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Stress and Strain Accumulation Due to a Long Dip-Slip Fault Movement in an Elastic-Layer over a Viscoelastic Half Space Model of the Lithosphere-Asthenosphere System

Sanjay Sen[1], Subrata Kr. Debnath[2]

[1]Department of Applied Mathematics, University of Calcutta, Kolkata, India
[2]Department of Basic Science and Humanities, Meghnad Saha Institute of Technology
(A Unit of Techno India Group), Kolkata, India
Email: dr.sanjaysen@rediff.com, skd.sxccal@gmail.com

ABSTRACT

Most of the earthquake faults in North-East India, China, mid Atlantic-ridge, the Pacific seismic belt and Japan are found to be predominantly dip-slip in nature. In the present paper a dip-slip fault is taken situated in an elastic layer over a viscoelastic half space representing the lithosphere-asthenosphere system. A movement of the dip-slip nature across the fault occurs when the accumulated stress due to various tectonic reasons e.g. mantle convection etc., exceeds the local friction and cohesive forces across the fault. The movement is assumed to be slipping in nature, expressions for displacements, stresses and strains are obtained by solving associated boundary value problem with the help of integral transformation and Green's function method and a suitable numerical methods is used for computation. A detailed study of these expressions may give some ideas about the nature of stress accumulation in the system, which in turn will be helpful in formulating an earthquake prediction programme.

Keywords: Aseismic Period; Dip-Slip Fault; Earthquake Prediction; Mantle Convection; Plate Movements; Stress Accumulation; Tectonic Process; Viscoelastic-Layered Model

1. Introduction

It is the observational fact that while some faults are strike slip (finite or infinite in length) in nature, there are faults (e.g., Sierra Nevada/Owens valley: Basin and Range faults, Rocky Mountains, Himalayas, Atlantic fault of central Greece—a steeply dipping fault with dip 60, 80 (deg)) where the surface level changes during the motion *i.e.* the faults are dip-slip in nature.

A pioneering work involving static ground deformation in elastic media was initiated by [1,2]. Ref. [3] did a wonderful work in analyzing the displacement, stress and strain for dip-slip movement. Later some theoretical models in this direction have been formulated by a number of authors like [4-30]. Ref. [31] has discussed various aspects of fault movement in his book. Ref. [32] has discussed stress accumulation near buried fault in lithosphere-asthenosphere system. The work of [33] can also

be mentioned in these connections.

In most of these works the medium were taken to be elastic and/or viscoelastic, but a layered model with elastic layer(s) over elastic or viscoelastic half space will be a more realistic one for lithosphere-asthenosphere system.

In the present case we consider a long dip-slip fault situated in an elastic layer over a viscoelastic half space which reaches up to the free surface. The medium is under the influence of tectonic forces due to mantle convection or some related phenomena. The fault is assumed to undergo a slipping movement when the stresses in the region exceed certain threshold values.

In our paper, we consider an elastic layer over a viscoelastic half space to represent the lithosphere-asthenosphere system, with constant rigidity (2.0×10^5 Mpa) and viscosity (10^{20} - 10^{21} pa·s) following the observational data mentioned by [34,35]. Analytical expressions for

displacements, stresses and strains in the system are obtained both before and after the fault movement using appropriate mathematical technique involving integral transformation and Green's function. Numerical computational works have been carried out with suitable values of the model parameters and the nature of the stress and strain accumulation in the medium have been investigated.

2. Formulation

We consider a long dip-slip fault F and width D situated in an elastic layer over a viscoelastic half space of linear Maxwell type.

A Cartesian co-ordinate system is used with a suitable point O on the strike of the fault as the origin,

Y_1 axis along the strike of the fault, Y_2 axis is as shown in **Figure 1** and y_3 axis pointing downwards. We choose another co ordinate system Y_1', Y_2' and Y_3' axes as shown in **Figure 1** below, so that the fault is given by $F : \left(y_2' = 0, 0 \le y_3' \le D \right)$. Let θ be the dip of the fault F and v_1, w_1 be the displacement components along y_2 and y_3 axes respectively for the layer and v_2, w_2 be that for the half-space. τ_{ij}^k, e_{ij}^k, are stress and strain, $k = 1$ for the layer and $k = 2$ for the half-space and $i, j = 2, 3$.
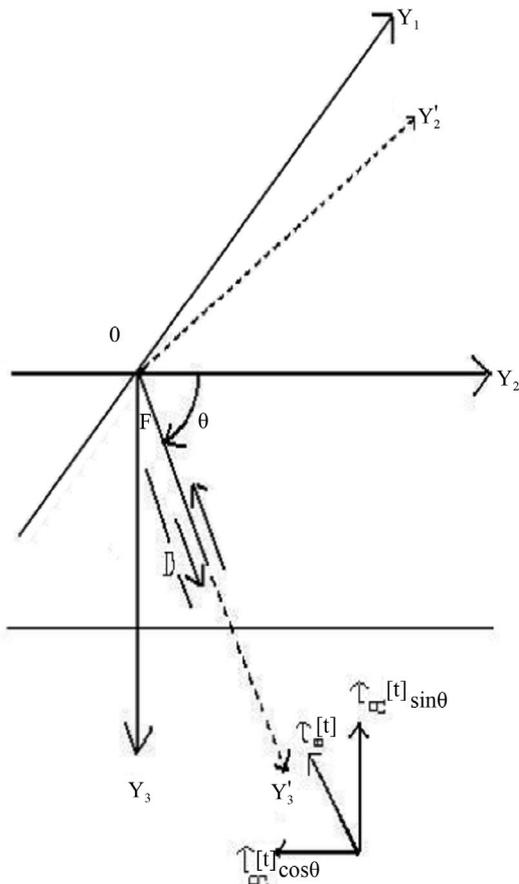
Figure 1. Section of the model by the plane $y_1 = 0$.

2.1. For an Elastic Medium the Constitutive Equations Are Taken as

For the layer: M1

$$\tau_{22}^1 = \mu_1 \frac{\partial v_1}{\partial y_2} \tag{2.1}$$

$$\tau_{23}^1 = \frac{1}{2} \mu_1 \left(\frac{\partial v_1}{\partial y_2} + \frac{\partial w_1}{\partial y_2} \right) \tag{2.2}$$

$$\tau_{33}^1 = \mu_1 \frac{\partial w_1}{\partial y_3} \tag{2.3}$$

2.2. For a Viscoelastic Maxwell Type Medium the Constitutive Equations Are Taken as

For half-space: M2

$$\left(\frac{1}{\eta_2} + \frac{1}{\mu_2} \frac{\partial}{\partial t} \right) \tau_{22}^2 = \frac{\partial}{\partial t} \left(e_{22}^2 \right) = \frac{\partial}{\partial t} \left(\frac{\partial v_2}{\partial y_2} \right) \tag{2.4}$$

$$\left(\frac{1}{\eta_2} + \frac{1}{\mu_2} \frac{\partial}{\partial t} \right) \tau_{23}^2 = \frac{\partial}{\partial t} \left(e_{23}^2 \right) = \left(\frac{1}{2} \right) \frac{\partial}{\partial t} \left(\frac{\partial v_2}{\partial y_3} + \frac{\partial w_2}{\partial y_2} \right) \tag{2.5}$$

$$\left(\frac{1}{\eta_2} + \frac{1}{\mu_2} \frac{\partial}{\partial t} \right) \tau_{33}^2 = \frac{\partial}{\partial t} \left(e_{33}^2 \right) = \frac{\partial}{\partial t} \left(\frac{\partial w_2}{\partial y_3} \right) \tag{2.6}$$

where, μ_1, μ_2 are the effective rigidity of the layer and the half-space respectively and η_2 is the effective viscosity of the half-space.

2.3. The Stresses Satisfy the Following Equations (Assuming No Changes in the External Body Force)

For M1:

$$\frac{\partial}{\partial y_2} \left(\tau_{22}^1 \right) + \frac{\partial}{\partial y_3} \left(\tau_{23}^1 \right) = 0 \tag{2.7}$$

$$\frac{\partial}{\partial y_2} \left(\tau_{32}^1 \right) + \frac{\partial}{\partial y_3} \left(\tau_{33}^1 \right) = 0 \tag{2.8}$$

where $\left((-\infty < y_2 < \infty), 0 \le y_3 \le H \right)$
For M2:

$$\frac{\partial}{\partial y_2} \left(\tau_{22}^1 \right) + \frac{\partial}{\partial y_3} \left(\tau_{23}^2 \right) = 0 \tag{2.9}$$

$$\frac{\partial}{\partial y_2} \left(\tau_{32}^2 \right) + \frac{\partial}{\partial y_3} \left(\tau_{33}^2 \right) = 0 \tag{2.10}$$

where $\left((-\infty < y_2 < \infty), y_3 \ge H, t \ge 0 \right)$ and assuming quasi-static deformation for which the inertia term are neglected.

The boundary conditions are taken as, with $t = 0$ representing an instant when the medium is aseismic state.

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$\tau_{22}^1(y_2,y_3,t)=0$ as $-\infty<y_2<\infty,0\le y_3\le H,t\ge0$ (2.11)

On the free surface $y_3=0, t\ge0$

$$\tau_{23}^1(y_2,y_3,t)=0 \qquad (2.12)$$

$$\tau_{33}^1(y_2,y_3,t)=0 \qquad (2.13)$$

Also as $y_3\to\infty\left(-\infty<y_2<\infty,t\ge0\right)$

$$\tau_{2'3'}^2(y_2,y_3,t)=\tau_\infty(t) \qquad (2.14)$$

$$\tau_{33}^2(y_2,y_3,t)=0 \qquad (2.15)$$

at $y_3=H,(-\infty<y_2<\infty,t\ge0)$

$$\tau_{33}^1(y_2,y_3,t)=\tau_{33}^2(y_2,y_3,t)=\tau_H\;(\text{say}) \qquad (2.16)$$

at $y_3=H,(-\infty<y_2<\infty,t\ge0)$

$$w_1(y_2,y_3,t)=w_2(y_2,y_3,t) \qquad (2.17)$$

[where $\tau_\infty(t)$ is the shear stress maintained by mantle convection and other tectonic phenomena throughout the medium].

2.4. The Initial Conditions Are

Let $\left((v)_2\right)_0,\left((w)_2\right)_0,\left(\tau_{rs}^2\right)_0$ and $\left(e_{rs}^2\right)_0 r,s=2,3$ be the value of $(v_2),(w_2),\left(\tau_{rs}^2\right)$ and $\left(e_{rs}^2\right)$ at time $t=0$ which are functions of y_2, y_3 and satisfy the relations (2.1)-(2.17).

a) Solutions in the absence of any fault dislocation [36,37]:

The boundary value problem given by (2.1)-(2.17), can be solved by taking Laplace transformation with respect to time "t" of all the constitutive equations and the boundary conditions. On taking the inverse Laplace transformation we get the solutions for displacement, stresses as:

For M1:

$v_1(y_2,y_3,t)$

$=(v_1)_0+\left[(y_3)/\mu_1\right]\times\left[\tau_\infty(t)-\tau_\infty(0)+\frac{\mu_1}{\eta_1}\times\int_0^t\tau_\infty(\tau)\mathrm{d}\tau\right]$

$w_1(y_2,y_3,t)=(w_1)_0+(y_3)\times\dfrac{1-\mu_1}{\mu_1}\times\tau_H$

$\tau_{22}^1(y_2,y_3,t)=\left(\tau_{22}^1\right)_0$

$\tau_{23}^1(y_2,y_3,t)$

$=\left(\tau_{23}^1\right)_0+\dfrac{1}{\mu_1}\times\left[\tau_\infty(t)-\tau_\infty(0)+\dfrac{\mu_1}{\eta_1}\times\int_0^t\tau_\infty(\tau)\mathrm{d}\tau\right]$ (A)

From the above solution we find that τ_{22}^1 remain unchanged from the initial one, while τ_{23}^1 increases line-

arly with time if we assume that $\tau_\infty(t)$ to be constant. We assume that the geological conditions as well as the characteristic of the fault is such that when $\tau_{2'3'}$ reaches some critical value, say $\tau_c<\tau_\infty(t)$ the fault F undergoes a sudden slip along the dip direction.

The magnitude of the sudden slip shall satisfy the following conditions as discussed by [13].

(C1) Its value will be maximum on the free surface.

(C2) The magnitude of the slip will decrease with y_3 as we move downwards and ultimately tends to zero near the lower edge of the fault.

$$\left(y_2'=0,y_3'=D\right)$$

b) Solutions after the fault movements [36,37]:

We assume that after a time T_1, the stress component $\tau_{2'3'}^2$ (which is the main driving force for the dip-slip motion of the fault) exceeds the critical value τ_c, and the fault F undergoes a sudden slip along the dip direction, characterized by a dislocation across the fault given in (**Appendix**).

We solve the resulting boundary value problem by modified Green's function method following [1,2,24] and correspondence principle (as shown in **Appendix**) and get the solution for displacements, stresses and strain as:

For M1:

$v_1(y_2,y_3,t)$

$=(v_1)_0+\left[y_3/\mu_1\right]\times\left[\tau_\infty(t)-\tau_\infty(0)+\dfrac{\mu_1}{\eta_1}\times\int_0^t\tau_\infty(\tau)\mathrm{d}\tau\right]$

$\int_F g(x_3')[(A_1/B_1)+(C_1/D_1)$

$-\sum_1^\infty\left(\dfrac{\alpha}{\beta}\right)^m\times A_m(t)\phi(y_2,y_3,\theta)\mathrm{d}x_3',\ 0\le y_3\le H$

$w_1(y_2,y_3,t)$

or,

$=(w_1)_0+(y_3)\times\dfrac{1-\mu_1}{\mu_1}\times\tau_H+\psi(y_2,y_3,t)(\text{say})$

where,

$\psi(y_2,y_3,t)$

$=(U/(2\times\pi))\times\int_F g(x_3')\left[(A_1/B_1)+(C_1/D_1)\right]$

$-\sum_1^\infty\left(\dfrac{\alpha}{\beta}\right)^m(\alpha/\beta)^m\times A_m(t)\phi(y_2,y_3,\theta)\mathrm{d}x_3'$

and

$A_m(t)=1+\sum_1^m\binom{m}{r}(2s/1-s)^r\left[1-e^{-a_1t}e_{r-1}(a_1t)\right]$

$s=\dfrac{\mu_2}{\mu_1},\alpha=\dfrac{\mu_1}{\mu_2}-1,\beta=\dfrac{\mu_1}{\mu_2}+1,a_1=\dfrac{\mu_1\mu_2}{(\mu_1+\mu_2)\times\eta_2}$

$$b_1 = \frac{2 \times \mu_1 \mu_2^2}{(\mu_1^2 - \mu_2^2) \times \eta_2}, \quad e_n(z) = 1 + \sum_1^n z^i, e_0(z) = 1,$$

$$B_{rn} = \binom{n}{r}, b_1^r, A_{rm} = \binom{m}{r} [(b_1/a_1)]^r$$

and

$$A_1 = (y_2) \sin \theta - (y_3) \cos \theta,$$

$$B_1 = \left[(x_3')^2 - 2(x_3')(y_2 \cos \theta + y_3 \sin \theta) + (y_2^2) + (y_3^2) \right],$$

$$C_1 = (y_2) \sin \theta + (y_3) \cos \theta, D_1$$

$$= ((x_3')^2) - 2(x_3')(y_2 \cos \theta - y_3 \sin \theta) + (y_2^2) + (y_3^2)$$

where,

$$\phi(y_2, y_3, \theta)$$
$$= ((A_1)_{-+}/(B_1)_{-+}) + ((A_1)_{--}/(B_1)_{--})$$
$$- ((C_1)_{+-}/(D_1)_{+-}) - ((C_1)_{++})/((D_1)_{++})$$

where,

$$(A_1)_{-+} = (y_2) \sin \theta - (2 \times m \times H + y_3) \cos \theta,$$

$$(B_1)_{-+} = [(x_3'^2) - 2(x_3')^2 (y_2 \cos \theta + (2 \times m \times H + y_3) \sin \theta)$$

$$(A_1)_{-+} = (y_2) \sin \theta - (2 \times m \times H + y_3) \cos \theta,$$

$$(B_1)_{-+} = \left[(x_3'^2) - 2(x_3')^2 (y_2 \cos \theta + (2 \times m \times H + y_3) \sin \theta) \right.$$
$$\left. + (y_2^2 + (2 \times m \times H + y_3)^2) \right],$$

$$(A_1)_{--} = (y_2) \sin \theta - (2 \times m \times H - y_3) \cos \theta,$$

$$(B_1)_{--} = \left[(x_3'^2) - 2(x_3')^2 (y_2 \cos \theta + (2 \times m \times H - y_3) \sin \theta) \right.$$
$$\left. + (y_2^2 + (2 \times m \times H - y_3)^2) \right],$$

$$(C_1)_{+-} = (y_2) \sin \theta + (2 \times m \times H - y_3) \cos \theta,$$

$$(D_1)_{+-} = ((x_3')^2 - 2(x_3')(y_2 \cos \theta - (2 \times m \times H - y_3) \sin \theta)$$
$$+ (y_2^2) + (2 \times m \times H - y_3)^2)$$

$$(C_1)_{++} = (y_2) \sin \theta + (2 \times m \times H + y_3) \cos \theta,$$

$$(D_1)_{++} = ((x_3')^2 - 2(x_3')(y_2 \cos \theta - (2 \times m \times H + y_3) \sin \theta)$$
$$+ (y_2^2) + (2 \times m \times H + y_3)^2)$$

$$\tau_{22}^1(y_2, y_3, t) = (\tau_{22}^1)_0$$

$$\tau_{23}^1(y_2, y_3, \theta, t)$$

$$= (\tau_{23}^1)_0 + \frac{1}{\mu_1} \times \left[\tau_\infty(t) - \tau_\infty(0) + \frac{\mu_1}{\eta_1} \times \int_0^t \tau_\infty(\tau) d\tau \right]$$

$$+ \mu_1 \times \frac{\partial}{\partial y_2} \psi(y_2, y_3, \theta, t) = (\tau_{23}^1)_0 + \frac{1}{\mu_1} \times [\tau_\infty(t) - \tau_\infty(0)$$

$$+ \frac{\mu_1}{\eta_1} \times \int_0^t \tau_\infty(\tau) d\tau \right] + \mu_1 \times \psi_2(y_2, y_3, \theta, t) (\text{say})$$

where, $$\psi_2(y_2, y_3, \theta, t) = \frac{\partial}{\partial y_2} \psi(y_2, y_3, \theta, t) \quad \text{(B)}$$

3. Numerical Computations

Following [38] and recent studies on rheological behavior of crust and upper mantle by [34,35] the values of the model parameters are taken as:

$\mu_1 = 3 \times 10^{11}$ dyne/cm^2, $\mu_2 = 3.5 \times 10^{11}$ dyne/cm^2,

$\eta_2 = 3 \times 10^{20}$ poise

D = Depth of the fault = 10 km [noting that the depth of all major earthquake faults are in between 10 - 15 km].

H = Thickness of the layer = 40 km. say (though the thickness varies from region to region of the Earth).

$t_1 = t - T_1$.

$\tau_\infty(t) = 2 \times 10^8$ dyne/cm^2 (200 bars), [post seismic observations reveal that stress released in major earthquake are of the order of 200 bars, in extreme cases it may be 400 bars]

$$(\tau_{23}^1)_0 = 5 \times 10^7 \text{ dyne/cm}^2 (50 \text{ bars})$$

and, $\tau_\infty(0) = 0$

We take the function $g(x_3') = W[((x_3')^2 - D^2)^2 / (D)^4]$, with W = 1 cm/year, satisfying the conditions stated in $(C_1) - (C_2)$.

We now compute the following quantities:

For layer M1:

$$W_1(y_2, y_3, \theta, t)$$

$$= w_1(y_2, y_3, \theta, t) - (w_1)_0 - (y_3) \times \frac{1 - \mu_1}{\mu_1} \times \tau_H \quad (3.1)$$

$$t_{23}^1(y_2, y_3, \theta, t)$$

$$= \tau_{23}^1(y_2, y_3, \theta, t) \quad (3.2)$$

$$- \frac{1}{\mu_1} \times \left[\tau_\infty(t) - \tau_\infty(0) + \frac{\mu_1}{\eta_1} \times \int_0^t \tau_\infty(\tau) d\tau \right]$$
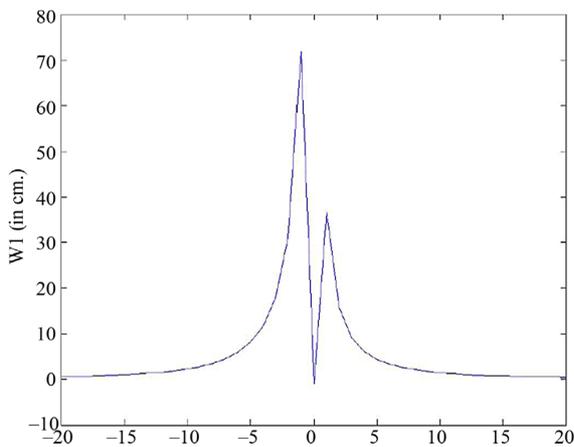
4. Results and Discussions

The parameters involved in the expressions for displacements and stresses have been assigned appropriate values available from observed data through repeated geodetic surveys prior to and after the seismic events in seismically active regions in South and North America and China. Numerical computations are carried out using this observed values of the parameters as discussed in §3.
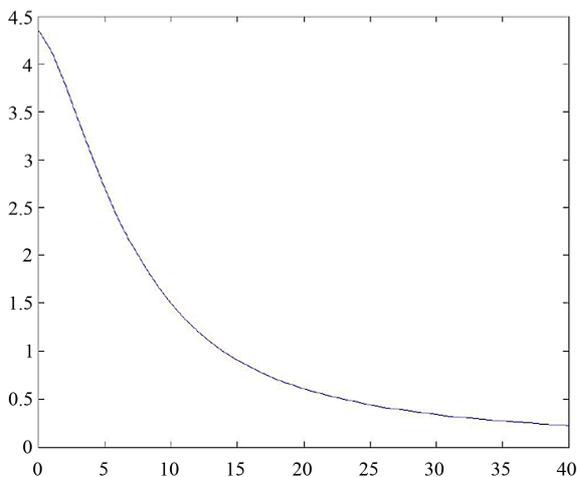
4.1. Variation of Vertical Component of Displacement Due to Sudden Slip across the Fault after t_1 = 1 Year

"Equation (3.1)" gives us the vertical component of displacement at (y_2, y_3) due to the movement across the fault

for different dip angle θ and at different time after the fault movement. We take $t_1 = 1$ year. In **Figure 2** the graph shows the nature of surface displacement W_1 against y_2, the distance from the strike of the fault with $\theta = 90$ (in deg). It is observed that the displacements are in opposite directions across the strike of the fault. Their magnitudes gradually decrease and tend to zero as we move away from the fault. This is quite expected as the effect of the fault movement gradually dies out with distance. The sudden changes of W_1 near $y_2 = 0$ is due to the dip-slip motion along the fault. This is in good conformity with the result shown in Paul Segal (2010). **Figure 3** shows the variation of W_1 with depth y_3 along the vertical through a point $y_2 = 5$ km. for a vertical fault with $\theta = 90$ (in deg). It shows that W_1 decreases sharply up to a depth of about 18 km. and thereafter diminishes to zero at a slower rate, and continuously decreases to zero for $y_3 > 100$ km.

4.2. Variation with Depth of the Main Driving Stress $t_{2'3'}$ in the Dip-Slip Direction Due to the Movement across F

Figures 4-7 show the variation of $t_{2'3'}$ with depth y_3 for various θ and some specific values of y_2.

In **Figure 4** it is found that for a dip-slip fault with dip angle $\theta = 30$ and along the line $y_2 = 8$ km, $t_{2'3'}$ undergoes a change (in one year) due to the slip movement across F. Initially there is a very small region of stress-release just below the free surface ($0 < y_3 < 4.5$ km). Thereafter, the rate of stress-release decreases up to a depth of 13 km and then stress begin to accumulate in the region from $y_3 = 13$ km to $y_3 = 20$ km after that the rate of stress accumulation gradually decreases continuously and becomes negligibly small at a depth of 50 km.

Figure 5 shows the variation of main driving stress component $t_{2'3'}$ with the depth of $y_2 = 8$ km, $t_1 = 1$ year

Figure 2. Variation of vertical component of displacement W_1 with y_2 for $y_3 = 0$ km, $\theta = 90$ (deg), $t_1 = 1$ year due to the fault movement.
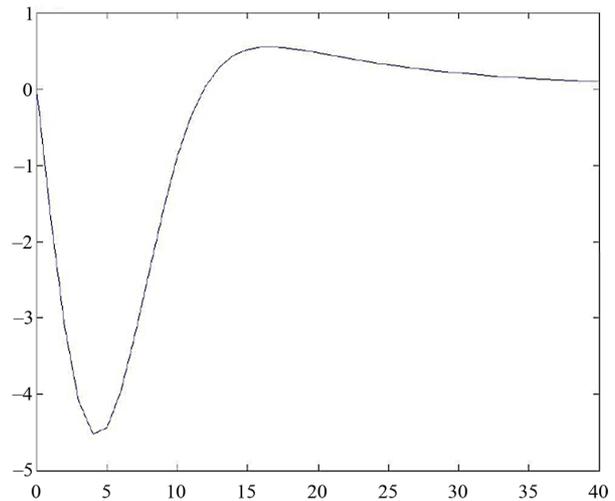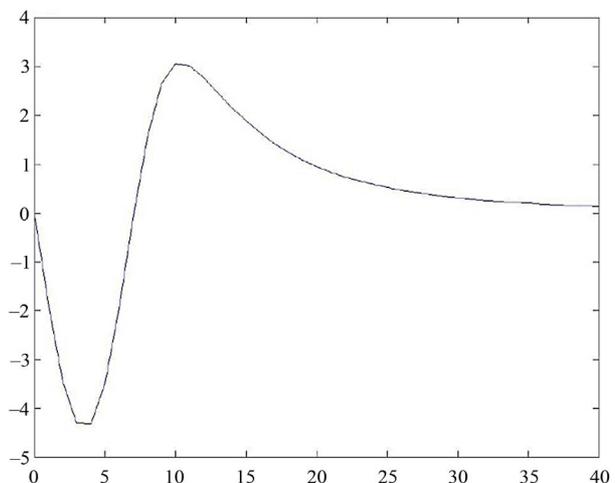

Figure 4. Variation of stress $t_{2'3'}$ with depth y_3 for $y_2 = 8$ km, $\theta = 30$ (deg), $t_1 = 1$ year due to the fault movement.

Figure 3. Variation of vertical component of displacement W_1 with depth y_3 for $y_2 = 2$ km, $\theta = 90$ (deg), $t_1 = 1$ year due to the fault movement.
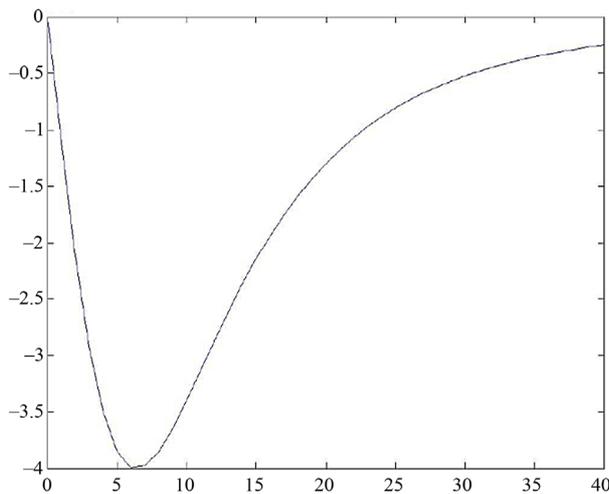

Figure 5. Variation of stress $t_{2'3'}$ with depth y_3 for $y_2 = 8$ km, $\theta = 45$ (deg), $t_1 = 1$ year due to the fault movement.

due to the slip movement across the fault at a deep angle $\theta = 45$ (degree). The rate of stress release increases up to a depth of 4 km. Then the rate decreases up to a depth of about 4 to 8 km. And then begin to accumulate from 8 to 20 km. with a maximum of 3 bar per year at a depth of about 10 km. And continuously decreases to zero at about 75 km from the free surface. **Figure 6** shows the variation of stress component $t_{2'3'}$ for $y_2 = 9$ km, $t_1 = 1$ year and dip angle $\theta = 60$ (degree) with the depth due to the slip across the fault.

We see stress releases in the region $0 \leq y_3 \leq 40$ km. with increasing rate up to $y_3 = 6.5$ km and then with decreasing rate and become 0 at a depth about 100 km.

For vertical dip-slip fault the nature is the same with less numerical values, which is explained by **Figure 7**.

For vertical dip-slip fault the nature is the same with less numerical values, which is explained by **Figure 2**.

Figure 6. Variation of stress $t_{2'3'}$ with depth y_3 for $y_2 = 9$ km, $t_1 = 1$ year, $\theta = 60$ (deg) due to fault movement.
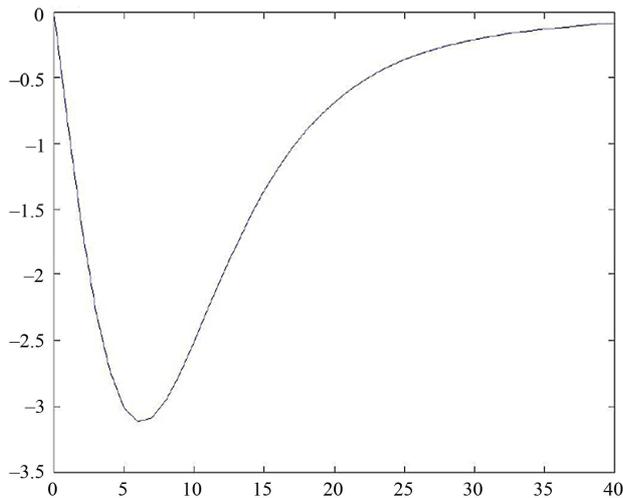
Figure 7. Variation of stress $t_{2'3'}$ with depth y_3 for $y_2 = 10$ km, $t_1 = 1$ year, $\theta = 90$ (deg) due to fault movement.

5. Conclusions

Thus in the above discussion we see that due to slip movement in the dip-slip fault, there are regions where stress get released and there are certain other regions where stress accumulates. The rate of stress release/accumulation depends essentially on the dip-angle θ and the distance y_2 from the fault.

If there were a second fault situated in the region where stress get released due to the movement across the first fault, the possibility of a movement across the second fault would likes to be deferred further. On the other hand, if the second fault be situated in the region where stress accumulates, the possibility of a movement across the second fault will be enhanced. The study of such interactions among neighboring faults is very important in seismically active regions where there are a number of neighboring faults exist.

6. Acknowledgements

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Appendix

Solutions after the Fault Movement

We assume that after a time T_1 the stress component $\tau_{2'3'}$ (which is the main driving force for the dip-slip motion of the fault) exceeds the critical value τ_c, the fault F undergoes a sudden slip. Then we have an additional condition characterizing the dislocation in w due to the slipping movement as:

$$\left[(w)\right]_{iF} = Ug(x'_3)H(t_1) \quad (1)$$

where, U is the maximum dislocation along the dip of the fault occurred at the free surface, it's value gradually diminishes with increasing direction of x'_3 governed by the function $g(x'_3)$ and $H(t_1)$ is the Heaviside function and $\left[(w)\right]_{iF} =$ The discontinuity of w_i across F given by

$$\left[(w)\right]_{iF} = \lim_{(y'_2 \to 0+)}(w)_i - \lim_{(y'_2 \to 0+)}(w)_i \quad (2)$$

$$(y'_2 = 0, 0 \le y'_3 \le D)$$

Taking Laplace transformation in (1) we get,

$$\overline{\left[(w)\right]_{iF}} = \frac{U}{p}g(y'_3) \quad (3)$$

where P is Laplace transformation variable and

$$\overline{\left[(w)\right]_{iF}} = \int_0^\infty \left[(w)\right]_{iF}\, e^{-pt}\mathrm{d}t\,(P > 0)$$

The fault-slip commences across F after time T_1. clearly,

$$\left[(w)\right]_{iF} = 0$$

for $t_1 \ge 0$, where $t_1 = t - T_1$, F is located in the region ($y'_2 = 0, 0 \le y'_3 \le D$).
We try to find the solution as:

$$(v)_t = \left((v)_i\right)_1 + \left((v)_i\right)_2, (w)_i = \left((w)_i\right)_1 + \left((w)_i\right)_2$$

$$\tau_{22}^i = \left(\tau_{22}^i\right)_1 + \left(\tau_{22}^i\right)_2, \tau_{23}^i = \left(\tau_{23}^i\right)_1 + \left(\tau_{23}^i\right)_2,$$

$$\tau_{33}^i = \left(\tau_{33}^i\right)_1 + \left(\tau_{33}^i\right)_2 \quad (4)$$

where $i = 1, 2$ and $\left((v)_1\right)_1, \left((w)_1\right)_1, \left(\tau_{rs}^i\right)_1$, are continuous everywhere in the model and are given by (A). While the second part $\left((v)_i\right)_2, \left(\tau_{rs}^i\right)_2$ are obtained by solving modified boundary value problem as stated below. We note that $\left((v)_2\right)_2$, is continuous even after the fault creep, so that $\left(\left[(v)_2\right]\right)_2 = 0$, while $\left((w)_2\right)_2$ satisfies the dislocation condition given by (2).
where, $i = 1, 2$ and $r, s = 2, 3$.

The resulting boundary value problem can now be stated as: $\left((v)_1\right)_2 \left((w)_1\right)_2 \left((v)_2\right)_2 \left((w)_2\right)_2$ satisfies 2D

Laplace equations as

$$\nabla^2\left(\overline{\left((w)_2\right)_2}\right) = 0\ \nabla^2\left(\overline{\left((v)_2\right)_2}\right) = 0$$

$$\nabla^2\left(\left((v)_1\right)_2\right) = 0, \nabla^2\left(\left((w)_1\right)_2\right) = 0 \quad (5)$$

where, $\overline{\left((w)_2\right)_2}$ is the Laplace transformation of $\left((w)_2\right)_2$, with the modified boundary conditions,
For M1:

$$\tau_{22}^1\left(y_2, y_3, \theta\right) = 0 \text{ as } |y_2| \to \infty, y_3 \ge 0 \quad (6)$$

$$\tau_{23}^1\left(y_2, y_3, \theta\right) = 0 \text{ as } y_2 \to \infty\,(-\infty < y_2 < \infty) \quad (7)$$

$$\tau_{33}^1(y_2, y_3, \theta) = 0 \text{ as } y_3 \to \infty\,(-\infty < y_2 < \infty) \quad (8)$$

and the other boundary conditions are as usual.
For M2:

$$\overline{\tau_{22}^1}\left(y_2, y_3, \theta, p\right) = 0 \text{ as } |y_2| \to \infty, y_3 \ge 0 \quad (9)$$

$$\overline{\tau_{23}^1}\left(y_2, y_3, \theta, p\right) = 0 \text{ as } y_2 \to \infty\,(-\infty < y_2 < \infty) \quad (10)$$

$$\overline{\tau_{33}^1}\left(y_2, y_3, \theta, p\right) = 0 \text{ as } y_3 \to \infty\,(-\infty < y_2 < \infty) \quad (11)$$

and the other boundary conditions are as usual.
We solve the above boundary value problem by modified Green's function method following [1,2,4], and the correspondence principle.

Let $Q(y_2, y_3)$ be any point in the layer, $Q_1(y_2, y_3)$ be any point in the half-space and $P(x_2, x_3)$ be any point in the fault, then we have,
For M1:

$$\overline{\left((w)_1\right)_2}(Q) = \int_F U \times g(x'_3)\left[G(y_2, y_3, x_2, x_3)\right]\mathrm{d}x'_3$$

where, $\left[G(y_2, y_3, x_2, x_3)\right] = \dfrac{\partial}{\partial x_2}G_1(y_2, y_3, x_2, x_3)$

and, $\left[G(y_2, y_3, x_2, x_3)\right] = [(A_1/B_1) + (C_1/D_1) - \displaystyle\sum_1^\infty$

$$\left(\frac{\mu_1 - \bar{\mu}_2}{\mu_1 + \bar{\mu}_2}\right)^m \times \phi(y'_2, y'_3, \theta)\mathrm{d}x'_3$$

where, $0 \le y_3 \le H$
and,

$$A_1 = (y_2)\sin\theta - (y_3)\cos\theta$$

$$B_1 = \left[\left(x_3'^2\right) - 2(x'_3)(y_2\cos\theta + y_3\cos\theta) + \left(y_2^2\right) + \left(y_3^2\right)\right]$$

$$C_1 = (y_2)\sin\theta + (y_3)\cos\theta$$

$$D_1 = \left((x'_3)^2\right) - 2(x'_3)(y_2\cos\theta - y_3\sin\theta) + \left(y_2^2\right) + \left(y_3^2\right)$$

where,

$$\phi(y'_2, y'_3, \theta)$$
$$= \left((A_1)_{-+}/(B_1)_{-+}\right) + \left((A_1)_{--}/(B_1)_{--}\right)$$
$$- \left((C_1)_{+-}/(D_1)_{+-}\right) - \left((C_1)_{++}/(D_1)_{++}\right)$$

where,

$$(A_1)_{-+} = (y_2)\sin\theta - (2 \times m \times H + y_3)\cos\theta,$$

$$(B_1)_{-+} = \left[(x_3'^2) - 2(x_3')^2 (y_2\cos\theta + (2 \times m \times H + y_3)\sin\theta) \right.$$
$$\left. + (y_2^2 + (2 \times m \times H + y_3)^2)\right],$$

$$(A_1)_{--} = (y_2)\sin\theta - (2 \times m \times H - y_3)\cos\theta,$$

$$(B_1)_{--} = \left[(x_3'^2) - 2(x_3')^2 (y_2\cos\theta + (2 \times m \times H - y_3)\sin\theta) \right.$$
$$\left. + (y_2^2 + (2 \times m \times H - y_3)^2)\right],$$

$$(C_1)_{+-} = (y_2)\sin\theta + (2 \times m \times H - y_3)\cos\theta,$$

$$(D_1)_{+-} = \left((x_3')^2 - 2(x_3')(y_2\cos\theta - (2 \times m \times H - y_3)\sin\theta) \right.$$
$$\left. + (y_2^2) + (2 \times m \times H - y_3)^2\right)$$

$$(C_1)_{++} = (y_2)\sin\theta + (2 \times m \times H + y_3)\cos\theta,$$

$$(D_1)_{++} = ((x_3')^2 - 2(x_3')(y_2\cos\theta - (2 \times m \times H + y_3)\sin\theta)$$
$$+ (y_2^2) + (2 \times m \times H + y_3)^2)$$