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A Design Method of Noncoherent Unitary Space-Time Codes

Li Peng, Qiuping Peng, Lingling Yang
Department of Electronics and Information Engineering, Huazhong University of Science and Technology, Wuhan National Laboratory for Optoelectronics, Wuhan, China
E-mail: pengli@mail.hust.edu.cn
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Abstract

We generalized an constructing method of noncoherent unitary space time codes (N-USTC) over Rayleigh flat fading channels. A family of N-USTCs with T symbol peroids, M transmit and N receive antennas was constructed by the exponential mapping method based on the tangent subspace of the Grassmann manifold. This exponential mapping method can transform the coherent space time codes (C-STC) into the N-USTC on the Grassmann manifold. We infered an universal framework of constructing a C-STC that is designed by using the algebraic number theory and has full rate and full diversity (FRFD) for t symbol periods and same antennas, where M, N, T, t are general positive integer. We discussed the constraint condition that the exponential mapping has only one solution, from which we presented an approach of searching the optimum adjustive factor α_{opt} that can generate an optimum noncoherent codeword. For different code parameters M, N, T, t and the optimum adjustive factor α_{opt}, we gave the simulation results of the several N-USTCs.[1]

Keywords: Noncoherent Uintary Space-Time Codes (N-USTC), Coherent Space-Time Codes (C-STC), Grassmann Manifold, Degree of Freedom, Exponenatial Map, Full Rate and Full Diversity (FRFD)

1. Introduction

The noncoherent unitary space-time code (N-USTC) in [1-4] provided a potential solution for the multiple antennas communication in fading channel that neither transmitter nor receiver knows the channel state imformation (CSI). This paper generalized an constructing method for a family of the N-USTCs based on the Grassmann manifold. The system models on noncoherent and cohenent channel are comparatively built. Starting from the basic theory of the Grassmann manifold [4], a basic thought of designing the Grassmannian unitary space-time matrix was described. That is the exponent mapping method [5,6] from the $M \times t$ C-STC to the $T \times M$ N-USTC for the MIMO system with M transmit and N receive antennas, where t and T are coherent and noncoherent symbol periods, respectively, and M, N, T, t are general positive integer, $T > t \ge M$ and $T = M + t$.

In order to map the $M \times t$ C-STC into the $T \times M$ N-USTC, firstly, one must consider how to construct the $M \times t$ C-STC. Many literatures [7-11] discussed multifarious methods of constructing the C-STCs. Enlightened by [7-11] and other literatures (omitted in reference as the limitation of length), we discussed a method of constructing the $M \times t$ universal C-STCs with FDFR based on the algebraic number theory. Therefore, we created four kinds of matrices: uncoded symbol matrix $\boldsymbol{S}$, linear combinatorial matrix $\boldsymbol{L}$, rotated matrix $\boldsymbol{R}$ and linear combinatorial symbol matrix $\boldsymbol{Z}$ that is $\boldsymbol{Z}=\boldsymbol{LSR}$ formed by the linear combinatorial technique of the symbols of constellations, such as q-PSK or q-QAM, and then we get the the coded matrix of a C-STC by transforming matrix $\boldsymbol{Z}$.

In the mapping process from the $M \times t$ C-STC into the $T \times M$ N-USTC, we discussed the constraint condition of the only one solution of the the exponent map, from which we discover that the optimum codeword of the Grassmannian N-USTC can be obtained by searching the optimum adjustive factor α_{opt}. Simulation tests show that for BPSK constellation symbols, when T is unchanged and antenna number M=N increases, the

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spectral efficiency increases and the performance of the bit error rate (BER) also advances, or when $M = N$ is unchanged and T increases, so do the spectral efficiency and the BER performance; for QPSK constellation symbols, when $M = N = 2$ and $T = 5$, the spectral efficiency achieves 2.4 bits/Hz/s but at the cost of sacrificing the BER performance.

2. System Model and Background Knowledge

2.1. System Model

We focus on the block fading channel model on which the fading coefficients are assumed to be constant during T periods of one codeword and to change independently from one codeword to the next. Under the assumption of no inter-symbol interference, the noncoherent channel model with codeword periods $T > M$ is

$$\boldsymbol{Y}_{TN} = \sqrt{T}\boldsymbol{X}_{TM}\boldsymbol{H}_{MN} + \boldsymbol{W}_{TN} \tag{1}$$

For the convenience of comparison and application later on, we simultaneously give the coherent channel model with codeword periods $t \geq M$:

$$\boldsymbol{Y}_{Nt} = \boldsymbol{H}_{NM}\boldsymbol{B}_{Mt} + \boldsymbol{W}_{Nt} \tag{2}$$

where $\boldsymbol{Y}$ is $T \times N$ received signal matrix for noncoherent model or $N \times t$ matrix for coherent model, $\boldsymbol{H}$ is $M \times N$ or $N \times M$ fading coefficients matrix and $\boldsymbol{W}$ is $T \times N$ or $N \times t$ additive noise matrix. Elements of $\boldsymbol{H}$ and $\boldsymbol{W}$ are assumed to be the independent and identically distributed complex Gaussian random variables respectively from distribution $CN(0,1)$ and $CN(0,\sigma^2)$. $\boldsymbol{X}_{TM}$ and $\boldsymbol{B}_{Mt}$ are noncoherent and coherent transmit signal matrices, respectively.

2.2. Grassmann Manifolds and Its Tangential Space

Manifold is a topologic space which is locally homeomorphic to the Euclidian space. More formally, Every point on n-dimensional manifold has a neighborhood homeomorphic to n-dimensional Euclidian space $\boldsymbol{R}^n$.

We consider a set of all M-dimension linear subspaces in T-dimension complex space. This set has the structure of manifold, called Grassmann manifold and denoted by $\boldsymbol{G}_{T,M}^{C}$, and its definition [12] is:

$$\boldsymbol{G}_{T,M}^{C} \triangleq \left\{ \langle \boldsymbol{\Phi} \rangle \middle| \boldsymbol{\Phi}^{\dagger}\boldsymbol{\Phi} = \boldsymbol{I}_M \right\} \tag{3}$$

where "$\dagger$" denotes transpose for real number or conjugate transpose for complex number; $\langle \boldsymbol{\Phi} \rangle$ denotes the subspace spanned by M column vectors in an $T \times M$ unitary matrix $\boldsymbol{\Phi}$. $\boldsymbol{G}_{T,M}^{C}$ can also be represented by the quotient space of the unitary group $\mathbf{U}(n)$ [12], *i.e.*

$$\boldsymbol{G}_{T,M}^{C} = \boldsymbol{U}(T) \big/ \left(\boldsymbol{U}(M) \times \boldsymbol{U}(T-M) \right) \tag{4}$$

As the real dimension of the unitary group $\mathbf{U}(n)$ is $\dim_R \boldsymbol{U}(n) = n^2$, one can obtain the real dimension of $\boldsymbol{G}_{T,M}^{C}$: $\dim_R \boldsymbol{G}_{T,M}^{C} = T^2 - M^2 - (T-M)^2 = 2M(T-M)$ [13] according to (4). So the complex dimension of $\boldsymbol{G}_{T,M}^{C}$ is $\dim_C \boldsymbol{G}_{T,M}^{C} = M(T-M)$ which means that the N-USTCs on $\boldsymbol{G}_{T,M}^{C}$ have $M(T-M)$ degrees of freedom, and the maximal symbol rate is $M(1-(M/T))$ [3].

Literature [5,6] introduces that the tangential space of any a point on $\boldsymbol{G}_{T,M}^{C}$ forms a set of matrices as follows:

$$\boldsymbol{\Delta}_{TM} = \boldsymbol{Q}\begin{pmatrix} 0 & \boldsymbol{B} \\ -\boldsymbol{B}^{\dagger} & 0 \end{pmatrix} \tag{5}$$

where $\boldsymbol{B} \in \boldsymbol{C}^{M \times (T-M)}$ and the point $\boldsymbol{Q}$ can be chosen arbitrarily, *i.e.*, for simplified calculation, one can choose $\boldsymbol{B} = \boldsymbol{I}_{TM} = \left[\boldsymbol{I}_{M \times M}, \boldsymbol{0}_{(T\text{-}M) \times M} \right]^{\dagger}$ as a reference subspace on $\boldsymbol{G}_{T,M}^{C}$. The dimension of the tangential space defined by (5) is also $M(T-M)$. According to the theory of Lie-group, *i.e.*, the point of the tangential space on $\boldsymbol{G}_{T,M}^{C}$ can be projected into the point of $\boldsymbol{G}_{T,M}^{C}$ by exponent map, the point $\boldsymbol{X}$ of $\boldsymbol{G}_{T,M}^{C}$ can be denoted by the exponent form of the tangential space:

$$\boldsymbol{X}_{TM} = \left[\exp\begin{pmatrix} 0 & \boldsymbol{B} \\ -\boldsymbol{B}^{\dagger} & 0 \end{pmatrix} \right] \boldsymbol{I}_{TM} \tag{6}$$

(6) shows a complicated computing task, but it can be simplified by the technique of the singular value decompose (SVD) of matrix. $\boldsymbol{B}$ is disposed by the SVD as follows:

$$\boldsymbol{B} = \boldsymbol{U}_{M \times M} \boldsymbol{\Lambda}_{M \times (T-M)} \boldsymbol{V}_{(T-M) \times (T-M)}^{\dagger} \tag{7}$$

where $\boldsymbol{U}$ and $\boldsymbol{V}$ are unitary matrices, and the form of $\boldsymbol{\Lambda}$ is:

$$\boldsymbol{\Lambda} = \begin{pmatrix} \lambda_1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & \ddots & 0 & \vdots & \ddots & \vdots \\ 0 & 0 & \lambda_M & 0 & \cdots & 0 \end{pmatrix} \tag{8}$$

where $\lambda_1, \cdots, \lambda_M$ are the singular values of matrix $\boldsymbol{B}$.

Putting (7) into (6), one can obtain the simplified $\boldsymbol{X}_{TM}$:

$$\boldsymbol{X}_{TM} = \begin{pmatrix} \boldsymbol{UCU}^{\dagger} \\ \boldsymbol{VSU}^{\dagger} \end{pmatrix}_{T \times M} \tag{9}$$

where $\boldsymbol{C}_{M \times M} = \begin{pmatrix} \cos\lambda_1 & & 0 \\ & \ddots & \\ 0 & & \cos\lambda_M \end{pmatrix}$,

$$\boldsymbol{S}_{(T-M)\times M}=\begin{pmatrix}\sin\lambda_1 & & 0\\ & \ddots & 0\\ 0 & & \sin\lambda_M\end{pmatrix}^{\dagger}.$$

3. Coherent Space-Time Codes

Most methods of constructing the C-STCs with FRFD are to efficaciously combine all information symbols s_i ($i=1,2,\cdots,tM$) to form the coding matrix $\boldsymbol{B}_{Mt}$, where all s_i belong to one of constellations, such as q-PSK or q-QAM, etc. If we adopt the technique of linear combination to design $\boldsymbol{B}_{Mt}$, then the rank and determinant properties of $\boldsymbol{B}_{Mt}(s_i)$ is equivalent to them of $\boldsymbol{B}_{Mt}(s_i)-\boldsymbol{B}_{Mt}(s_j)$, where $i,j\in[1,Mt]$ and $i\neq j$. Let r be the minimal rank available of any codeword matrix $\boldsymbol{B}_{Mt}(s_i)$. According to the design criterion of determinate in [10], under the linear combination of all s_i of forming $\boldsymbol{B}_{Mt}$, we can obtain a FRFD matrix and its rank $r=M$, so the maximum coding gain $\left(\prod_{j=1}^{r}\lambda_j\right)^{1/r}=\left(\prod_{j=1}^{M}\lambda_j\right)^{1/M}$ can be guaranteed. Being enlightened by using the algebraic number theory to construct the C-STCs in [8,10,11], we investigate how to design the universal coherent matrix $\boldsymbol{B}_{Mt}$ with FRFD for M,N,t being general positive integer. The applied design step is shown as follows.

a) We first create three kinds of matrices: uncoded symbol matrix $\boldsymbol{S}$, linear combinatorial matrix $\boldsymbol{L}$ (also named left-multiplied matrix) and rotated matrix $\boldsymbol{R}$ (also named right-multiplied matrix), they have next general forms:

$$\boldsymbol{S}=\begin{bmatrix}s_1 & s_{M+1} & \cdots & s_{(t-1)M+1}\\ s_2 & s_{M+2} & \cdots & s_{(t-1)M+2}\\ \vdots & \vdots & \ddots & \vdots\\ s_M & s_{2M} & \cdots & s_{Mt}\end{bmatrix},$$

$$\boldsymbol{L}=\begin{bmatrix}1 & \theta & \theta^2 & \cdots & \theta^{M-1}\\ 1 & j\theta & j^2\theta^2 & \cdots & j^{M-1}\theta^{M-1}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 1 & j^{M-1}\theta & j\theta^2 & \cdots & j^{M-2}\theta^{M-1}\end{bmatrix},$$

$$\boldsymbol{R}=\begin{bmatrix}1 & 0 & \cdots & 0\\ 0 & \phi & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \phi^{t-1}\end{bmatrix}$$

where $s_1,s_2,\cdots,s_{tM}$ take from the constelltions; let $\phi^t=\theta$; choosing j makes the determinate of matrix $\boldsymbol{L}$ be unequal to zero. Let $\theta=e^{i\omega}$ which is an algebraic number [10], here $i=\sqrt{-1}$ and ω is a parameter of needing the optimization design so that ω is searched in $(0,\pi/2)$ to maximize the coding gain.

b) For $\boldsymbol{S}$ left-multiplied by $\boldsymbol{L}$ and right-multiplied by $\boldsymbol{R}$, one can get the linear combinatorial symbol matrix $\mathbf{Z}_{M\times t}$ like (10) as follows:

c) In the linear combinatorial symbol matrix $\mathbf{Z}_{M\times t}$, circulant-right-shifting the second row one time, the third row two times,…, the final row (*i.e.*, the M th row) $M-1$ times, respectively, one can get the coded matrix like (11) as follows:

$$\boldsymbol{Z}_{M\times t}=\boldsymbol{LSR}=\begin{bmatrix}1 & \theta & \theta^2 & \cdots & \theta^{M-1}\\ 1 & j\theta & j^2\theta^2 & \cdots & j^{M-1}\theta^{M-1}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 1 & j^{M-1}\theta & j\theta^2 & \cdots & j^{M-2}\theta^{M-1}\end{bmatrix}\begin{bmatrix}s_1 & s_{1+M} & \cdots & s_{1+M(t-1)}\\ s_2 & s_{2+M} & \cdots & s_{2+M(t-1)}\\ \vdots & \vdots & \ddots & \vdots\\ s_M & s_{2M} & \cdots & s_{Mt}\end{bmatrix}\begin{bmatrix}1 & 0 & \cdots & 0\\ 0 & \phi & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \phi^{t-1}\end{bmatrix}$$

$$=\begin{bmatrix}s_1+\theta s_2+\ldots+\theta^{M-1}s_M & \phi\left(s_{1+M}+\theta s_{2+M}+\cdots+\theta^{M-1}s_{2M}\right) & \cdots & \phi^{t-1}\left(s_{1+M(t-1)}+\theta s_{2+M(t-1)}+\cdots+\theta^{M-1}s_{Mt}\right)\\ s_1+j\theta s_2+\ldots+j^{M-1}\theta^{M-1}s_M & \phi\left(s_{1+M}+j\theta s_{2+M}+\cdots+j^{t-1}\theta^{t-1}s_{2M}\right) & \cdots & \phi^{t-1}\left(s_{1+M(t-1)}+j\theta s_{2+M(t-1)}+\cdots+j^{M-1}\theta^{M-1}s_{Mt}\right)\\ \vdots & \vdots & \ddots & \vdots\\ s_1+j^{M-1}\theta s_2+\ldots+j^{M-2}\theta^{M-1}s_M & \phi\left(s_{1+M}+j^{M-1}\theta s_{2+M}+\cdots+j^{M-2}\theta^{M-1}s_{2M}\right) & \cdots & \phi^{t-1}\left(s_{1+M(t-1)}+j^{M-1}\theta s_{2+M(t-1)}+\cdots+j^{M-2}\theta^{M-1}s_{Mt}\right)\end{bmatrix} \quad (10)$$

$$\boldsymbol{B}_{Mt}=\begin{bmatrix} s_1+\theta s_2+\cdots+\theta^{M-1}s_M & \phi\left(s_{1+M}+\theta s_{2+M}+\cdots+\theta^{M-1}s_{2M}\right) & \cdots & \begin{matrix}\phi^{t-1}\left(s_{1+M(t-1)}+\theta s_{2+M(t-1)}\right.\\ \left.+\cdots+\theta^{M-1}s_{Mt}\right)\end{matrix} \\ \begin{matrix}\phi^{t-1}\left(s_{1+M(t-1))}+j\theta s_{2+M(t-1)}\right.\\ \left.+\cdots+j^{M-1}\theta^{M-1}s_{Mt}\right)\end{matrix} & s_1+j\theta s_2+\cdots+j^{M-1}\theta^{M-1}s_M & \cdots & \begin{matrix}\phi^{t-2}\left(s_{1+M(t-2)}+j\theta s_{2+M(t-2)}\right.\\ \left.+\cdots+j^{M-1}\theta^{M-1}s_{M(t-1)}\right)\end{matrix} \\ \vdots & \vdots & \ddots & \vdots \\ \begin{matrix}\cdots\phi^{t-1}\left(s_{1+M(t-1)}+j^{M-1}\theta s_{2+M(t-1)}\right.\\ \left.+\cdots+j^{M-2}\theta^{M-1}s_{Mt}\right)\end{matrix} & s_1+j^{M-1}\theta s_2+\cdots+j^{M-2}\theta^{M-1}s & \begin{matrix}\phi\left(s_{1+M}+j^{M-1}\theta s_{2+M}\right.\\ \left.+\cdots+j^{M-2}\theta^{M-1}s_{2M}\right)\cdots\end{matrix} & \end{bmatrix} \quad (11)$$

Several examples of the C-STCs are presented as follows. Let $M=t=2$, according to (10), we have:

$$\boldsymbol{Z}_{2\times2}=\boldsymbol{LSR}=\begin{bmatrix}1 & \theta\\ 1 & j\theta\end{bmatrix}\begin{bmatrix}s_1 & s_3\\ s_2 & s_4\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & \phi\end{bmatrix}=\begin{bmatrix}s_1+\theta s_2 & \phi\left(s_3+\theta s_4\right)\\ s_1+j\theta s_2 & \phi\left(s_3+j\theta s_4\right)\end{bmatrix}$$

If $j=1$, then $\left|\boldsymbol{Z}_{2\times2}\right|=0$ which means the linear combinatorial form of information symbols is lost. If $j=-1$, the linear combinatorial operation is retained. For $\phi^2=\theta=\mathrm{e}^{i(\pi/4)}$, we can get the 2×2 C-STC matrix which is same as those in [5,6]

$$\boldsymbol{B}_{2\times2}=\begin{bmatrix}s_1+\theta s_2 & \phi\left(s_3+\theta s_4\right)\\ \phi\left(s_3-\theta s_4\right) & s_1-\theta s_2\end{bmatrix}$$

Again let $M=3$, $t=4$, we have

$$\boldsymbol{Z}_{3\times4}=\boldsymbol{LSR}=\begin{bmatrix}1 & \theta & \theta^2\\ 1 & j\theta & j^2\theta^2\\ 1 & j^2\theta & j\theta^2\end{bmatrix}\begin{bmatrix}s_1 & s_4 & s_7 & s_{10}\\ s_2 & s_5 & s_8 & s_{11}\\ s_3 & s_6 & s_9 & s_{12}\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \phi & 0 & 0\\ 0 & 0 & \phi^2 & 0\\ 0 & 0 & 0 & \phi^3\end{bmatrix}$$

when $\phi^4=\theta=\mathrm{e}^{i\frac{\pi}{6}}$, $j=\mathrm{e}^{i\frac{2\pi}{3}}$, $\boldsymbol{Z}_{3\times4}$ is full rank. Thus we can get the 3×4 C-STC matrix like (12) as follows:

$$\boldsymbol{B}_{3\times4}==\begin{bmatrix}s_1+\theta s_2+\theta^2 s_3 & \phi\left(s_4+\theta s_5+\theta^2 s_6\right) & \phi^2\left(s_7+\theta s_8+\theta^2 s_9\right) & \phi^3\left(s_{10}+\theta s_{11}+\theta^2 s_{12}\right)\\ \phi^3\left(s_{10}+j\theta s_{11}+j^2\theta^2 s_{12}\right) & s_1+j\theta s_2+j^2\theta^2 s_3 & \phi\left(s_4+j\theta s_5+j^2\theta^2 s_6\right) & \phi^2\left(s_7+j\theta s_8+j^2\theta^2 s_9\right)\\ \phi^2\left(s_7+j^2\theta s_8+j\theta^2 s_9\right) & \phi^3\left(s_{10}+j^2\theta s_{11}+j\theta^2 s_{12}\right) & s_1+j^2\theta s_2+j\theta^2 s_3 & \phi\left(s_4+j^2\theta s_5+j\theta^2 s_6\right)\end{bmatrix} \quad (12)$$

Similarly, we can get $\boldsymbol{B}_{2\times3}$, $\boldsymbol{B}_{2\times4}$, $\boldsymbol{B}_{2\times5}$ and $\boldsymbol{B}_{3\times3}$ which and all above will be applied to simulation testing later on.

4. Noncoherent Space-Time Codes

Literatures [5,6] introduce the design criterion of the Grassmann N-USTC. Let Ω_i and Ω_j denote the subspaces spanned by the column vectors of $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$, respectively. Let $\left(\theta_1,\theta_2,\cdots,\theta_M\right)$ denote the principal angles between Ω_i and Ω_j, then the chordal product distance between two points $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$ on $\boldsymbol{G}_{T,M}^{C}$ is:

$$\Theta_{i,j}=\prod_{m=1}^{M}\sin^2\theta_m \quad (13)$$

The design criterion of the Grassmann N-USTC C is to make the minimal chordal product distance achieve the maximum, *i.e.* $\max\limits_{C}\min\limits_{X_i\neq X_j\in C}\Theta_{i,j}$. It is known from the expression (6) that the product $\boldsymbol{\Theta}_{i,0}$ of the chordal distances between the subspace Ω_i and the reference subspace Ω_0 is equal to the product of all singular values in matrix $\boldsymbol{B}_i$ whose M column vectors span the subspace Ω_i. The design criterion of a C-STC is to maximize its coding gain, which is equal to maximizing the minimum product of singular values of codeword matrix. Therefore we can use the matrix $\boldsymbol{B}_{Mt}$ of (11) to design the matrix $\boldsymbol{B}$ in (6).

The exponential map from $\boldsymbol{B}_{Mt}$ to $\boldsymbol{X}_{TM}$ must be the monotone and reversible, which requires that the exponential map of (6) is the reversible map, *i.e.*, (9) exists the reversible matrix. So $\cos\lambda_m$ and $\sin\lambda_m$ in (9) should be the monotone function, then the constraint condition of λ_m is:

$$\max_m \lambda_m \left(B_{Mt}\right) \le \pi/2, \quad m = 0,1,\cdots,M-1 \tag{14}$$

where $\lambda_m\left(B_{Mt}\right)$ is the m th singular value of any codeword B_{Mt} which is equal to the m th principal angle between any Ω and Ω_0. Therefore, a conceivable skill is that taking a scale α, called the adjustive factor which multiplies the codeword matrix B_{Mt}, can guarantee the map to be monotone and reversible. Thus (6) can be rewritten as follows:

$$\boldsymbol{X}_{TM} = \left[\exp\begin{pmatrix} 0 & \alpha\boldsymbol{B} \\ -\left(\alpha\boldsymbol{B}\right)^{\dagger} & 0 \end{pmatrix}\right]\boldsymbol{I}_{TM} \tag{15}$$

Obviously, α only affects the singular value of the matrix B_{Mt}. Let $\left(0,\alpha_{\max}\right)$ denote a range of α values, the method of optimum searching α is described as follows.

Let $\boldsymbol{X}_i, \boldsymbol{X}_j \in \boldsymbol{G}_{T,M}^{C}$ be two distinct N-USTC codewords, the SVD of the matrix $\boldsymbol{X}_j^{\dagger}\boldsymbol{X}_i$ is $\boldsymbol{X}_j^{\dagger}\boldsymbol{X}_i = \boldsymbol{U\Sigma V}^{\dagger}$, where $\boldsymbol{\Sigma}$ is a diagonal matrix formed by the singular values $\lambda_1,\cdots,\lambda_M$. The M principal angles between the subspaces Ω_i and Ω_j spanned respectively by $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$ are $\theta_m = \cos^{-1}\lambda_m$, $m = 0,1,\cdots,M-1$, substituting $\theta_m = \cos^{-1}\lambda_m$ for the expression (13), we can get:

$$\Theta_{i,j} = \prod_{m=1}^{M}\left(1-\lambda_m^2\right) \tag{16}$$

Under the condition of making $\Theta_{i,j}$ maximize, by searching α in $\left(0,\alpha_{\max}\right)$, we can get the optimum α_{opt}.

Now we design $\boldsymbol{X}_{TM}$ by mapping exponentially $\boldsymbol{B}_{Mt}$ to $\boldsymbol{G}_{T,M}^{C}$. The design step is:

(a) For M transmit antennas and t coherent periods, according to (10) and (11), design the C-STC matrix $\boldsymbol{B}_{Mt}$ from the information symbol $s_1, s_2, \cdots, s_{tM}$.

(b) Substitute $\boldsymbol{B}_{Mt}$ for $\boldsymbol{B}$ in (15), search the optimum adjustive factor α, and construct the exponential mapping matrix $\boldsymbol{E} = \begin{pmatrix} 0 & \alpha\boldsymbol{B}_{Mt} \\ -\left(\alpha\boldsymbol{B}_{Mt}\right)^{\dagger} & 0 \end{pmatrix}$.

(c) According to (7) and the above $\boldsymbol{E}$ matrix, applying the SVD to $\left(\alpha_{opt}\boldsymbol{B}_{Mt}\right)$, we get the noncoherent codeword $\boldsymbol{X}_{TM}$ like (9), and $T = M + t$. □

5. Examples and Numerical Simulation Results

According to the above presented method, this section gives several examples of the N-USTCs whose numerical simulation curves are shown as **Figure 1**. Let $D = M\left(T-M\right)$ denote the degree of freedom. Suppose modulation is q-PSK with symbol number p. So the spectral efficiency of the N-USTC is $\eta_{T\times M} = \left(\log_2 p\right)\cdot D/T$ bits/Hz/s.

Example 1: Compare two curves of solid line with black dot and dash line with circle in **Figure 1**. For system $M = N = 2$ and QPSK modulation with $p = 4$, let $t = 2$, we construct the C-STC $\boldsymbol{B}_{2\times 2}$, where $\phi^2 = \theta = \mathrm{e}^{i(\pi/4)}$, $j = -1$. When $t = 3$, we get $\boldsymbol{B}_{2\times 3}$, where $\phi^3 = \theta = \mathrm{e}^{i(\pi/4)}$, $j = -1$. As $T = M + t$, having $T = 4$ for $t = 2$ and $T = 5$ for $t = 3$, corresponding to $D = 4$ and $D = 6$, we compute $\eta_{4\times 2} = 2$ bits/Hz/s and $\eta_{5\times 2} = 2.4$ bits/Hz/s, respectively. We map the C-STC $\boldsymbol{B}_{2\times 2}$ into the N-USTC $\boldsymbol{X}_{4\times 2}$ on $G_{4,2}^{C}$ and $\boldsymbol{B}_{2\times 3}$ into $\boldsymbol{X}_{5\times 2}$ on $G_{5,2}^{C}$ which correspond to the optimum adjustive factor $\alpha_{opt,Q}^{4\times 2} = 0.29$ and $\alpha_{opt,Q}^{5\times 2} = 0.25$, respectively. At 10^{-5} bit error rate (BER), $\boldsymbol{X}_{5\times 2}$ outperform $\boldsymbol{X}_{4\times 2}$ about 3 dB. Obviously, under all parameter being same except T increasing, the N-USTC BER performance and the spectral efficiency are improved.

Example 2: Compare four curves of solid line with black square, dash line with white square, solid line with black diamond and dash line with white diamond in **Figure 1**. For system $M = N = 2$ and BPSK modulation with $p = 2$, let $t = 2,3,4,5$, get $T = 4,5,6,7$ and $D = 4,6,8,10$, so $\eta_{4\times 2} = 1$, $\eta_{5\times 2} = 1.2$, $\eta_{6\times 2} = 1.33$ and $\eta_{7\times 2} = 1.43$ bits/Hz/s, respectively. We map $\boldsymbol{B}_{2\times 2}$, $\boldsymbol{B}_{2\times 3}$, $\boldsymbol{B}_{2\times 4}$ and $\boldsymbol{B}_{2\times 6}$ into $\boldsymbol{X}_{4\times 2}$, $\boldsymbol{X}_{5\times 2}$, $\boldsymbol{X}_{6\times 2}$ and $\boldsymbol{X}_{7\times 2}$ whose factors are $\alpha_{opt,B}^{4\times 2} = 0.41$, $\alpha_{opt,B}^{5\times 2} = 0.36$, $\alpha_{opt,B}^{6\times 2} = 0.32$ and $\alpha_{opt,B}^{7\times 2} = 0.29$. At 10^{-5} BER, the performance of the N-USTC improve about 0.5 - 1.0 dB and the spectral efficiency increases along with T increasing.

Example 3: Compare two curves with solid line with black triangle and dash line with white triangle in **Figure 1**. For system $M = N = 3$ and BPSK modulation with $p = 2$, let $t = 3,4$, get $T = 6,7$ and $D = 9,12$, so $\eta_{6\times 3} = 1.5$ and $\eta_{7\times 3} = 1.71$ bits/Hz/s, respectively. We map $\boldsymbol{B}_{3\times 3}$ and $\boldsymbol{B}_{3\times 4}$ into $\boldsymbol{X}_{6\times 3}$ and $\boldsymbol{X}_{7\times 3}$ whose fac-

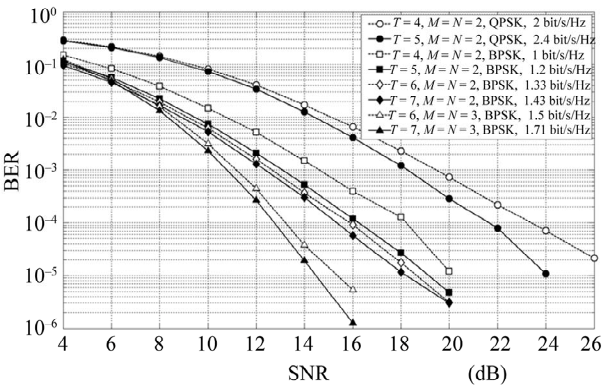

Figure 1. Performance comparison of several N-USTCs.

tors are $\alpha_{opt,B}^{6\times3}=0.24$ and $\alpha_{opt,B}^{7\times3}=0.22$. At 10^{-5} BER, the performance of the N-USTC improve about 0.8 dB from $T=6$ to $T=7$, and the spectral efficiency also increases 0.2 bits/Hz/s.

6. Conclusions

A specific step that maps the coherent space-time matrix into the noncoherent space-time matrix by means of the exponent form of the tangential space of Grassmann manifold was summed up for designing the N-USTCs. Especially, our work makes the structural parameters M, N, T, t with regard to both the N-USTC based on the Grassmann manifold and the C-STC based on the algebraic number theory be able to be designed more flexibly. We also discovered that in the discussed family of Grassmannian N-USTC, the optimum codeword can be obtained by searching the optimum adjustive factor α_{opt}. It is noticed that the design of the parameter j in left-multiplied matrix $\boldsymbol{L}$ is open problem, we will track this problem in the future.

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