

Joint Power Allocation and Beamforming for Cooperative Networks

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Abstract

In this paper, we investigate power allocation problem with the use of transmit beamforming in a dual hop MISO (multiple input single output) relay channel. We consider either amplify and forward (AF) or decode and forward (DF) cooperative protocols at the relay and optimize the power allocated to the relay and the source, under total transmit power constraint, to minimize the bit error rate (BER) of relaying system. Cooperative communication is viewed as a method for increasing diversity gain and reducing end to end path loss. The use of relay can create a virtual antenna array so that it allows users to exploit the advantages of multiple input multiple output (MIMO) techniques. In this work, we solve cooperative ratio, which is defined as the ratio power used for cooperative transmission over the total power. This approach is then compared to an equal power assignment method and its performance enhancement is verified by simulation results.

Keywords: Amplify and Forward, Decode and Forward, Transmit Beamforming, Power Allocation, MISO, Cooperative Communication

1. Introduction

In recent years, cooperative diversity is considered as a potential transmit strategy for wireless networks. The basic idea of cooperative transmission is to allow several transmit nodes in the network to help in order to create a virtual antenna array and exploit spatial diversity at the destination [1-3]. It has been shown in the literature [4], [5] that cooperative communication can avoid the difficulties of implementing actual antenna arrays and convert the single input single output (SISO) system into a virtual multiple input multiple output (MIMO) system. In this way, cooperation between users allows them to exploit the diversity gain and others advantages of MIMO system in a SISO wireless network. Several protocols have been proposed to achieve the gains promised by the use of node cooperation. Two main relay protocols are widely known: amplify and forward (AF) and decode and forward (DF). AF means that the received signal is multiplied by an amplification gain and then retransmitted by the relay without performing any decoding. In contrast to this, the signal is decoded at the relay and re-encoded for retransmission in (DF) strategy.

An important question in cooperative communication

is power allocation. Given the same total power, how much should be allocated to source information transmission and how much to relaying information transmission. In [6], efficient power allocation strategy is investigated in an orthogonal AF network to satisfy the target SNR requirement. In [7], the achievable information capacity is analyzed to obtain optimal power allocation and partner selection with the total power constraints for SISO relay channels. In [8], optimal power allocation is studied to minimize the total energy consumption satisfying the BER target of the cooperative system. Most current research considers transmit beamforming and distributed beamforming in relaying system. In [9], the performance of the DF protocol in multiple relay channels with maximal ratio combining (MRC) and transmit beamforming has been analyzed. In [10], the performance of a two hop relay network with transmit beamforming at the source and MRC at the destination has been analyzed. Also, distributed beamforming over multiple relays is widely studied. For example, several distributed beamforming techniques using AF strategy have been recently proposed in [11,12] that minimize the total relay transmit power subject to the receive signal-to-noise ratio (SNR) constraint.

In this paper, we focus on the optimal cooperative ratio in the sense to minimize the average BER. For both AF and DF relays, we solve cooperative ratio, which is defined as the ratio power used for cooperative transmission over the total power.

In particular, we focus on AF/DF cooperation scheme in an environment with one source equipped with multiple antennas and using beamforming transmitting to the destination through a relay equipped both with single antenna.

The remainder of this paper is organized as follows: Section 2 deals with system model in consideration. In Section 3, we formulate an optimal power allocation solution for AF protocol and then in Section 4 for DF protocol. Section 5 presents simulations results. The paper finally draws the conclusions in Section 6.

2. System Model

Consider a cooperative MISO networks, where the source s equipped with N antennas and using transmit beamforming and communicates with a destination d with the help of relay r equipped both with a single antenna. We assume that the source does not have a direct link to the destination due to the large distance or the fading obstacle. The whole transmission is accomplished in two phases: in the first phase, the source transmits to the relay, and the relay receives. In the second phase, the relay amplifies (AF) or decode (DF) the received signal and forwards to the destination. We assume that the relay is located in the straight line between the source and the destination. The distance between s to r , s to d and r to d are denoted as d_{sr} , d_{sd} and d_{rd} respectively. We define a ratio r as follows: $d_{sr} = r d_{sd}$ and $d_{sr} = (1 - r) d_{sd}$. In the first phase, the source uses beamforming to transmit to the relay. The source-relay r signal model y_{sr} is given by:

$$y_{sr} = \sqrt{P_s K d_{sr}^{-\alpha}} h_{sr} w x + n_{sr} \quad (1)$$

where $h_{sr} = [h_{1r}, h_{2r}, \dots, h_{Nr}]$ is $(1 \times N)$ channel vector from the source to the relay with complex Gaussian entries with zero mean and unit variance. α is the path loss exponent and K is a constant due to path loss at the reference distance. w is $(N \times 1)$ beamforming vector and satisfies $(w^* w = 1)$, where $(\cdot)^*$ denotes the conjugate transpose. x is the modulated transmitted symbol, and n_{sr} is the channel noise drawn from an ensemble of independent and identically distributed (i.i.d) complex Gaussian random variables with zero mean and variance σ_n^2 . P_s is the transmitting energy used at the source. During the second phase, the relay transmits to the destination. The relay-destination y_{rd} signal model is given as:

$$y_{rd} = \sqrt{P_r K d_{rd}^{-\alpha}} h_{rd} x_r + n_{rd} \quad (2)$$

where h_{rd} is the channel gain of relay-destination path which is complex gaussian with zero mean and unit variance and n_{rd} is the channel noise drawn from an ensemble of i.i.d complex Gaussian random variables with zero mean and variance σ_n^2 . P_r is the transmitting energy used at the relay and x_r is the relay signal. Either AF or DF can be used in this phase. We discuss them separately in the next two sections.

3. Amplify and Forward and Optimum Power Allocation

If the relay use AF protocol,

$$x_r = G y_{sr} \quad (4)$$

The relay-destination y_{rd} signal model is rewritten as:

$$y_{rd} = \sqrt{P_r K d_{rd}^{-\alpha}} h_{rd} G y_{sr} + n_{rd} \quad (5)$$

where

$$G = \frac{1}{\sqrt{P_s K d_{sr}^{-\alpha} |h_{sr} w|^2 + \sigma_n^2}} \quad (6)$$

The end to end SNR (Signal to Noise Ratio) at the destination is given by:

$$\gamma_{AF1} = \frac{\frac{P_s K d_{sr}^{-\alpha} |h_{sr} w|^2}{\sigma_n^2} \frac{P_r K d_{rd}^{-\alpha} |h_{rd}|^2}{\sigma_n^2}}{\frac{P_r K d_{rd}^{-\alpha} |h_{rd}|^2}{\sigma_n^2} + \frac{1}{G^2 \sigma_n^2}} \quad (7)$$

The end to end SNR can be simplified as:

$$\gamma_{AF} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1} \quad (8)$$

where $\gamma_1 = \frac{P_s K d_{sr}^{-\alpha}}{\sigma_n^2} |h_{sr} w|^2$ and $\gamma_2 = \frac{P_r K d_{rd}^{-\alpha}}{\sigma_n^2} |h_{rd}|^2$.

The form of the end to end SNR in (8) can be upper bounded by:

$$\gamma_{AF} \leq \frac{\bar{\gamma}_1 \bar{\gamma}_2}{\bar{\gamma}_1 + \bar{\gamma}_2 + 1} \quad (9)$$

where $\bar{\gamma}_1 = E[\gamma_1]$ and $\bar{\gamma}_2 = E[\gamma_2]$. When the signal is transmitted with coherent beamforming, the beamforming

weight is $w = \frac{h_{sr}^*}{\|h_{sr}\|}$ and the beamforming gain is

$E[|h_{sr} w|^2] = N$, we can rewrite $\bar{\gamma}_1$ and $\bar{\gamma}_2$ as:

$$\bar{\gamma}_1 = \frac{P_s K d_{sr}^{-\alpha} N}{\sigma_n^2} \quad \text{and} \quad \bar{\gamma}_2 = \frac{P_r K d_{rd}^{-\alpha}}{\sigma_n^2}$$

The goal of optimal power allocation is to maximize the total system SNR under the total power constraint P

as:

$$\begin{aligned} & \max \gamma_{AF}(P_s, P_r) \\ & \text{s.t. } P_s + P_r \leq P \end{aligned}$$

The constraint can be reformulated by introducing new variable β , $0 \leq \beta \leq 1$ and $P_s = \beta P$ and $P_r = (1 - \beta)P$. The average end to end SNR can then be calculated as follows:

$$\bar{\gamma}_{AF} = \frac{\beta PKd_{sr}^{-\alpha} N(1 - \beta) PKd_{rd}^{-\alpha} \gamma_0}{\beta PKd_{sr}^{-\alpha} N + (1 - \beta) PKd_{rd}^{-\alpha} + \sigma_n^2} \quad (10)$$

where $\gamma_0 = \frac{1}{\sigma_n^2}$.

Optimum power allocation can be performed by taking the first derivation of Equation (10) and equate it to zero.

$$\frac{\partial \bar{\gamma}_{AF}}{\partial \beta} = 0 \quad (11)$$

This will result in polynomial degree 2 in the form of

$$a\beta^2 + b\beta + c = 0$$

where

$$\begin{aligned} a &= -NP\gamma_0 Kd_{sr}^{-\alpha} + P\gamma_0 Kd_{rd}^{-\alpha} \\ b &= -2(P\gamma_0 Kd_{rd}^{-\alpha} + 1) \\ c &= P\gamma_0 Kd_{rd}^{-\alpha} + 1 \end{aligned} \quad (12)$$

Solving the above second degree polynomial, optimum power allocation ratio is obtained as the positive real root which lies between zero and one.

For a BPSK modulated signal, the BER is calculated as follows:

$$BER = Q(2\gamma_{AF}) \quad (13)$$

4. Decode and Forward and Optimum Power Allocation

As in the case of AF the relay receives the broadcast from the source in Phase 1, but instead simply amplifying and forwarding the signal, the relay tries to decode the signal and then forward it. If the relay decodes the signal correctly ($x_r = x$).

Considering that the message will be received correctly at destination when transmission of both phases is correct, the end to end error probability can be found using:

$$\begin{aligned} P_e^{e-e} &= 1 - \left((1 - P_e^{sr}) \times (1 - P_e^{rd}) \right) \\ &= P_e^{sr} + P_e^{rd} - P_e^{sr} \times P_e^{rd} \end{aligned} \quad (14)$$

where P_e^{sr} and P_e^{rd} are respectively the instantaneous probability of decoding error at source-relay link and

relay-destination link and can be calculated as follows:

$$P_e^{sr} = Q\left(\sqrt{2P_s\gamma_0 Kd_{sr}^{-\alpha} \|h_{sr}\|^2}\right) \quad (15)$$

$$P_e^{rd} = Q\left(\sqrt{2P_r\gamma_0 Kd_{rd}^{-\alpha} \|h_{rd}\|^2}\right) \quad (16)$$

where

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right) dt \quad (17)$$

Since $\|h_{sr}\|^2$ is the sum of squared N i.i.d. zero mean circularly symmetric complex Gaussian random variables, $\chi = \|h_{sr}\|^2$ has a chi-square distribution with $2N$ degrees of freedom and $X = \|h_{rd}\|^2$ has an exponential distribution.

After applying Chernoff Bound to (15) and (16) and integrating them with respect to distributions of their terms, we obtain the average error probability:

$$\begin{aligned} \overline{P_e^{rd}} &= \int_0^\infty \frac{1}{2} \exp(-P_r Kd_{rd}^{-\alpha} \gamma_0 X) \exp(-X) dX \\ &= \frac{1}{2(1 + P_r Kd_{rd}^{-\alpha} \gamma_0)} \end{aligned} \quad (18)$$

We recall that the Chernoff Bound for a Gaussian random variable is:

$$Q(\sqrt{x}) \leq \frac{1}{2} e^{-\frac{x}{2}} \quad (19)$$

$$\begin{aligned} \overline{P_e^{sr}} &= \int_0^\infty \frac{1}{2} \exp(-P_s Kd_{sr}^{-\alpha} \gamma_0 \chi) p(\chi) d\chi \\ &= \frac{1}{2^{(N+1)} (P_s Kd_{sr}^{-\alpha} \gamma_0 + 0.5)^N} \end{aligned} \quad (20)$$

The pdf (probability density function) of the random variable χ is given by:

$$p(\chi) = \frac{1}{2^N \Gamma(N)} \chi^{N-1} \exp\left(-\frac{\chi}{2}\right) \quad (21)$$

Substituting (18) and (20) into (14), we obtain the average end to end error probability as:

$$\begin{aligned} \overline{P_e^{e-e}} &= \frac{1}{2^{(N+1)} \left(\beta PKd_{sr}^{-\alpha} \gamma_0 + \frac{1}{2} \right)^N} \\ &+ \frac{1}{2(1 + (1 - \beta) PKd_{rd}^{-\alpha} \gamma_0)} \\ &- \frac{1}{2^{(N+2)} (\beta PKd_{sr}^{-\alpha} \gamma_0)^N (1 + (1 - \beta) PKd_{rd}^{-\alpha} \gamma_0)} \end{aligned} \quad (22)$$

Again, optimal power allocation can be similarly

found by replacing P_s and P_r respectively by βP and $(1-\beta)P$. Optimum power allocation can be found from Equation (22). Taking the first derivation and equate it to zero, the optimization problem translates into solving polynomial degree $N + 1$. In this paper, we consider $N = 4$. So, power allocation problem consists of solving a polynomial degree equal to 5

$$\begin{aligned}
 &(a\beta^5 + b\beta^4 + c\beta^3 + d\beta^2 + e\beta + f = 0) \\
 &a = 64(P\gamma_0 K d_{rd}^{-\alpha})(P\gamma_0 K d_{sr}^{-\alpha})^5 \\
 &b = 160(P\gamma_0 K d_{rd}^{-\alpha})(P\gamma_0 K d_{sr}^{-\alpha})^4 \\
 &c = 160(P\gamma_0 K d_{rd}^{-\alpha})(P\gamma_0 K d_{sr}^{-\alpha})^3 \\
 &d = 80(P\gamma_0 K d_{rd}^{-\alpha})(P\gamma_0 K d_{sr}^{-\alpha})^2 \\
 &\quad - 16(P\gamma_0 K d_{rd}^{-\alpha})(P\gamma_0 K d_{sr}^{-\alpha}) \\
 &e = 42(P\gamma_0 K d_{rd}^{-\alpha})(P\gamma_0 K d_{sr}^{-\alpha}) \\
 &\quad + 32(P\gamma_0 K d_{rd}^{-\alpha})^2(P\gamma_0 K d_{sr}^{-\alpha}) \\
 &f = -16(P\gamma_0 K d_{rd}^{-\alpha})^2(P\gamma_0 K d_{sr}^{-\alpha}) \\
 &\quad - 24(P\gamma_0 K d_{rd}^{-\alpha})(P\gamma_0 K d_{sr}^{-\alpha}) \\
 &\quad - 8(P\gamma_0 K d_{sr}^{-\alpha}) + (P\gamma_0 K d_{rd}^{-\alpha})^5
 \end{aligned} \tag{23}$$

5. Simulation Results

In this section, we evaluate the performance of our scheme in term of end-to-end BER. We consider equal and optimal power allocations for both AF and DF and evaluate their performance assuming that the relay is allowed to be in any position along a straight line between the source and the destination. The simulation is conducted for $N = 4$ transmit antennas at the source and we recall that the relay and the destination are equipped both of a single receive antenna.

Throughout the simulation, the parameters d_{sd} and α are set to 10 km and 3.6 respectively. We define the SNR (Signal to Noise Ratio) as the ratio between system total power P and noise variance σ_n^2 and we replace d_{sr} by $r d_{sd}$ and d_{rd} by $(1-r) d_{sd}$ in the different expressions of BER.

Figure 1 shows the average bit error rate (BER) versus r for AF protocol at SNR = 25 dB. It is observed that amplify and forward with optimal power allocation can bring significant performance improvement compared to equal power allocation with and without beamforming (BF) especially when the relay is away from the source. The lowest BER is achieved when $r = 0.7$ whereas with equal power allocation and without beamforming (single

antenna at the source) $r = 0.5$ and $r = 0.6$ with beamforming at the source.

Figure 2 displays the average BER versus r for DF protocol at SNR = 25 dB. Similar observations about BER performance compared to equal power allocation can also be made. It is interesting to see that the amelioration in performance is still observed as r increases. The lowest BER is achieved by optimal power allocation when $r = 0.8$

Figure 3 shows the difference in average BER between AF and DF evaluated at SNR = 25 dB. We note that DF protocol outperform AF protocol for r from 0.01 to 0.99.

6. Conclusions

In this work, we studied the performance of optimal power allocation with transmit beamforming in two hop AF/DF relay. We first derive an analytical study of optimal power allocation of the proposed system. Next, our

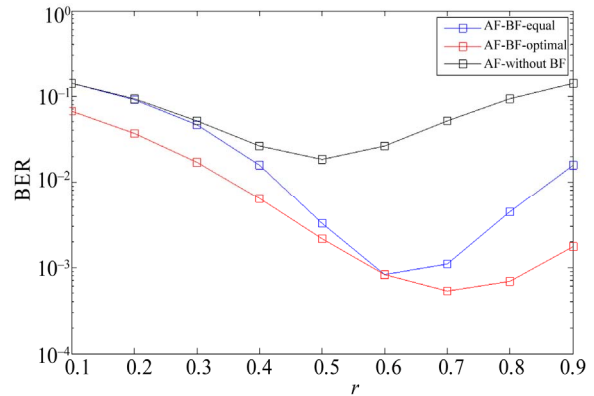


Figure 1. BER performance of AF with equal and optimal power allocation.

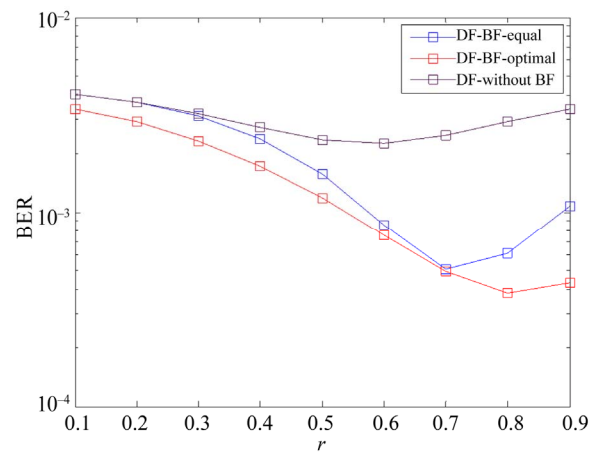


Figure 2. BER performance of DF with equal and optimal power allocation.

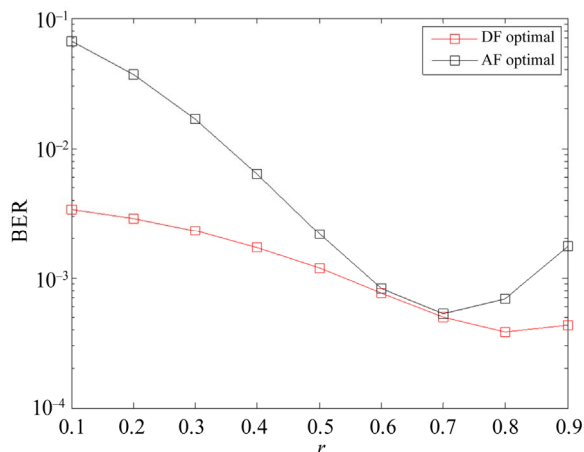


Figure 3. BER performance of AF and DF with equal and optimal power allocation.

analytical results are confirmed by simulations. It is shown that by using our optimized method, we provide better performance than equal power allocation with and without beamforming. Especially, when the relay is away from middle point between the source and the destination, optimal power allocation technique should be used to ensure better performance. Moreover, in our case the BER can be made low by moving the relay closer to the destination.

7. References

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