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# Consensus Control for a Kind of Dynamical Agents in Network

**Hongwang Yu**

School of Science, Nanjing Audit and University, Nanjing, China
Email: whyu0312@sohu.com

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## Abstract

**This paper discusses consensus control for a kind of dynamical agents in network. It is assumed that the agents distributed on a plane and their location coordinates are measured by remote sensor and transmitted to its neighbors. By designing the linear distributed control protocol, it is shown that the group of agents will achieves consensus. The simulations are given to show the effectiveness of our theoretical result.**

## Keywords

**Distributed Control, Graph Laplacian, Dynamical Agents**

## 1. Introduction

Distributed coordination of network of dynamic agents has attracted a great attention in recent years. Modeling and exploring these coordinated dynamic agents have become an important issue in physics, biophysics, systems biology, applied mathematics, mechanics, computer science and control theory [1]-[11]. How and when coordinated dynamic agents achieve aggregation is one of the interesting topics in the research area. Such problem may also be described as a consensus control problem.

To describe the collective behavior of agents in a large scale network, the agent in the network usually is modeled by a very simple mathematical model, which is an approximation of real objects. Saber and Murray [3] [4] proposed a systematical framework of consensus problems in networks of dynamic agents. In their work the dynamics of the agent is modelled by a simple scalar continuous-time integrator $\dot{x} = u$, the convergence analysis is provided in different types of the network topologies. Following the work of [3] [4], Guangming Xie [10] study the case where the agent is a point-mass distributed in a line, and its dynamics is described by the Newton's law $ma = F$. In their work the dynamic agents connected by a network, which is characterized by a graph and each agent is *Lyapunov* stable. They show that by means of a simple linear control protocol based on the structure of the graph, the dynamical agents will eventually achieve aggregation, *i.e.* all agents will gradually move into a fixed position, meanwhile their velocities converge to zero.

In our work a similar problem is studied under the condition that the agents move in a plane. The agents may represent the vehicles or mobile robots spread over a wild area and they communicate by means of some remote sensors with certain error. When the agents are moving in a plane, the collective behavior conditions will depend on the communicated error and the algebraic characterization of the communicated network topology, as well as the dynamical behavior of agents.

This paper is organized as follows. In Section 2, we recall some properties on graph theory and give the problem formulation. In Section 3 the main results of this paper are given and some simulation results are presented in Section 4. Final section is a conclusion.

## 2. Preliminaries

Consider a network of dynamical agents defined by a graph $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$. The node set $\mathcal{V}$ consists of dynamical agents $p_i; i\in \underline{M}$. The dynamics of $p_i$ for $i\in\underline{M}$ is described as follows.

Let $x_i=(x_{i1},x_{i2})^\tau\in R^2$ be the coordinate of dynamical agent $p_i$ in $R^2$, then the dynamical equation of agent $p_i$ is

$$\begin{aligned}\dot{x}_i &= v_i\\ m_i\dot{v}_i &= kv_i+u_i\\ y_i &= F\begin{bmatrix}x_i\\ v_i\end{bmatrix}\end{aligned}\tag{1}$$

where $x_i=(x_{i1},x_{i2})^\tau$ indicates the location of agent $p_i$ in the plane, $v_i=(v_{i1},v_{i2})^\tau$ represents the velocity of the *i*-th agent and $m_i$ is its mass and $k=\begin{bmatrix}k_{11} & k_{12}\\ k_{21} & k_{22}\end{bmatrix}$ is a dynamical feedback matrix of the agent. *F* is an observation matrix of the agent by some remote sensor.

In what follows we simply assume that $m_i=1$ for all $i\in\underline{M}$ and $p_i=x_i$. Let $F=[C\ \ 0]$ which means that the location information of the *i*-th agent is only measured by some remote sensor and is transmitted to its neighbors through the network. The matrix *C* is assumed to be an orthogonal matrix in the form $C=\begin{bmatrix}1 & \delta\\ -\delta & 1\end{bmatrix}$. The parameter $\delta$ will indicates that the network transmitted error or the coordinates used for sensor could be different from that of the agents.

For the dynamic agent (1) in network we have following assumption.

**Assumption 2.1** *The dynamics* (1) *is Lyapunov stable when it disconnected with its neighbors, meaning that the dynamical agent as an autonomous will gradually stop by moving a finite distance for any non-zero initial velocity* $v_i(0)$.

The collective behavior of dynamical agents in network can be described by $x(t):=(x_1^\tau(t),x_2^\tau(t),\cdots,x_M^\tau(t))^\tau\in R^{2M}$; $t\ge 0$. We denote the initial locations and the initial velocities of the system as $x(0)=(x_1^\tau(0),\cdots,x_M^\tau(0))^\tau$, $v(0)=(v_1^\tau(0),v_2^\tau(0),\cdots,v_M^\tau(0))^\tau$ respectively.

In this work, we discuss the collective behavior of the dynamical agents under a decentralized control law in the form that

$$u_i=K_i\left(y_{j_1},y_{j_2},\cdots,y_{j_{l_i}}\right)\tag{2}$$

where indexes $\{j_1,j_2,\cdots,j_{l_i}\}\subset\underline{M}$.

We claim that a group of dynamical agents associated with $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ asymptotically achieve the collective behavior under control protocol (2). That is to say, for any initial conditions of the agents $x_i(0)\in R^2$, $v_i(0)\in R^2$; $i\in\underline{M}$, there will exist a fixed position $x^*\in R^2$, which depends on the initial condition, such that for $i\in\underline{M}$

$$\lim_{t\to\infty} x_i(t) = x^*, \qquad \lim_{t\to\infty} v_i(t) = 0 \tag{3}$$

In our work, let (2) be

$$u_i = \sum_{x_j \in N_i} a_{ij}\left(y_j - y_i\right) \tag{4}$$

where $N_i$ is the set of neighbors of agent $p_i$.

**Remark 1:** *If we choose* $k_{11} = k_{22} = k, k_{12} = k_{21} = 0$ *and* $\delta = 0$, *then the two-dimension agent systems* (1) *with the control protocol* (4) *can be decoupled into two identical linear systems of the form*

$$\dot{x}_{is} = v_{is}$$

$$m_i \dot{v}_{is} = kv_{is} + \sum_{p_j \in N_i} a_{ij}\left(x_{js} - x_{is}\right)$$

for $s = 1, 2$. *i.e.* $\dim x_i = 1$, and it was discussed in [12].

## 3. Collective Behaviors of Dynamical Agents

Consider a group of dynamical agents in network associated with a graph $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$. The node set $\mathscr{V}$ consists of dynamic $p_i; i \in \underline{M}$. The dynamical $p_i$ for each $i \in \underline{M}$ is described by linear dynamical equation (1) satisfying Assumption 2.1. Under control protocol (4) the dynamical equation of agent $p_i$ is written by

$$\begin{aligned} \dot{x}_i &= v_i \\ \dot{v}_i &= kv_i + \sum_{p_j \in N_i} a_{ij}\left(y_j - y_i\right) \end{aligned} \tag{5}$$

Denote $\xi_i = \left(x_i^\tau, v_i^\tau\right)^\tau = \left(x_{i1}, x_{i2}, v_{i1}, v_{i2}\right)^\tau$, $i \in \underline{M}$, then (5) is written in

$$\dot{\xi}_i = A\xi_i + B\sum_{p_j \in N_i} a_{ij}\left(\xi_j - \xi_i\right) \tag{6}$$

where $A = \begin{bmatrix} 0_{2\times2} & I_2 \\ 0_{2\times2} & k \end{bmatrix}$, $B = \begin{bmatrix} 0_{2\times2} & 0_{2\times2} \\ C & 0_{2\times2} \end{bmatrix}$.

Let $\xi = \left(\xi_1^\tau, \xi_2^\tau, \cdots, \xi_M^\tau\right)^\tau$, then the dynamic network is of the following form.

$$\dot{\xi} = \Omega\xi \tag{7}$$

where

$$\Omega = I_M \otimes A - L \otimes B \tag{8}$$

and *L* is the aforementioned *Laplacian* associated with the graph $\mathscr{G}$.

The collective behavior problem of dynamical agents can be described in $\chi$-consensus asymptotical consensus stability ([3]). Let $\chi: R^{4M} \to R^2$ be a map, for $\xi(0) := \left(x_1(0), v_1(0), x_2(0), v_2(0), \cdots, x_M(0), v_M(0)\right)^\tau \in R^{4M}$, $\chi: \xi(0) \mapsto x\left(\in R^2\right)$. The group of dynamical agents is called $\chi$-consensus asymptotically stable under control protocol (4) if let $\chi\left(\xi(0)\right) = x^*$ for a given $\xi(0)$, then for each agent in network its state variables meets the properties of (3).

As dynamics (7) is a standard linear time-invariant dynamical system, its trajectory can be described by

$$\xi(t) = \exp(\Omega t)\xi(0) \tag{9}$$

The consensus asymptotical stability implies that the matrix $\exp(\Omega t)$ converges to a constant matrix, thus we will explore some properties of the matrix $\Omega$.

**Lemma 3.1** *The matrix* $\Omega$ *has two eigenvectors associated with zero eigenvalue. Let* $v_r, v_l$ *be the right and left eigenvectors* (*denoted by matrices*) *of matrix* $\Omega$ *associated with zero eigenvalue, respectively. Then*

$$v_r = \frac{1}{\sqrt{M}}\mathbf{1}_M \otimes \begin{bmatrix} k^{-1} \\ 0_{2\times2} \end{bmatrix}, \qquad v_l^\tau = \frac{1}{\sqrt{M}}\mathbf{1}_M \otimes \begin{bmatrix} k^\tau \\ -I_2 \end{bmatrix} \tag{10}$$

and $v_l v_r = I_2$, where $\mathbf{1}_M = (1,1,\cdots,1)^\tau \in R^M$

**Proof:** It is well known that the graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ is connected if and only if its *Laplacian* satisfies that $rank(L) = M - 1$. Moreover, $\mathbf{1}_M = (1,1,\cdots,1)^\tau \in R^M$ is an eigenvector of *L* associated with eigenvalue $\lambda = 0$, *i.e.*, $L \cdot \mathbf{1}_M = 0 \cdot \mathbf{1}_M$. Then, there is only one zero eigenvalue of *L*, all the other ones are positive and real. By the definition of (8) one has

$$\Omega \cdot \frac{1}{\sqrt{M}} \mathbf{1}_M \otimes \begin{bmatrix} k^{-1} \\ 0_{2\times 2} \end{bmatrix} = \frac{1}{\sqrt{M}} (I_M \otimes A - L \otimes B) \cdot \mathbf{1}_M \otimes \begin{bmatrix} k^{-1} \\ 0_{2\times 2} \end{bmatrix}$$
$$= \frac{1}{\sqrt{M}} \mathbf{1}_M \otimes 0_{4\times 2} - \frac{1}{\sqrt{M}} 0_M \otimes \begin{bmatrix} 0_{2\times 2} \\ Pk^{-1} \end{bmatrix}$$

Thus, $v_r = \frac{1}{\sqrt{M}} \mathbf{1}_M \otimes \begin{bmatrix} k^{-1} \\ 0_{2\times 2} \end{bmatrix}$ represented two right-eigenvectors of $\Omega$ associated with zero-eigenvalue. Similarly, it is easy to check $v_l = \frac{1}{\sqrt{M}} \mathbf{1}_M^\tau \otimes [k - I_2]$ represents two left-eigenvectors of $\Omega$ and $v_l v_r = I_2$. □

The following Lemma is key to our work.

**Lemma 3.2** *If the control gain k in dynamical agent* (1) *satisfies Assumption* 2.1, *and* $\delta$ *in the C of* (4) *satisfies*

$$\delta_1 < \delta < \delta_2, \tag{11}$$

with

$$\delta_1 = \frac{abc - \sqrt{\Delta}}{2\lambda(a^2 + b^2)}, \quad \delta_2 = \frac{abc + \sqrt{\Delta}}{2\lambda(a^2 + b^2)} \tag{12}$$

where $a = k_{21} - k_{12}$, $b = -k_{11} - k_{22}$, $c = k_{11}k_{22} - k_{12}k_{21}$, $\Delta = (abc)^2 + 4\lambda(a^2 + b^2)b^2 c$ and $\lambda = \lambda_M$ denotes the biggest eigenvalue of matrix *L*, then it is hold that

$$\lim_{t\to\infty} \exp(\Omega t) = v_r v_l \tag{13}$$

**Proof:** Denote the eigenvalues of *L* by $0 = \lambda_1 < \lambda_2 \le \lambda_3 \le \cdots \le \lambda_M$, and let $\Lambda$ be the Jordan form associated with *L*, there exists an orthogonal matrix *W* such that $W^\tau L W = \Lambda = \mathrm{diag}\{\lambda_1, \lambda_2, \cdots, \lambda_M\}$.

One can verify the following formulae.

$$(W^\tau \otimes I_4) \cdot \Omega \cdot (W \otimes I_4) = (W^\tau \otimes I_4)(I_M \otimes A - L \otimes B)(W \otimes I_4)$$
$$= I_M \otimes A - \Lambda \otimes B$$
$$= \mathrm{diag}\{A - \lambda_1 B, A - \lambda_2 B, \cdots, A - \lambda_M B\}$$

The dynamical behavior of the network (7) is characterized by the eigenvalues of $A - \lambda_i B$ for $i \in \{1, 2, \cdots, M\}$.

First we discuss the block with $\lambda_1 = 0$. By Assumption 2.1, one has $k_{11} + k_{22} < 0, k_{12}k_{21} < k_{11}k_{22}$ and $rank(A - \lambda_1 B) = rank\ A = 2$, its four characteristic eigenvalues must satisfy $s_1 = s_2 = 0$, $Re(s_3) < 0$, $Re(s_4) < 0$..

For $\lambda_i > 0$, one has $A - \lambda_i B = \begin{bmatrix} 0_{2\times 2} & I_2 \\ -\lambda_i C & k \end{bmatrix}$. As $rank(C) = 2$, $rank(A - \lambda_i B) = 4$. Therefore, $\Omega$ has only two zero eigenvalues.

Consider the characteristic polynomial of $A - \lambda_i B; i \in \underline{M}$

$$\pi_{A-\lambda_i B}(s) = det(sI - (A - \lambda_i B)) = \begin{vmatrix} s & 0 & -1 & 0 \\ 0 & s & 0 & -1 \\ \lambda_i & \delta\lambda_i & s - k_{11} & -k_{12} \\ -\delta\lambda_i & \lambda_i & -k_{21} & s - k_{22} \end{vmatrix} = s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4$$

where

$$\begin{aligned} &a_1 = -k_{11} - k_{22},\ a_2 = k_{11}k_{22} - k_{12}k_{21} + 2\lambda_i, \\ &a_3 = \delta\lambda_i\left(k_{21} - k_{12}\right) - \lambda_i\left(k_{11} + k_{22}\right), \\ &a_4 = \lambda_i^2\left(1 + \delta^2\right) \end{aligned} \tag{14}$$

Construct the *Routh* array of $\pi_{A-\lambda_i B}(s)$

$$\begin{array}{c|ccc} s^4 & 1 & a_2 & a_4 \\ s^3 & a_1 & a_3 & 0 \\ \hline s^2 & b_1 & b_2 & 0 \\ s^1 & c_1 & 0 & \\ s^0 & d_1 & & \end{array}$$

with $b_1 = \dfrac{a_1a_2 - a_3}{a_1}, b_2 = d_1 = a_4 = \lambda_i^2\left(1-\delta^2\right), c_1 = \dfrac{b_1a_3 - a_1b_2}{b_1} = \dfrac{a_1a_2a_3 - a_3^2 - a_1^2a_4}{a_1b_1}$. By the *Routh-Hurwith* criterion, for stability it is necessary that $a_1 > 0, b_1 > 0, c_1 > 0, d_1 > 0$. Therefore, the dynamical network is stable if the following inequalities hold

$$\begin{cases} a_1 > 0 \\ a_4 > 0 \\ a_1a_2 - a_3 > 0 \\ a_1a_2a_3 - a_3^2 - a_1^2a_4 > 0 \end{cases} \tag{15}$$

By (14) one has

$$\begin{aligned} a_1a_2 - a_3 &= -\left(k_{11} + k_{22}\right)\cdot\left(k_{11}k_{22} - k_{12}k_{21} + 2\lambda_i\right) - \lambda_i\left[\delta\left(k_{21} - k_{12}\right) - \left(k_{11} + k_{22}\right)\right] \\ &= -\left(k_{11} + k_{22}\right)\cdot\left(k_{11}k_{22} - k_{12}k_{21}\right) - \lambda_i\left[\delta\left(k_{21} - k_{12}\right) + k_{11} + k_{22}\right] \end{aligned} \tag{16}$$

and

$$\begin{aligned} a_1a_2a_3 - a_3^2 - a_1^2a_4 &= -\left\{\left(k_{11} + k_{22}\right)\cdot\left(k_{11}k_{22} - k_{12}k_{21}\right) + \lambda_i\left[\delta\left(k_{21} - k_{12}\right) + k_{11} + k_{22}\right]\right\} \\ &\quad \times\lambda_i\left[\delta\left(k_{21} - k_{12}\right) - \left(k_{11} + k_{22}\right)\right] - \left(k_{11} + k_{22}\right)^2\lambda_i^2\left(1 + \delta^2\right) \\ &= -\left\{\left(k_{11} + k_{22}\right)\cdot\left(k_{11}k_{22} - k_{12}k_{21}\right)\cdot\left[\delta\left(k_{21} - k_{12}\right) - \left(k_{11} + k_{22}\right)\right]\right\}\lambda_i \\ &\quad + \lambda_i^2\delta^2\left[\left(k_{11} + k_{22}\right)^2 + \left(k_{21} - k_{12}\right)^2\right] \end{aligned} \tag{17}$$

The inequalities (15) can be rewritten as the following form by using the conditions of Lemma 3.2 and the Equations (16)-(17).

$$\begin{cases} b\left(\lambda + c\right) - \lambda a\delta > 0 \\ b^2c + abc\delta - \lambda\left(a^2 + b^2\right)\delta^2 > 0 \end{cases} \tag{18}$$

We can further show that the second inequality in above implies the first one. Obviously, it is true when $a = 0$. If $a > 0$, one gets

$$\begin{cases} \delta < \dfrac{b\left(\lambda + c\right)}{a\lambda} \\ \delta_1 < \delta < \delta_2 \end{cases}$$

where $\delta_i, i = 1, 2$ are defined in (12).

Thus, one can consider the following inequalities

$$\frac{b(\lambda+c)}{a\lambda} > \frac{abc+b\sqrt{a^2c^2+4\lambda(a^2+b^2)c}}{2\lambda(a^2+b^2)} \Leftrightarrow \frac{\lambda+c}{a} > \frac{ac+\sqrt{a^2c^2+4\lambda(a^2+b^2)c}}{2(a^2+b^2)}$$

$$\Leftrightarrow 2\lambda(a^2+b^2)+a^2c+2b^2c > a\sqrt{a^2c^2+4\lambda(a^2+b^2)c}$$

$$\Leftrightarrow 4\lambda^2(a^2+b^2)^2+4b^4c^2+8\lambda(a^2+b^2)b^2c+4a^2b^2c^2>0$$

The last inequality obviously holds. Therefore, the solution of (18) leads $\delta_1<\delta<\delta_2$.

If $a<0$, one can obtain $\delta>\frac{b(\lambda+c)}{a\lambda}$ and $\delta_1<\delta<\delta_2$. So we can get that $\delta_1>\frac{b(\lambda+c)}{a\lambda}$ with a similar computing process. It shows that $\delta_1<\delta<\delta_2$ is the solution set of the inequalities (15) for any *a*.

Therefore, $A-\lambda_i B;\ 2\le i\le M$ are *Hurwitz*.

By $\Theta:=(\theta_1,\theta_2,\cdots,\theta_{4M-1},\theta_{4M})\in R^{4M\times 4M}$ one denotes right-eigenvectors of $\Omega$ associated with eigenvalues $\gamma_1,\gamma_2,\cdots,\gamma_{2M}$, respectively. Thus,

$$\Omega\Theta=\Theta\begin{bmatrix} 0_2 & 0 & 0 & \cdots & 0 & 0\\ 0 & J_1 & 0 & \cdots & 0 & 0\\ 0 & 0 & J_2 & \cdots & 0 & 0\\ \vdots & & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & 0 & \cdots & J_{M-1} & 0\\ 0 & 0 & 0 & \cdots & 0 & J_M \end{bmatrix}=\Theta J$$

where $J_1$ denote the Jordan form of two order associated with the eigenvalues $\gamma_1$, and $\gamma_2$. $J_i$ denote the Jordan form of four order associated with the eigenvalues $\gamma_{4i-3}$, $\gamma_{4i-2}$, $\gamma_{4i-1}$ and $\gamma_{4i}$ for all $i=2,3,\cdots,M$.

Let $\Theta^{-1}=\tilde{\Theta}:=\left(\tilde{\theta}_1^\tau,\tilde{\theta}_2^\tau,\cdots,\tilde{\theta}_{4M-1}^\tau,\tilde{\theta}_{4M}^\tau\right)^\tau\in R^{4M\times 4M}$, where $\tilde{\theta}_i; i\in \underline{4M}$ are $4M$ row left-eigenvectors of $\Omega$, correspondingly.

$$\tilde{\Theta}\Omega=\begin{bmatrix} 0_2 & 0 & 0 & \cdots & 0 & 0\\ 0 & J_1 & 0 & \cdots & 0 & 0\\ 0 & 0 & J_2 & \cdots & 0 & 0\\ \vdots & & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & 0 & \cdots & J_{M-1} & 0\\ 0 & 0 & 0 & \cdots & 0 & J_M \end{bmatrix}\tilde{\Theta}=J\tilde{\Theta}$$

As $0>Re(\gamma_3)\ge Re(\gamma_4)\ge\cdots\ge Re(\gamma_{4M-1})\ge Re(\gamma_{4M})$, one has

$$\lim_{t\to\infty} e^{Jt}=\begin{bmatrix} I_2 & 0_{2\times(4M-2)}\\ 0_{(4M-2)\times 2} & 0_{(4M-2)\times(4M-2)}\end{bmatrix}$$

and

$$\lim_{t\to\infty}\exp(\Omega t)=\Theta\begin{bmatrix} I_2 & 0_{2\times(4M-2)}\\ 0_{(4M-2)\times 2} & 0_{(4M-2)\times(4M-2)}\end{bmatrix}\tilde{\Theta}$$

Let $v_l=(\theta_1\ \ \theta_2)$ and $v_r=\begin{pmatrix}\tilde{\theta}_1\\ \tilde{\theta}_2\end{pmatrix}$, one has

$$\lim_{t\to\infty}\exp(\Omega t)=v_r v_l.$$

Due to the fact that $\Theta\cdot\Theta^{-1}=\Theta\cdot\tilde{\Theta}=I_{4M\times 4M}$, $v_r$ and $v_l$ satisfy the property $v_l v_r=I_2$. □

**Theorem 3.1** *Under conditions of Lemma* 3.2 *the control protocol* (4) *globally and asymptotically achieves* *the collective behavior of the dynamic agents.*

**Proof:** As $\xi(t)=\exp(\Omega t)\xi(0)$ and $\lim_{t\to\infty}\exp(\Omega t)=v_r v_l$, it follows that

$$\lim_{t\to\infty}\xi(t)=\lim_{t\to\infty}\exp(\Omega t)\xi(0)=v_r v_l \xi(0)$$

$$=\frac{1}{M}\mathbf{1}_M\otimes\begin{bmatrix}k^{-1}\\0_{2\times2}\end{bmatrix}\cdot\mathbf{1}_M^{\tau}\otimes\begin{bmatrix}k & -I_2\end{bmatrix}\xi(0)=\frac{1}{M}\mathbf{1}_M\mathbf{1}_M^{\tau}\otimes\begin{bmatrix}I_2 & -k^{-1}\\0_{2\times2} & 0_{2\times2}\end{bmatrix}\cdot\begin{bmatrix}x_1(0)\\v_1(0)\\\vdots\\x_M(0)\\v_M(0)\end{bmatrix}$$

Therefore,

$$\lim_{t\to\infty}x_i(t)=\frac{1}{M}\left\{\sum_{j=1}^{M}\left[x_j(0)-k^{-1}v_j(0)\right]\right\} \tag{19}$$

and it is obvious that

$$\lim_{t\to\infty}v_i(t)=0,\quad i\in\{1,2,\cdots,M\} \tag{20}$$

This implies the protocol (5) globally asymptotically achieve aggregation. □

**Corollary 3.1** *If the control gain k satisfies* $k_{12}=k_{21}$ *and* $\lambda\delta^2<(k_{11}k_{22}-k_{12}k_{21})$, *then the control protocol* (4) *globally and asymptotically achieves the collective behavior of the dynamic agents.*

Under Assumption 2.1 one has $k_{12}k_{21}<k_{11}k_{22}$. Thus, by carefully examining (12) one finds that $c>0$ and it further implies that $\delta_1<0$ and $\delta_2>0$ in (11). Thus we have the following.

**Corollary 3.2** *The dynamical agents achieve collective behavior if* $|\delta|\ll 1$ *in control protocol* (4). *Again, the* $\chi$ *-map is defined by* (19) *and* (20).

## 4. Simulations

We study some examples to show that our results are effective. The network of dynamic agents is described in **Figure 1**.

We can obtain the Laplacian matrix $L$ of the graph $\mathscr{G}$ of **Figure 1** and its eigenvalues are $\lambda_1=0$, $\lambda_2=\lambda_3=\frac{1}{2}(7-\sqrt{13})$, $\lambda_4=4$, $\lambda_5=\lambda_6=\frac{1}{2}(7+\sqrt{13})$.

We consider that the dynamic agent (1) in the network has $k=\begin{bmatrix}-1 & 0.1\\0.2 & -1\end{bmatrix}$ and observation matrix $C=\begin{bmatrix}1 & 0.3\\-0.3 & 1\end{bmatrix}$. Thus, it is *Lyapunov* stable and satisfies Assumption 2.1. One can get $a=0.1$, $b=2$,

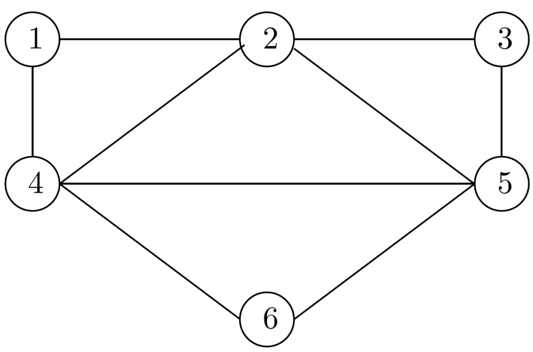


**Figure 1.** A undirected graph $\mathscr{G}$ with $M$ = 6 nodes.

$c = 0.98$, $\sqrt{\Delta} = 16.5699$, and the $\delta = 0.3$ belongs to the range of parameters *i.e.* $\delta_1 = -0.467617 < \delta < 0.478812 = \delta_2$.

When a control protocol (4) is applied into the agents in network, the collective behavior of dynamic agents takes place according to our result.

**Figure 2** gives simulation results of the collective behavior of the agents with initial conditions $x_{11}(0) = x_{12}(0) = 15$, $x_{21}(0) = x_{22}(0) = 25$, $x_{31}(0) = 2$, $x_{32}(0) = 20$, $x_{41}(0) = 1$, $x_{42}(0) = 10$, $x_{51}(0) = 1$, $x_{52}(0) = 2$, $x_{61}(0) = 25$, $x_{62}(0) = 1$, and the initial velocities $v_{11}(0) = 12$, $v_{12}(0) = 18$, $v_{21}(0) = 25$, $v_{22}(0) = 18$, $v_{31}(0) = 15$, $v_{32}(0) = 25$, $v_{41}(0) = 12$, $v_{42}(0) = 15$, $v_{51}(0) = 12$, $v_{52}(0) = 13$, $v_{61}(0) = 20$, $v_{62}(0) = 15$.

It is found that when the agents approach to $x^* = \begin{bmatrix} 29.6 \\ 33.1 \end{bmatrix}$, the speeds of agents tend to zero.

## 5. Conclusion

We discuss the consensus control of dynamical agents in network which associated with a graph $\mathcal{G}$. When the

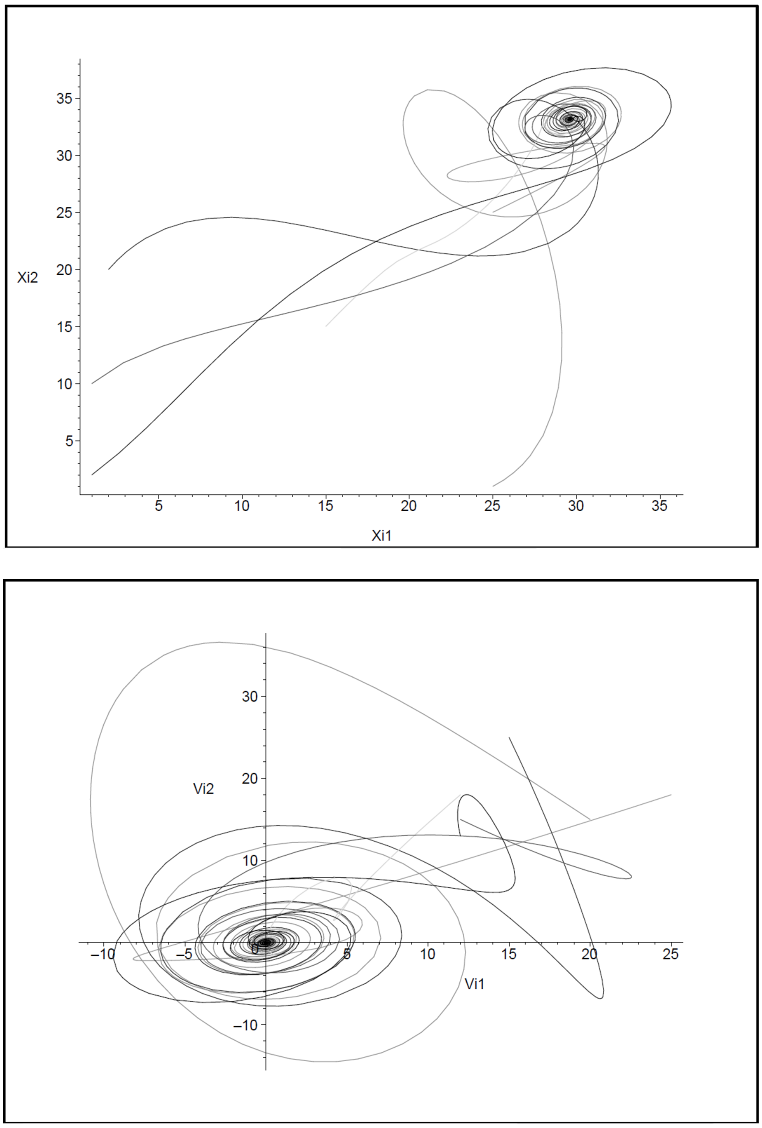


**Figure 2.** State and velocity trajectories of the agents in $\mathcal{G}$.

agents are moving in a plane, the aggregation of the dynamical agents are depended on not only the communicated error, but also the algebraic characterization of the communicated network graph and the dynamical properties of agents.

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