International Journal of Geosciences, 2011, 2, 541-548
doi:10.4236/ijg.2011.24057 Published Online November 2011 (http://www.SciRP.org/journal/ijg)

Scientific Research

Geostatistical Correlation of Aquifer Potentials in Abia State, South-Eastern Nigeria

Magnus Uzoma Igboekwe[1*], Cyril Ngozi Nwankwo[2]
[1]*Department of Physics, Michael Okpara University of Agriculture, Umudike, Nigeria*
[2]*Department of Physics, University of Port Harcourt, Port Harcourt, Nigeria*
E-mail: {*igboekwemu, cyrilnn*}*@yahoo.com*
Received February 7, 2011; *revised August* 16, 2011; *accepted September* 28, 2011

Abstract

In this paper, a collection of statistical correlation methods is used in the study of aquifer potentials in Abia State of south-eastern Nigeria. The Physiology, geomorphology and hydrogeology of the area are first presented. Sixty-six Vertical Electrical Sounding (VES) data sets are used to determine the aquifer. Demographic studies are then carried out in 220 communities in order to determine the relationship between population size on one hand and a unit draw-down of wells due to groundwater abstraction on the other. The relationship between geological Formation, aquifer potentials and depth of boreholes are then calculated using Pearson's correlation matrix. Results show that the mean population of persons appears to be higher in Bende-Ameki Formation **(**of Eocene-Oligocene age**)** and the late Tetiary-Early Quaternary Coastal Plain Sands, than in the Cretaceous shale Formation of Asata Nkporo. The mean population of persons sitting on these Formations is 31,200, 18,370 and 5400 respectively. Furthermore, it is observed that a population increase of about 50 persons in a community in Abia State is accompanied by a unit volume (1 m^3) draw-down of wells due to groundwater abstraction. It is therefore concluded that population size is positively correlated with groundwater abstraction, aquifer potentials and geological Formation favouring aquifer in Abia State.

Keywords: Geostatistics, Pearson's Correlation, Groundwater, Krigging, Aquifer, Bende-Ameki Formation, Coastal Plain Sands.

1. Introduction

With the creation of Abia state in 1991 in south-eastern part of Nigeria and the consequent increase in population density in the cities [1], there arose a corresponding increase in demand for potable water for the teaming population. There has been scarcity of improved water schemes before and even after the creation of the state. Many of the boreholes drilled at different places in these cities have become abortive or dried up because of heavy draw-down by the teaming population.

The result of the draw-down or out right failure of the wells as well as inadequacy of water supply from improved schemes is the consumption of poor drinking water by the people [2]. It therefore has become necessary to study the geostastitical correlation of aquifer potentials in the state in order to determine the relationship between population size and a unit draw-down of water boreholes due to groundwater abstraction. This paper is therefore aimed at finding the correlationship in population size, groundwater abstraction, aquifer potentials (or geological Formation favouring aquifer) and depth of boreholes.

Physiography, geomorphology, geology and hydrogeology: Abia state is located between lat 4˚49.30'N - 6˚02'N and between long 7˚08'E - 8˚04'E in the south-eastern part of Nigeria (**Figure 1**). It has an area of about 5833.77 km^2, which is roughly 9% of Nigeria's land mass. It is bounded in the north by Enugu state, in the south by River state, in the east by Cross River and Akwa Ibom states and in the west by Imo state.

Population: Using the 1991 census figures, Abia state has a population of 2,293,978 inhabitants [3], residing in both the rural and urban areas in the state. This is about 393 inhabitants per square km on the average, which is about 3.5 times the mean population in Nigeria. While some parts like Ngwa is below the average, Umuahia and Aba are each above the mean population density with about 2000 inhabitants per square km respectively.

Climate: Abia State enjoys an equatorial climate con-

Copyright © 2011 SciRes.

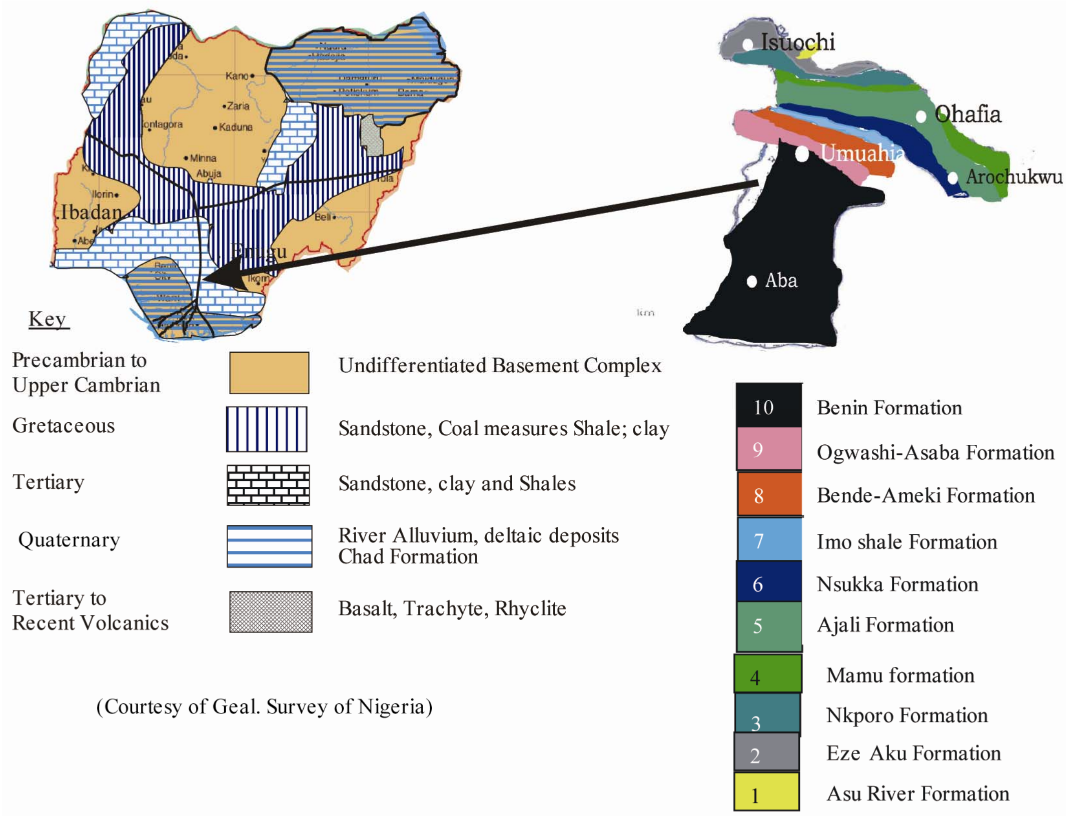

Figure 1. Geographic location and geologic map of Abia State.

sisting mainly of two major seasons: Rainy season, (March-October) and Dry season (November-February) each year. The north east trade wind from Sahara Desert and the southerly humid marine air mass from the Atlantic Ocean cause the seasonal variation in the climate of Abia state. The number of sunshine hours in the state is 3600 hours per year.

Topography: South of Abia state is low lying. The south-eastern part is about 122 m above sea level. The average elevation in the entire state is about 152 m above mean sea level.

Geology: There are two principal geological Formations in the state namely Bende-Ameki and the Coastal Plain Sands otherwise known as Benin Formation. The Bende-Ameki Formation of Eocene to Oligocene age consists of medium–coarse-grained white sand stones, (**Figure 1**). The late Tetiary-Early Quaternary Benin Formation is the most predominant [4] and completely overlies the Bende-Ameki Formation with a south westward dip. The Formation is about 200m thick [5]. The lithology is unconsolidated fine-medium-coarse-grained cross bedded sands occasionally pebbly with localized clay and shale [6].

Hydrogeology: The two principal geological Formations have a comparative groundwater regime. They both have reliable groundwater that can sustain regional borehole production. The Bende-Ameki Formation has less groundwater when compared to the Benin Formation. The numerous lenticular sand bodies within the Bende-Ameki Formation are not extensive and constitute minor aquifer with narrow zones of sub-artesian condition. Specific capacities are in the range of 3 - 6 m^3 per meter per hour, [1,7]. On the other hand, the high permeability of Benin Formation, the overlying lateritic earth, and the weathered top of this Formation as well as the underlying clay shale member of Bende-Ameki series provide the hydrogeological condition favouring the aquifer formation in the area.

Drainage: Abia state is drained by five important rivers namely: Imo, Esu, Akpoha, Igu and Aba River. The drainage is however dominated by two main rivers: the Imo River on the west and Cross-River on the east.

Rainfall: The rainfall duration in the state can be classified into the wet and the dry seasons. Abia state enjoys a copious rainfall during rainy (monsoon) season. The mean monthly rainfall during this season [4] is 335 mm and falls to 65 mm during the dry season. The annual rainfall is between 2000 mm and 2250 mm south of Abia and between 1250 and 2000 mm north-east of Abia. There are about 240 rain days towards the north of

Copyright © 2011 SciRes.

Ovim-Abriba axis and about 255 rain days south of it.

2. Methods and Results

Geostatistics is a collection of statistical methods, which are traditionally used in geo-sciences. These methods describe special auto correlation among sample data. They are recently adopted in ecology and appear to be very useful in this new area. Geostastitical analysis applications provide sophisticated interpolation techniques, which can search for and identify data anomalies, then analyze the data for statistical trends. This analysis provides extreme flexibility. It allows the explorationist to cross-validate the data quantitatively evaluating the accuracy of geological model and predictions. Geostatistical and geospatial GIS processing in combination provide a very powerful exploration tool for identifying, evaluating and preventing geospatial relationship that could otherwise have gone unnoticed [8]. Using both geospatial and geostastitical analysis, we can test whether the distribution of water bore-holes show a close spatial relationship to one, two, or all three sets of geologic and demographic parameters that would be expected by chance. If this correlation is statistically strong, we can then use the model to make spatial predictions on the aquifer potential of the study area.

The following steps are involved in geostatistical analysis: estimation of correlogram, estimation of parameters of the correlogram model, estimation of the surface (map) using point krigging and estimation of mean value using block krigging.

The Pearson's product-moment correlation coefficient (ρ_{xy}) expresses the degree of correlation between the variables x and y with expected values Ux and Uy and standard deviation σ_x and σ_x defined as

$$\rho_{xy} = \frac{\operatorname{Cov}(x, y)}{\sigma_x \sigma_y} = \frac{E(X - \mu x)(Y - \mu y)}{\sigma_x \sigma_y} \tag{1}$$

where E is the expected value operator and Cov means covariance. Since

$$\mu_x = E(x), \ \sigma_x^2 = E(x^2) - E^2(x) \tag{2}$$

and likewise for Y, we may also write

$$\rho_{X,Y} = \frac{E(XY) - E(X)E(Y)}{\sqrt{E(X^2) - E^2(X)}\sqrt{E(Y^2) - E^2(Y)}} \tag{3}$$

The correlation is defined only if both the standard deviations are finite and both of them are nonzero. It is noteworthy here that it is a corollary of the Cauchy-Schwarz inequality that the correlation cannot exceed 1 in absolute value. Also, correlation is 1 in the case of an increasing linear relationship, –1 in the case of a decreasing linear relationship, and some value in between in all other cases, indicating the degree of linear dependence between the variables. The closer the coefficient is to –1 or 1, the stronger the correlation between the variables. If variables are independent then the correlation is 0 but the converse is not true because the correlation coefficient detects only linear dependencies between two variables.

A correlation between two variables is diluted in the presence of measurement error around estimates of one or both variables in which case disattenuation provide a more accurate coefficient.

If we have a series of n measurements of X and Y written as x_1 and y_1 where 1, 2, ··· n, then the Pearson product-moment correlation coefficient can be used to estimate the correlation of x and y. In this case Pearson coefficient is also known as "sample correlation coefficient". In such case, Pearson correlation coefficient is written as:

$$\begin{aligned} r_{xy} &= \frac{\sum x_i y_i - n\overline{xy}}{(n-1)s_x s_y} \\ &= \frac{n\sum x_i y_i - \sum x_i \sum y_i}{\sqrt{n\sum x_i^2 - \left(\sum x_i\right)^2}\sqrt{n\sum y_i^2 - \left(\sum y_i\right)^2}} \end{aligned} \tag{4}$$

$$r_{xy} = \frac{\sum (x_i - \overline{x})(y_i - \overline{y})}{(n-1)s_x s_y} \tag{5}$$

where x and y are the sample means of X and Y. S_x and S_y are the sample standard deviation of X and Y and the sum is from i = 1 to n of population correlation. We may rewrite the equation thus.

$$\begin{aligned} r_{xy} &= \frac{\sum x_i y_i - n\overline{xy}}{(n-1)s_x s_y} \\ &= \frac{n\sum x_i y_i - \sum x_i \sum y_i}{\sqrt{n\sum x_i^2 - \left(\sum x_i\right)^2}\sqrt{n\sum y_i^2 - \left(\sum y_i\right)^2}} \end{aligned} \tag{6}$$

Here the absolute value of the sample correlation must be less than or equal to 1.

The square of the sample correlation coefficient, which is also known as the coefficient of determination, is the fraction of the variance in y_i that is accounted for by a linear fit of x_i to y_i This is written

$$r_{xy}^2 = 1 - \frac{S_x^2/y}{S_y^2} \tag{7}$$

where S_x^2/y is the square of the error of a linear regression of x_i on y_i by the equation $y = a + bx$:

$$S_{x/y}^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(y_i - a - bx_i\right)^2 \tag{8}$$

Copyright © 2011 SciRes.

and S_x^2 is just the variance of y:

$$S_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2 \tag{9}$$

Since the sample correlation coefficient is symmetric in x_i and y_i, Gayen, [9] was able to get the same value for a fit of y_i to x_i:

$$r_{xy}^2 = 1 - \frac{S_{x/y}^2}{S_y^2} \tag{10}$$

This equation also gives an intuitive idea of the correlation coefficient for higher dimensions. Just as the above sample correlation coefficient is the fraction of variance accounted for by the fit of a 1-dimensional linear sub manifold to a set of 2-dimensional vectors (x_i, y_i), so we can define a correlation coefficient for a fit of an m-dimensional linear manifold to a set of n-dimensional vectors. For example, if we fit a plane $z = a + bx + cy$ to a set of data (x_i, y_i, z_i) then the correlation coefficient of z to x and y is

$$r^2 = 1 - \frac{S_z^2/xy}{S_z^2} \tag{11}$$

This form expresses the method of simple linear regression. In this method, X is the vector of independent variables x_i and Y of the dependent variables, y_i and a simple linear relationship between X and Y is sought through a least squares method on the estimate of y:

$$Y = X\beta + \varepsilon \tag{12}$$

Then, the equation of the least-squares line can be derived to be of the form

$$\left(Y - \bar{Y}\right) = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - \left(\sum x_i\right)^2}\left(X - \bar{X}\right) \tag{13}$$

This equation can be rearranged in the form.

$$\left(Y - \bar{Y}\right) = \frac{rs_y}{s_x}\left(X - \bar{X}\right) \tag{14}$$

where r has the familiar form mentioned in Equation (4) above.

The Correlation matrix of n random variables $X_1, \cdots, X_n$ is the $n \times n$ matrix whose i, j entry is Corr(X_i, X_j). If the measures of correlation used are product-moment coefficients, the correlation matrix is the same as the covariance matrix of the standardized random variables X_i/SD(X_i) for $i = 1, \cdots, n$. Consequently, it is necessarily a positive-semidefinite matrix. The correlation matrix is symmetric because the correlation between X_i and X_j is the same as the correlation between Xi and Xj.

An example of a correlation matrix is of the form:

$$\begin{array}{ccccc} & Y_1 & Y_2 & Y_3 & Y_1 \\ Y_1 & 1 & x_i & x_2 & x_3 \\ Y_2 & x_i & 1 & x_4 & x_5 \\ Y_3 & x_2 & x_4 & 1 & x_6 \\ Y_4 & x_3 & x_5 & x_6 & 1 \end{array} \tag{15}$$

3. Data Acquisition

Sixty-six (66) VES sounding data sets were collected throughout Abia state using the ABEM SAS 4000 Terrameter and analyzed using the Resist software [10]. **Figure 2** shows a sample geoelectrical section along AA/ for the results of seven geoelectric soundings namely: VES # 8, 7, 9, 1, 10, 4, 5 conducted in the southern part of Umuahia of Abia state (a sedimentary area with the prolific Coastal Plain Sands—geologic Formation—favouring good aquifer). It begins from Umuobia Olokoro (point A) and ends at Amizi Oloko (point A′). This section shows the presence of four (sometimes interrupted) geoelectric layers namely the top soil, underlain by brown-reddish laterite. Other layers lying immediately below the laterite are the fine-medium-coarse-grained sands (the aquiferous zone) and lastly the conductive (clay) layer. The litholog of a borehole at Michael Okpara University of Agriculture, Umudike (**Figure 4**) shows these lithologic units much more distinctly.

A typical sounding curve obtained at Abia state University Uturu (ABSU) is also shown in **Figure 3**. It is from such curves as this and the corresponding layer parameters that the geoelectrical sections are developed.

Sequel to the above resistivity survey, demographic studies on two hundred and twenty (220) towns and villages in Abia state were then carried out. The communities were grouped into eight study zones and data were collected under the following headings: population, existing water schemes, possible depth of bore-hole, geological formation and aquifer potentials. The Vertical Electrical Sounding (VES) points were used to determine the aquifer while demographic figures were used to determine the statistical parameters. Of the 220 communities surveyed, 162 of them enjoy good aquifer potential *i.e.* almost three quarters (74%) have substantial aquifers. Similarly 158 *i.e.* 72% of the communities sit on Coastal Plain Sands (or Benin Formation). The findings are summarized in **Table 1**.

4. Discussion of Results

The distribution of mean population by some background The distribution of mean population by some background characteristics is shown on **Table 1**. As the Table shows,

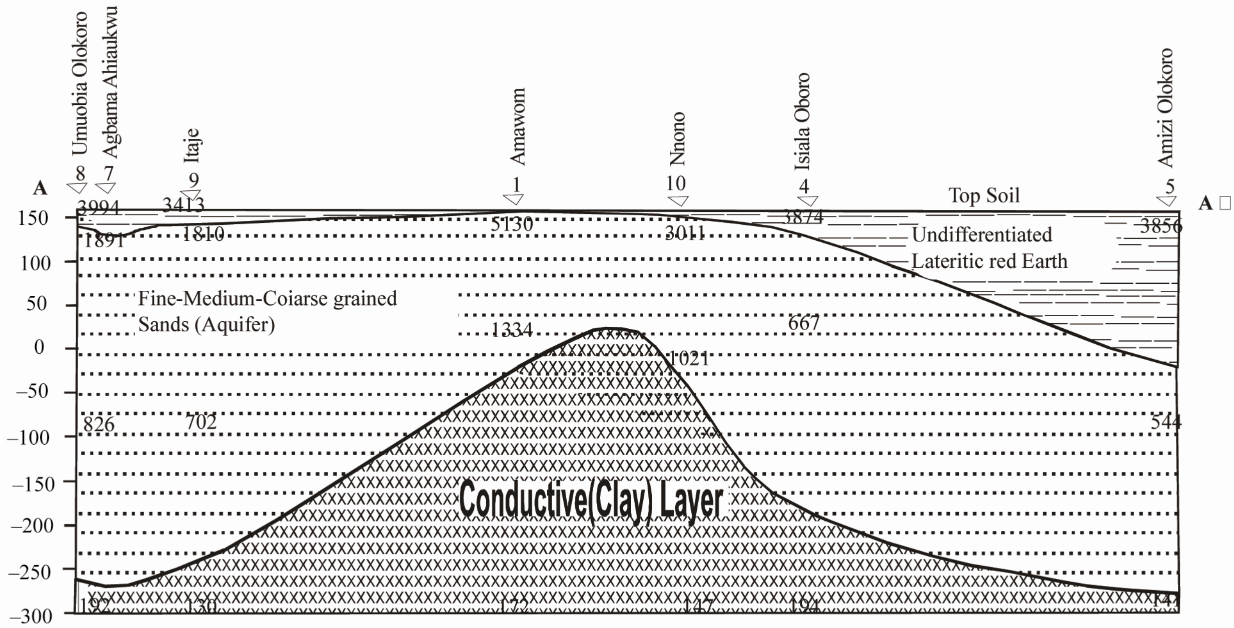

Figure 2. Geoelectrical section along section AA (showing good aquifer formation).

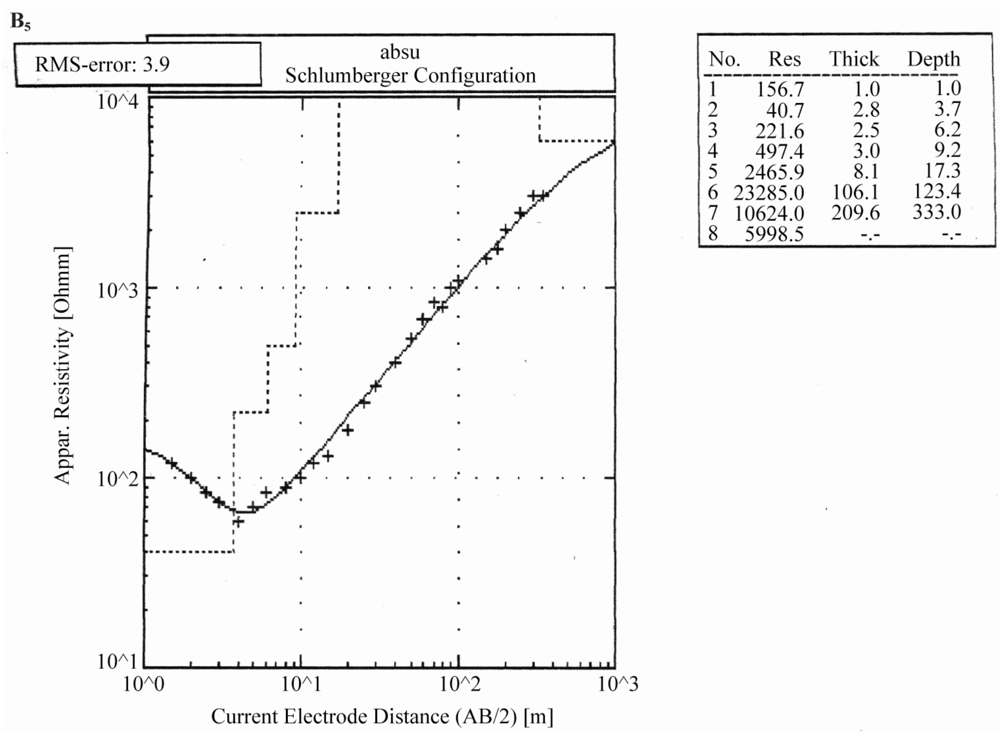

Figure 3. Computer modeled curve of VES B5.

the mean population is higher in places with good and moderate (or fair) aquifer potentials *i.e.* the places with difficult potentials appear to be less conducive for habitation.

On geological formation, the mean population appears to be highest for Bende-Ameki with about 31,200 persons, followed by the Coastal Plain Sands. Asata Nkporo on the other hand has the lowest mean population of about 5400 persons. Ordinarily, population is usually linked with surface water. However, population is linked with aquifer in this case to show that there are many streams which feed the aquifer in the areas with high

Copyright © 2011 SciRes.

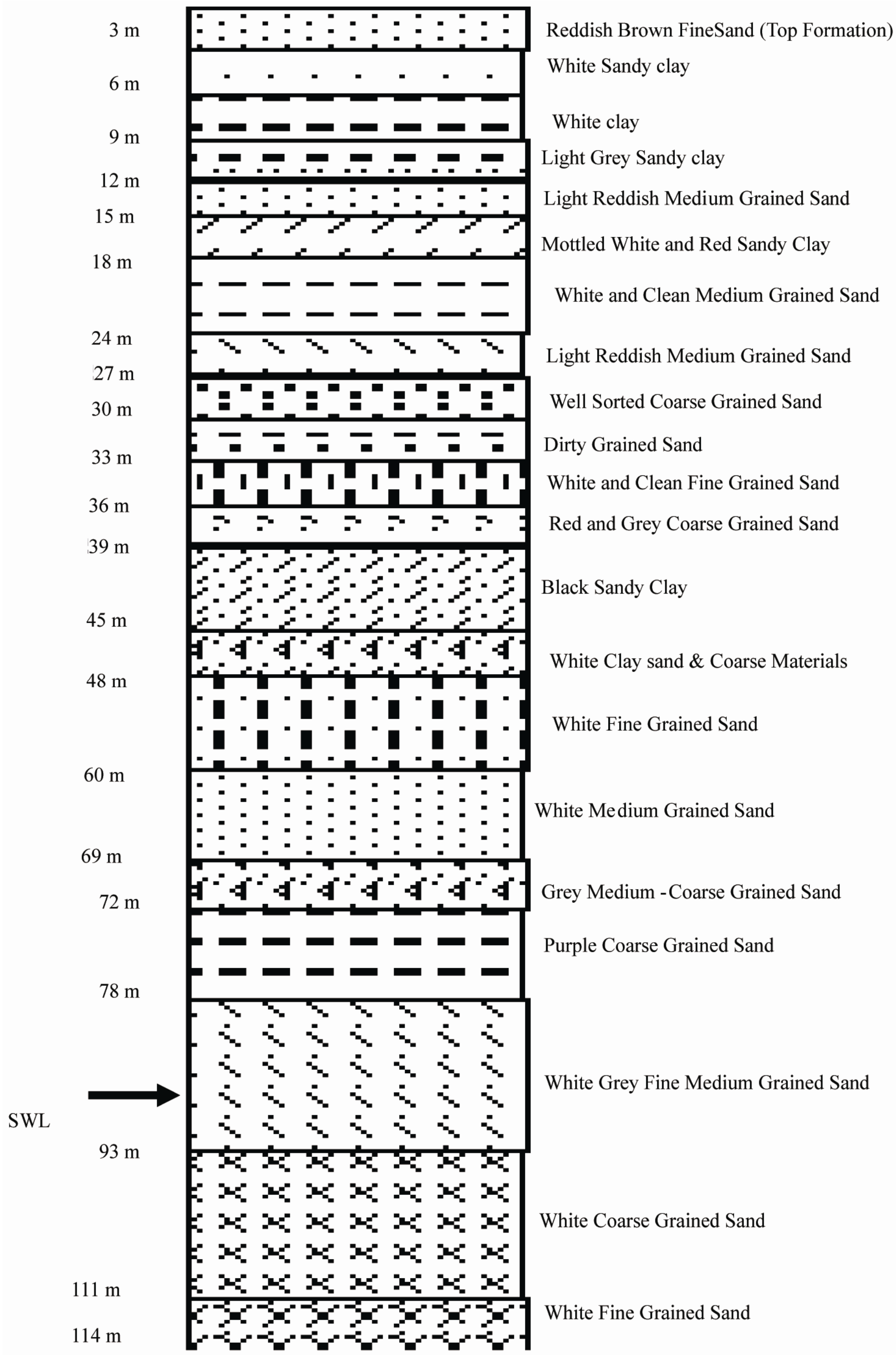

Figure 4. Litholog of a borehole at M.O. University, Umudike.

mean population.

The correlation matrix showing the relationship among the population, aquifer potential, geological Formation and depth is shown in **Table 2**.

From the table, it is obvious that geological formation is positively correlated with the depth of a well and aquifer potential. On the other hand, population appears to be negatively correlated with depth.

To test the significance of the relationship between population on one hand and depth, geological formation and aquifer potential on the other, a multiple regression analysis was conducted.

Copyright © 2011 SciRes.

Table 1. Mean population by some background characteristics.

	NO. OF COMMUNITIES	MEAN POPULATION	% POPULATION	% COMMUNITIES
AQUIFER POTENTIAL				
GOOD	162	18,552	81	73.6
FAIR	1	13,000	1	0.4
MODERATE	8	28,875	6	4
DIFFICULT	49	9143	12	22
GEOLOGICAL FORMATION				
COASTAL PLAIN SANDS	158	18,370	79	72
LOWER COAL MEASURES	34	9765	9	15
UPPER COAL MEASURES	9	10,111	2	4
ASATA MKPORO	5	5400	0.7	2.3
BENDE-AMAEKI	10	31,200	8.4	5
IMO SHALE	1	8000	0.2	0.4
AGWU NDIABOR	2	7500	0.4	0.9
AJALI	1	10,000	0.3	0.4
POSSIBLE DEPTH (M)				
60	93	20,194	53	42
65 - 130	27	11,296	8.6	12
140	17	18,588	9.0	8
150	36	17,139	17.4	16
160 - 230	23	9229	6.0	11
240	13	8115	3.0	6
250 - 260	11	9818	3.0	5

Table 2. Correlation matrix of population and some background characteristics.

	Population	Depth	Geo. Form	Aquifer Pot.
Population	1	–0.217	–0.043	–0.116
Depth	–0.217	1	0.510	0.588
Geo. Form	–0.043	0.510	1	0.757
Aquifer Pot.	–0.116	0.588	0.757	1

Results of the analysis show that population is significantly related only to the depth of a well. This is the general trend shown by the Equations (1)-(15). Hence the regression equation is given by

$$Y_i = 22561 - 49.8X_i \quad (16)$$

where Y_i = Population and X_i = Depth.

This indicates that on the average, an increase of about 50 persons in the population of a community in Abia State is accompanied by a decrease of about one metre depth of water in a well.

It should be noted that for wells of uniform diameter, the depth is directly proportional to the volume of groundwater contained. Therefore, a population increase of about 50 persons in a community in Abia State is accompanied by a unit volume (1 m^3) draw-down of wells due to groundwater abstraction.

Generally, the aquifer in Abia state is the prolific Coastal Plain Sands and depth to boreholes range from 60 m to 100 m in Aba, Obingwa and Ukwa Local Government Areas and from 140 to 250 m around Umuahia, Isialangwa and Ikwauno Local Government Areas.

5. Conclusions

The relationships between geological Formation, aquifer potentials and depth to water boreholes have been calculated in this work using Pearson's correlation matrix. Results from resistivity soundings show that most parts of Abia state are blessed with good aquifer potentials. Demographic studies conducted here also show that the mean population appears to be higher in places with good and moderate aquifer potentials than in those with difficult ones. That is to say, the mean population appears to be higher in Bende-Ameki Formation of Eocene to Oligocene age and the late Tetiary-Early Quaternary Benin Formation, than in the Cretaceous shale Formation of Asata Nkporo. The mean population of persons sitting on these Formations is 31,200, 18,370 and 5400 respectively. It is further observed that a population increase of about 50 persons in a community in Abia State is accompanied by a unit volume (1 m^3) draw-down of wells due to groundwater abstraction. It is therefore concluded

Copyright © 2011 SciRes.

that population size is positively correlated with groundwater abstraction, aquifer potentials and geological Formation favouring aquifer in Abia State.

6. References

[1] M. U. Igboekwe, "Geoelectrical Exploration for Groundwater Potentials in Abia State, Nigeria," Ph.D. Dissertion, Michael Okpara University of Agiruclture, Umudike, 2005, p. 131.

[2] M. U. Igboekwe, V. V. S Gurunadha Rao, and E. E. Okwueze, "Groundwater Flow Modeling of Kwa Ibo River Watershed, South-Eastern Nigeria," *Hydrological Processes*, Vol. 22, No. 10, 2008, pp. 1523-1531. doi:10.1002/hyp.6530

[3] NPC (National Population Commission of Nigeria,) "National Population Commission Census Figures for Abia State, Nigeria," Abuja, 1991.

[4] P. D. C. Mbonu, J. O. Ebeniro, C. O. Ofoegbu and A. S. Ekine, "Geoelectric Sounding for the Determination of Aquifer Characteristics in Parts of the Umuahia Area of Nigeria," *Geophysics*, Vol. 56 , No. 2, 1991, pp. 284-291. doi:10.1190/1.1443042

[5] Ebilah-Salmon and Partners in Association with Esokay Ltd., Abia State Rural Water Supply Project, Feasibility Report and Preliminary Engineering Design Report, 1993, p. 183.

[6] M. U. Igboekwe, E. E. Okwueze and C. S. Okereke, "Delineation of Potential Aquifer Zones from Geoelectric Soundings in Kwa Ibo River Watershed, South-Eastern, Nigeria," *Journal of Engineering and Applied Sciences*, Vol. 1, No. 4, 2006, pp. 410-421.

[7] Ebilah-Salmon and Partners. "Investigation of the Existing Water Supply Facilities within the University Complex. Geophysical Report and Recommendations for Reactivation and Future Exploitation for Potable Water Supply, Federal University of Agriculture, Umudike, 1994, p. 25.

[8] J. L. Rogers and W. A. Nicewander, "Thirteen Ways to Look at the Correlation Coefficient," *The American Statistician*, Vol. 42, 1988, pp. 59-66. doi:10.2307/2685263

[9] A. K. Gayen, "The Frequency Distribution of the Product- Moment Correlation Coefficient in Random Samples of Any Size Drawn from Non-Normal Universes," *Biometrica*, Vol. 38, No. 1-2, 1951, pp. 219-247.

[10] D. L. B. Jupp and K. Vozoff, "Stable Interative Methods for Inversion of Geophysical Data," *Geophysical Journal of the Royal Astronomical Society*, Vol. 24, No. 3, pp. 957-976.

Copyright © 2011 SciRes.