exclusion mechanism, either smaller particles than the pore or larger particles, do not perform deep bed filtration.

The pore size exclusion assumes that the particles can only enter larger pores, so, only a fraction of porosity will be accessible for particles, *i.e.* the water flux carrying particles of a fixed size is just a fraction of the overall water flux via porous media.

Here, analytical solution will shows for a small pore size variation medium, only the intermediate size particles perform deep bed filtration. In this case, the population velocity is particle size-dependent. The averaged equations for deep bed filtration of intermediates size particles differ from the classical deep bed filtration.

## 2. Deep Bed Filtration

The deep bed filtration system consists of equations for the particle mass balance, for the particle capture kinetics and of Darcy's law [4,5]

$$\begin{aligned} \frac{\partial c(X, T)}{\partial T} + \frac{\partial c(X, T)}{\partial X} &= -\frac{1}{\phi} \frac{\partial c(X, T)}{\partial T}, \\ \frac{\partial \sigma(X, T)}{\partial T} &= \lambda(\sigma) \phi c(X, T), \\ U &= -\frac{k_o k(\sigma)}{\mu L} \frac{\partial p}{\partial X}, \end{aligned} \quad (1)$$

where  $\lambda(\sigma)$  is the dimensionless filtration coefficient,  $c(X, T)$  is the suspended particle concentration,  $\sigma(X, T)$

is the deposited particle concentration, and the formation damage function  $k(\sigma)$ , shows the permeability declines due to particle deposition.

If we assume the suspension as an incompressible fluid, the velocity  $U$  is independent of  $X$  and we can solve the third Equation (1) (dynamical model) independently.

In the case of constant filtration coefficient, the particle penetration depth equals  $\frac{1}{\lambda}$ , but here, as we focused on the size exclusion capture, the phenomenological model (1) does not account for particle size distributions (the larger the particles, the smaller are the pores, and the higher is the capture rate).

Particles do not move with the carrier water velocity, although we have continuity Equation (1). In one dimensional deep bed filtration, suspended concentration shock that moves with carrier water velocity, the suspended and captured concentrations are equal to zero ahead of this shock [4].

### 3. Advective Velocity

In order to discuss particle transport and determine the average velocity of particle suspension, the velocity distribution at the scale of the each pore must be considered. By approximating each pore as a capillary tube (a rough analogy), the velocity distribution for the fluid will be parabolic with a no slip condition at the walls.

A particle will not be able to travel the same pathways as the carrier water, because the particle center of mass will be excluded from the immediate region of the wall. They will also be excluded from pores smaller than the particle.

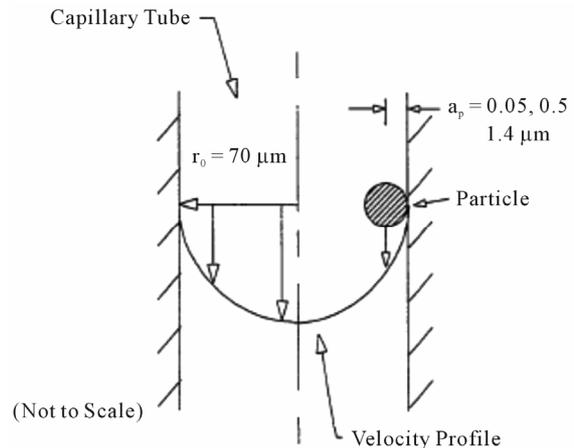
The result of this exclusion based upon size is that the particles will take on an average velocity which is greater than that of the carrier water [6].

The particle flowing through a capillary tube and subsequent size exclusion is shown in **Figure 1**.

### 4. Derive the Equations

In size exclusion mechanism, some particles are captured by the rock from the suspension, *i.e.* if the large particle arrives at a small pore,  $r_p < r_s$ , it is captured and plugs the pore; and a small particle  $r_p < r_s$  passes the pore without being captured (both large and small particles, do not perform deep bed filtration).

The geometric model structure of the pore space is as follows: the porous space is a bundle of parallel capillary; the flux through each pore is proportional to the fourth power of its radius; complete mixing takes place at length scale, *i.e.* there is a nonzero probability for a particle moving through any point  $x$  to get into any pore at the point  $x+l$ .



**Figure 1. Graphic representation of the size exclusion principle for a particle flowing through a capillary tube.**

The complete mixing of different size particles occurs in the chambers. The capture occurs at the thin pore inlet, where large particles arrive. So an inlet cross-section of each parallel capillary section acts as a sieve.

A particle with the radius  $r_s$  passes through the pore with radius  $r_p$  only if the particle radius is smaller than the pore radius,  $r_p < r_s$ . Therefore, small pores are inaccessible pore volume. We introduce the accessibility factor  $\gamma$  for particles with radius  $r_s$  as a fraction of pore volume with capillary radii larger than  $r_s$ :

$$\gamma(r_s, x, t) = \frac{\int_{r_s}^{\infty} r_p^2 H(r_p, x, t) dr_p}{\int_0^{\infty} r_p^2 H(r_p, x, t) dr_p} \quad (2)$$

Let us define the flux  $J(r_s, x, t) dr_s * dr_p$  of particles with specific radius  $r_s$  via pores with a specific radius  $r_p$  and also the total flux  $J(r_s, x, t) dr_s$  of particles with radii in the interval  $[r_s, r_s + dr_s]$ :

$$J(r_s, x, t) dr_s = \left[ \int_{r_s}^{\infty} J(r_s, r_p, x, t) dr_p \right] dr_s \quad (3)$$

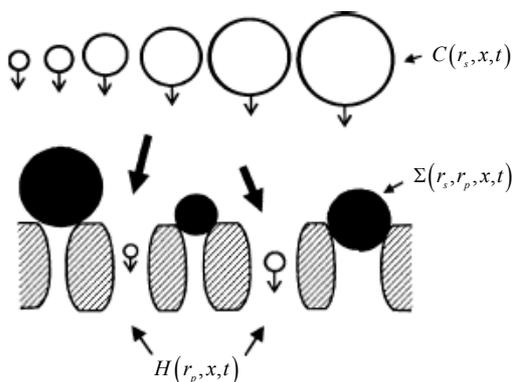
The flux of particles with radius  $r_s$  via pores with smaller radius ( $r_p < r_s$ ) equal zero. Therefore, the water flux carrying  $r_s$ -particles is lower than the overall water flux in the porous medium.

$$J(r_s, x, t) dr_s = UC(r_s, x, t) \frac{\int_{r_s}^{\infty} H(r_p, x, t) r_p^4 dr_p}{\int_0^{\infty} H(r_p, x, t) r_p^4 dr_p} dr_s \quad (4)$$

Introducing the fraction of the total flux that carries particles with radius  $r_s$ :

$$\alpha(r_s, x, t) = \frac{\int_{r_s}^{\infty} r_p^4 H(r_p, x, t) dr_p}{\int_0^{\infty} r_p^4 H(r_p, x, t) dr_p} \quad (5)$$

So following formula is the flux of particles with radii



**Figure 2. Schema of the large particle entrapment by small pores.**

varying from  $r_s$  to  $r_s + dr_s$  :

$$J(r_s, x, t) dr_s = U\alpha(r_s, x, t)C(r_s, x, t)dr_s \quad (6)$$

Formula for the flux reduction and accessibility factors ((2) and (5)) can be derived for regular pore networks using effective medium or percolation theories [7].

#### 4.1. Fraction of Particles Trapped and Retained According to Sharma and Yortsos (1987)

To derive local rates for particle removal due to mechanical entrapment, they focused on a representative volume of the porous medium with a statistically large number of pores. They assume fluid flows through the medium at a constant superficial velocity  $q$ , firstly. Then, they denote by  $n$  the average number of pore throats a fluid particle encounters in the volume element before emerging from it. If  $\Delta t$  is the time taken for the fluid to traverse the volume element, then:

$$n = \frac{q\Delta t}{\phi l_p} \quad (a.1)$$

where:  $n$  = number of steps;  $\phi$  = porpsity;  $q$  = fluid superficial velocity,  $L \cdot T^{-1}$ ;  $l_p$  = effective pore length,  $L$ .

Pore length ( $l_p$ ) is constant. As the fluid carries suspended particles, a certain fraction of the latter is trapped by the pore throats at each of the  $n$  steps. If the fraction of particles of size in the interval  $(r_s < r < r_s + dr_s)$  trapped at each step is  $P(r_s)$ , the mass balance on particles of this size at the conclusion of the  $i$ th step reads as follows,

$$(\rho_s f_s) = (\rho_s f_s)_{i-1} [1 - P(r_s)] \quad (a.2)$$

$$P(r_s) = \frac{\text{no. of particles in } (r_s, r_s + dr_s) \text{ trapped in } i\text{th step}}{\text{no. of particles in } (r_s, r_s + dr_s) \text{ before } i\text{th step}} \quad (a.3)$$

where:  $\rho$  = concentration, no.  $L^{-3}$ ;  $f$  = size distribution,  $L^{-1}$ ;  $P(r_s)$  = fraction of particles retained per step.

They proceed by assuming that the above probability of

trapping is constant at each particle step. At the end of  $n$  steps the fraction of particles trapped by the sequence of  $n$  steps,  $P_i$ , assumed independent, is given by

$$P_i = P(r_s) \sum_{i=1}^n [1 - P(r_s)]^{i-1} = 1 - [1 - P(r_s)]^n \quad (a.3)$$

In the case of low concentrated suspensions, the pore space fraction occupied by retained particles is negligibly small if compared with the overall pore space. Therefore, the porosity is assumed to be constant. The population balance equation is derived as the following form:

$$\phi \frac{\partial [\gamma(r_s, x, t)C(r_s, x, t)]}{\partial t} + \frac{\partial [U\alpha(r_s, x, t)C(r_s, x, t)]}{\partial x} = - \frac{\partial \Sigma(r_s, x, t)}{\partial t} \quad (7)$$

The number of particles with size in the interval  $[r_s, r_s + dr_s]$  captured in pores with radius in the interval  $[r_p, r_p + dr_p]$  per unit of time is called the particle-capture rate. This rate is proportionality coefficient is called the filtration coefficient  $\lambda'(r_s, r_p)$  :

$$\lambda'(r_s, r_p) = 0 : r_p > r_s \quad (8)$$

Finally, for incompressible aqueous suspension and in a closed system for three unknowns  $C(r_s, x, t)$ ,  $\Sigma(r_s, x, t)$  and  $H(r_p, x, t)$  we will have:

$$\frac{\partial H(r_p, x, t)}{\partial t} = -U \frac{r_p^4 H(r_p, x, t)}{\int_0^\infty r_p^4 H(r_p, x, t) dr_p} \int \lambda'(r_s, r_p) C(r_s, x, t) dr_s \quad (9)$$

Introduction of dimensionless variables

$$X = \frac{x}{L}, T = \frac{Ut}{L\phi}, \lambda = \lambda' L \quad (10)$$

So:

$$\frac{\partial H(r_p, X, T)}{\partial T} = -\phi \frac{r_p^4 H(r_p, X, T)}{\int_0^\infty r_p^4 H(r_p, X, T) dr_p} \int \lambda'(r_s, r_p) C(r_s, X, T) dr_s \quad (11)$$

The boundary condition at the core inlet correspond to the injection of water with a given particle size distribution  $C^{(0)}(r_s, T)$ . The injected  $r_s$ -particle flux is equal to  $C^{(0)}(r_s, T)U$ . The inlet core/reservoir cross-section acts as a sieve. The injected  $r_s$ -particles are carried into the porous medium by a fraction of water flux via accessible pores-  $\alpha^{(0)}(r_s, T)U$  (Figure 2). The injected  $r_s$ -particles carried by water flux via inaccessible pores  $[1 - \alpha^{(0)}(r_s, T)]U$  are deposited at the outer surface of the inlet and form the external filter cake from the very

beginning of injection. For particles larger than any pore, there are no accessible pores and the flux reduction factor is zero. So, all these particles are retained at the inlet cross-section, contributing to external filter cake growth. On the other hand, for particles smaller than the smallest pore, they will enter the porous medium without being captured. (deep bed filtration will not perform in both condition). The particles retained at the outer surface of the inlet large particles do not restrict access of newly arriving particles to the core inlet before the transition time [7] Finally,

$$h(X, T) = h_0(X) - \sigma(X, T) \tag{12}$$

Equation (12) shows that one particle can plug only one pore and vice versa.

### 4.2. Filtration in a Single Pore Size Medium

Distribution of suspended particles and pores in a single pore size medium are illustrated bellow.

**Figure 3(a)** shows the pore size distribution (Dirac's delta function) at  $T = 0$  and the particle size distribution in the injected suspension at  $X = 0$ . If we consider the propagation of small particles with  $r_s < r'_p$ . For this case, formulae (2) and (5) show that  $\alpha = \gamma = 1$ ; *i.e.* all pores are accessible for small particles, and there is no flux reduction.

Therefore, small particles are transported with the velocity of carrier water without being captured (no pores will be plugged by small particles).

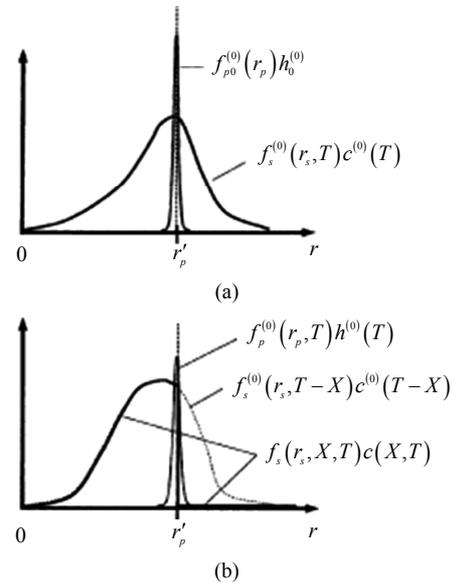
Now consider the propagation of large particles  $r_s > r'_p$ . In this case, from Equations (2) and (5) it follows that  $\alpha = \gamma = 0$ . Therefore, none of the pores is accessible for large particles, and there is no large particle flux. So, all large particles are deposited in the inlet cross-section (they never arrive at the core out-let). It was also observed in a laboratory study [8] where size exclusion was the dominant capture mechanism.

It is important to highlight that, depending on the size, the particles in uniform pore size medium either pass or are trapped. Therefore, the deep bed filtration, where there exists an average penetration length for each size particle, does not happen in the case of particulate flow in a single size porous media. The penetration length is zero for large particles, and is infinite for small particles.

### 5. Highlighted Assumptions

It was assumed that the aqueous suspension is incompressible so the velocity  $U$  in Equation (1) is independent of  $X$  and we can solve the third Equation (1) independently (dynamical model separates from the kinematics model).

There were no deposited particles and plugged pores at the beginning of deep bed filtration. There are no sus-



**Figure 3. (a) initial and boundary concentration distributions for pores and suspended particles; (b) particle distribution for any  $X$  and  $T$ ; pore distribution at the inlet cross section for  $T > 0$ .**

pended particles ahead of the injected water front.

The porosity is assumed constant in the case of low concentrated suspensions.

We assumed that the particles retained at the outer surface of the inlet large particles do not restrict access of newly arriving particles to the core inlet before the transition time [9]. The external cake does not form a solid matrix before the transition time and cannot capture the particles from the injected suspension.

### 6. Conclusions

Particles are not captured during flow through pore system, but there is a sequence of particle capturing sieves perpendicular to the flow direction.

Absence of particles in the pores that are smaller than the particles, results in reduction of the particle carrying water flux if compared with the overall water flux. So, only a fraction of the pore space is accessible for particles.

The analytical solution for flow in a single pore size  $r'_p$  medium shows that capture free advection of small particles ( $r_s < r'_p$ ) takes place, and large particles ( $r_s > r'_p$ ) do not penetrate into the porous medium (there is no deep bed filtration in a uniform pore size medium).

Large particles never arrive at the core outlet. It was observed in a laboratory study [8] where size exclusion was the dominant capture mechanism.

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## Table of Symbols

$c$  = suspended particle concentration in carrier fluid;

$\sigma$  = particle retained concentration;

$k_{det}$  = detachment rate coefficient;

$U$  = flow velocity;

$U_s$  = fluid velocity;

$U_p$  = particle velocity;

$U_o$  = fluid center line velocity;

$r$  = radial distance;

$r_o$  = capillary radius;

$a_p$  = particle radius;

$p$  = dynamic pressure;

$x$  = longitudinal distance;

$D_L$  = longitudinal dispersion coefficient ( $L \cdot T^{-1}$ );

$D$  = free fluid molecular diffusion coefficient of solute ( $L^2 \cdot T^{-1}$ );

$V_s$  = fluid interstitial velocity ( $L \cdot T^{-1}$ ); and

$Pe$  = Peclet number =  $\frac{V_s \cdot d_g}{D}$

$Pe_D$  = dynamic Peclet number =  $\frac{V_s \cdot d_g}{D_L}$

$C$  = particle concentration ( $M \cdot L^{-3}$ );

$x$  = longitudinal position ( $L$ ); and

$\lambda$  = filter coefficient ( $L^{-1}$ ).

$w_s$  = particle settling velocity;

$V_s$  = fluid interstitial velocity;

$\rho_{pf}$  = densities of particle and fluid, respectively;

$g$  = gravitational acceleration;

$H$  = Hamakar constant (ergs);

$N_G$  = gravitational group =  $\eta_G$

$D_{LP}$  = particle longitudinal dispersion coefficient ( $L^2 T^{-1}$ );

$D_p$  = particle molecular diffusion coefficient in a free fluid ( $L^2 T^{-1}$ );

$V_p$  = particle velocity ( $L^2 T^{-1}$ );

$d_g$  = media grain diameter ( $L$ ); and

$d_p$  = particle diameter ( $L$ )