

# Low Complexity Discrete Bit-Loading for OFDM Systems with Application in Power Line Communications

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## Abstract

Adaptive bit-loading algorithms can improve the performance of OFDM systems significantly. The tradeoff between the performance of the algorithm and its computational complexity is essential for the implementation of loading algorithms. In this paper, we present a low complexity non-iterative discrete bit-loading algorithm to maximize the data rate subject to specified target BER and uniform power allocation. Simulation results show that the proposed algorithm outperforms the equal-BER loading and achieves similar rates to incremental allocation, yet with much lower complexity.

**Keywords:** Bit-Loading, OFDM, Power Line Communications

## 1. Introduction

Power Line Communication (PLC) technology has received a great amount of research interest in the last decade. This technology exploits the already existing and ubiquitous power line distribution infrastructure to provide a broadband multimedia connectivity solution to and within the home or office. Orthogonal Frequency Division Multiplexing (OFDM) has been a major candidate for PLC systems as well as many other broadband communication systems. This is mainly due to its robustness to multipath, selective fading and different kinds of interference. Examples of its applications include Digital Audio Broadcasting (DAB), Terrestrial digital TV (DVB-T), wireless LANs and Wi-Max.

In a conventional OFDM system, all subcarriers use a fixed constellation size. Therefore, their overall error probability is dominated by the subcarriers that have the worst signal-to-noise ratios (SNR). When channel state information is available, the performance of OFDM can be significantly improved by using adaptive modulation [1]. In adaptive modulation, different parameters including data rate, transmit power, instantaneous bit-error-rate (BER), constellation size and channel code or scheme can be adjusted according to the subchannel fading conditions.

Several bit/power loading algorithm can be found in the literature [2-15]. Most of these algorithms can be classified, based on their objective function, into two

categories: margin-adaptive (MA) algorithms (e.g. [7-9]) that strive to minimize the transmitted power subject to data rate and BER constraints, and rate-adaptive (RA) algorithms (e.g. [10-14]) that strive to maximize the data rate subject to power and BER constraints. In addition, some algorithms that have different objectives can also be found in the literature. For example, the algorithm proposed by Goldfeld *et al.* [12] is aimed at minimizing the probability of error. In all these algorithms, there is generally a tradeoff between the algorithm performance and the computational complexity. Optimum bit/power loading can be achieved by the well-known water-filling approach. Some loading algorithms can also achieve near-optimal solutions using incremental allocation (e.g. [11-14]). However, the cost in terms of computational complexity associated with both approaches is excessive. To reduce the complexity of bit allocation, closed-form expressions for BER or channel capacity approximations can be exploited. This method, however, requires rounding of the constellation size to integer numbers, hence deviates the allocation from optimality. Wyglinski *et al.* [11] proposed an optimum bit-loading with reduced complexity. However, the method is still rather computationally complex especially when the number of subcarriers is large, because it includes an extra iterative algorithm to find the initial peak BER in addition to the main algorithm.

In this paper, we attempt to solve the rate maximization problem with target overall BER constraint and uni-

form power allocation. A simple discrete bit-loading algorithm that approaches the maximum throughput with minimal complexity is presented. The algorithm performance is verified in a widely-accepted power line channel model [16].

## 2. Adaptive Bit Loading

### 2.1. Problem Formulation

The proposed loading algorithm aims to solve the following rate-adaptive problem given a target mean BER  $P_T$  and a fixed energy distribution across all the subcarriers:

$$\text{Maximize } \sum_{i=1}^N b_i \quad \text{subject to } \bar{P} = \frac{\sum_{i=1}^N b_i P_i}{\sum_{i=1}^N b_i} \leq P_T \quad (1)$$

where  $b_i$  and  $P_i$  are the number of bits and BER of the  $i$ th subcarrier respectively.  $N$  and  $\bar{P}$  are the number of used subcarriers and their mean BER respectively. As in other studies, it is assumed that perfect knowledge of the channel gains is available to both the transmitter and the receiver.

Different loading algorithms trying to solve this problem have been discussed in the previous section. These algorithms, however, are either too complex or do not achieve maximum throughput.

### 2.2. Proposed Loading Algorithm

The BER of square MQAM with Gray bit mapping can be approximated [1]:

$$P_i = 0.2 \exp\left\{-\frac{1.6\gamma_i}{2^{b_i} - 1}\right\}, \quad i = 1, 2, \dots, N \quad (2)$$

where  $\gamma_i = \frac{P_i |H_i|^2}{\sigma_i^2}$  is the  $i$ th subchannel SNR, with  $P_i$ ,

$H_i$  and  $\sigma_i^2$  being the signal power, channel gain and noise power of the  $i$ th subcarrier respectively. Accordingly, the number of bits that can be carried in subchannel  $i$  is given by:

$$b_i = \log_2 \left\{ 1 + \frac{\gamma_i}{\Gamma_i} \right\}, \quad i = 1, 2, \dots, N \quad (3)$$

where  $\Gamma_i$  is the SNR gap representing how far the system is from achieving capacity and can be defined from (2) and (3) as:

$$\Gamma_i = -\frac{\ln(5 \cdot P_i)}{1.6}, \quad i = 1, 2, \dots, N \quad (4)$$

The average number of bits per subcarrier in one

OFDM symbol can then be written as:

$$\begin{aligned} \bar{b} &= \left(\frac{1}{N}\right) \sum_{i=1}^N \log_2 \left\{ 1 - \frac{1.6\gamma_i}{\ln(5 \cdot P_i)} \right\} \\ &= \left(\frac{1}{N}\right) \sum_{i=1}^N \log_2 \left\{ \prod_{i=1}^N \left( 1 - \frac{1.6\gamma_i}{\ln(5 \cdot P_i)} \right) \right\} \\ &= \log_2 \left\{ 1 - \frac{1.6\gamma_{mc}}{\ln(5 \cdot \bar{P})} \right\} \end{aligned} \quad (5)$$

where  $\gamma_{mc}$  is the multichannel SNR which characterizes the set of  $N$  subchannels by an equivalent single AWGN that achieves the same data rate with the same error probability [17]. **Figure 1** illustrates the concept of the multichannel SNR. From (5), the multichannel SNR can be defined by:

$$\gamma_{mc} = \frac{\ln(5 \cdot \bar{P})}{1.6} \left\{ 1 - \left[ \prod_{i=1}^N \left( 1 - \frac{1.6\gamma_i}{\ln(5 \cdot P_i)} \right) \right]^{(1/N)} \right\} \quad (6)$$

The proposed algorithm initially computes for each subcarrier the maximum number of bits  $b_i$  that gives a value of  $P_i$  below  $P_T$ . The resulting overall BER  $\bar{P}$  will generally be below  $P_T$  by a large margin. To exploit this margin, the algorithm then computes the number of extra bits that can be added to the OFDM symbol without violating the BER constraint  $P_T$ . The extra bits are added to the subcarriers that will have the minimum effect in the overall BER. This is done by evaluating  $\Delta P$  for each subcarrier where

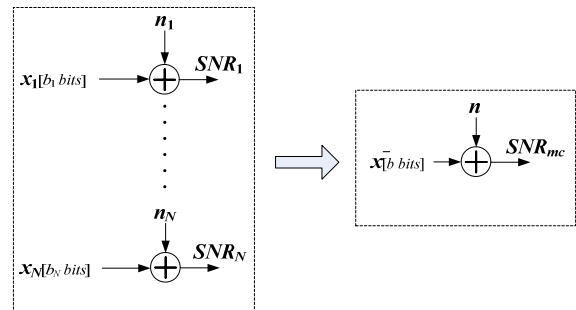
$$\Delta P_i = b_i (P_i^+ - P_i) \quad (7)$$

and  $P_i^+$  is the BER of the  $i$ th subchannel when the constellation size is shifted to the immediately higher constellation.

The proposed algorithm takes the following steps:

1) Given SNR values  $\gamma_i$ , find the largest signal constellation  $b_i$  for each subcarrier for which  $P_i$  is below  $P_T$ .

2) Calculate the current values of  $\bar{b}$  and  $\bar{P}$  and com-



**Figure 1.** The concept of the multichannel SNR [17].

pute the multichannel SNR  $\gamma_{mc}$  using (6).

3) Use  $\gamma_{mc}$  to find the maximum average number of bits  $\bar{b}_{max}$  per subcarrier that satisfies  $P_T$  :

$$\bar{b}_{max} = \log_2 \left\{ 1 - \frac{1 \cdot 6 \cdot \gamma_{mc}}{\ln(5 \cdot P_T)} \right\} \quad (8)$$

4) Find the number of extra bits  $I = N \cdot (\bar{b}_{max} - \bar{b})$  that can be added to the OFDM symbol.

5) Calculate  $\Delta P_i$  for all subchannels that have  $b_i$  below the maximum constellation size. Sort subchannels according to their  $\Delta P_i$  in increasing order.

Add  $I$  extra bits to the subchannels that have the lowest  $\Delta P_i$  by shifting their constellation to the immediately higher size.

### 3. Simulation Results

#### 3.1. Channel Model

Power line networks differ significantly in topology, structure, and physical properties from conventional communication channels such as twisted pair, coaxial, or fiber-optic cables [16]. Because they were not specifically designed for data transmission, power lines provide a harsh environment for higher frequency communication signals. Zimmermann and Dostert [16] proposed a practical channel model that is suitable for describing the transmission behavior of power line channels. The model is based on practical measurements of actual power line networks and is given by the channel transfer function:

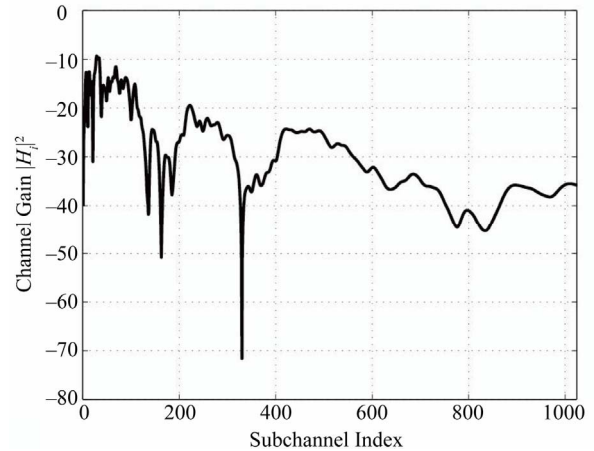
$$H(f) = \sum_{i=1}^{N_p} \underbrace{c_i}_{\text{weighting factor}} \cdot \underbrace{e^{-(a_0+a_1 f^k) d_i}}_{\text{attenuation portion}} \cdot \underbrace{e^{-j2\pi f(d_i/v_p)}}_{\text{delay portion}} \quad (9)$$

where  $N_p$  is the number of multipaths,  $c_i$  and  $d_i$  are the weighting factor and length of the  $i$ th path respectively. Frequency-dependant attenuation is modeled by the parameters  $a_0$ ,  $a_1$  and  $k$ .

In the model, the first exponential presents attenuation in the PLC channel, whereas the second exponential, with the propagation speed  $v_p$ , describes the echo scenario. This PLC multipath channel model is used here where parameters of the 15-path channel are given in [16]. Based on this power line channel model, the sub-channel gain versus the subchannel index is illustrated in **Figure 2** for 1024 subcarriers.

#### 3.2. Results

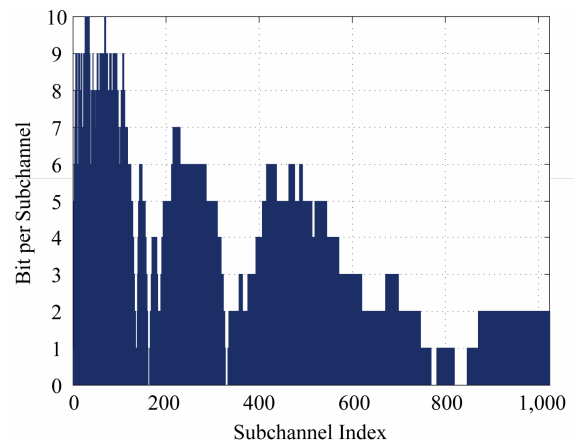
The performance of the proposed bit-loading algorithm is evaluated by computer simulations against incremental and equal-BER loading algorithms based on the achieved



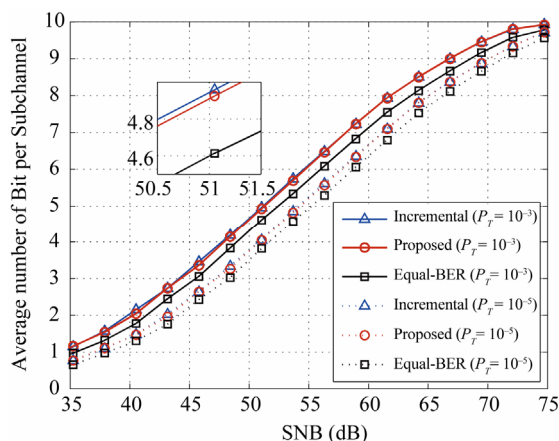
**Figure 2.** Channel gain for the 15-path power line channel.

average number of bits per subchannel. In incremental loading, all subcarriers are initially allocated the maximum signal constellation and then bits are incrementally removed from the subcarrier with the worst BER until the overall BER satisfies  $P_T$ . In equal-BER allocation, a constant BER threshold (*i.e.*,  $P_T$ ) is set for all subcarriers and each subcarrier is allocated the maximum number of bits for which its  $P_i$  is below  $P_T$ . In all the algorithms the system employs OFDM with 1024 subcarriers in the frequency band 1.8 - 30 MHz and the approximation (2) is used. Each subchannel can be assigned to carry a maximum number of 10 bits.

**Figure 3** shows the number of bits allocated to each subcarrier using the proposed algorithm when the target BER constraint  $P_T$  is equal to  $10^{-5}$ . The performance of the proposed algorithm, measured in average number of bits per subchannel, is depicted in **Figure 4** and compared to incremental and equal-BER loading algorithms for two different values of target BER ( $P_T = 10^{-3}, 10^{-5}$ ). The figure shows that the proposed algorithm and incremental loading have similar performance, whereas the



**Figure 3.** Bit allocation based on the proposed algorithm.



**Figure 4.** Performance of the proposed loading algorithm as compared to incremental and default loading methods in the 15-path power line channel.

**Table 1.** Mean Computation times (ms) for different values of SNR (CPU: Intel(R) Core(TM)2 Duo 3 GHz ).

Algorithm	SNR Values			
	40 dB	50 dB	60 dB	70 dB
Incremental	2.80	2.90	3.30	3.10
Equal-BER	0.30	0.33	0.38	0.40
Proposed	0.60	0.65	0.74	0.76

equal-BER loading achieves lower rates. To compare the computational complexity of these algorithms, the computation time (in milli-seconds) taken by each algorithm to reach the final allocation is measured. To insure fairness, each of those algorithms was implemented using Matlab and executed 100 times in the same workstation.

**Table 1** illustrates the mean computation times for a 1024-subcarrier OFDM system with  $P_T$  of  $10^{-5}$  for different values of SNR. It should be noted that the proposed algorithm is non-iterative, which results in a significant reduction in algorithm computational complexity as compared to the iterative incremental loading. Although the computation time of the proposed algorithm is less than a quarter of that of the incremental loading, both of these algorithms have indistinguishable performances in terms of average number of bits per subchannel. The proposed method takes about twice the time taken by the equal-BER to reach the final allocation. However, it achieves a considerable improvement of more than 300 bits per OFDM symbol over the equal-BER loading method.

## 4. Conclusions

This paper presents a simple non-iterative discrete bit-loading algorithm striving to maximize the data rate subject to target BER constraint and uniform power dis-

tribution. The algorithm was tested in a practically-proven power line communication channel model using computer simulations. Results show that the proposed algorithm improves the data rates achieved by the equal-BER loading with a small cost in complexity. When compared to the incremental loading, the proposed algorithm achieves similar rates, but with much lower computational complexity.

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