

# PCA Application in Channel Estimation in MIMO-OFDM System

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## Abstract

Initial estimation is a considerable issue in channel estimation techniques, since all of the following processes depends on it, which in this paper its improvement is discussed. Least Square (LS) method is a common simple way to estimate a channel initially but its efficiency is not as significant as more complex approaches. It is possible to enhance channel estimation performance by using some methods such as principal component analysis (PCA), which is not prevalent in channel estimation, and its adaptation to channel information can be challenging. PCA method improves initial estimation performance by projecting data onto direction of eigenvectors by means of using simple algebra. In this paper, channel estimation is examined in Multiple Input Multiple Output-Orthogonal Frequency Division Multiplexing (MIMO-OFDM) system, with significant advantages such as an acceptable performance in frequency selective fading channel. Moreover the proposed channel estimation method manipulates the benefits of MIMO channel by using the information, gained by all channels to estimate the information of each receiver.

**Keywords:** Channel Estimation, MIMO-OFDM, PCA, Pilot Aided Technique

## 1. Introduction

Multiple Input Multiple Output system, uses multiple antennas at both receiver and transmitter, to increase channel capacity [1] and link reliability. Also OFDM technique is appropriate for frequency selective environments due to its channel conversion to parallel flat fading subchannels [2] simplifying receiver structure, additionally it leads to high spectral efficiency. The combination of OFDM and MIMO system takes benefits from both techniques.

Channel information is needed at receiver for signal detection and its accuracy affects on overall system performance. Different channel estimation methods have been used to gain channel information, one is the blind method [3], which uses statistical information of channel. The other one is the training based channel estimation [4], using known training data. The latter method is chosen as it is applicable in fast time varying channels and mobile wireless systems. In this approach, pilots are inserted

among the data at the transmitter, extracted at receiver to estimate channel and to compensate channel fading. Applying an interpolator is necessary, while pilots are sent through some subcarriers. Interpolator has different types [5] but for an initial channel estimation, a linear one, can be sufficient.

In this paper the initial data aided channel estimation method, LS [6] is applied, as it is simple and applicable. In the next step, PCA, a linear mapping method [7], is used to improve channel estimation, by using the eigenvectors with the largest eigenvalues and its objective is to minimize projection error. Due to the importance of some of its properties like width and length, PCA can be a good choice in this research to improve channel estimation.

In this paper the initial channel estimation technique, proposed in [8] is extended, but at first the method, used in [8], is described briefly in Section 3.1, and its improvement by PCA method is explained in 3.2, finally results are shown in Section 4, followed by conclusion.

## 2. MIMO-OFDM System Description

A  $2 \times 2$  MIMO-OFDM system is shown in **Figure 1**. In the OFDM system, first binary data is mapped to symbols by modulation and pilots are inserted among subcarriers, in equal space according to sampling theory [9], which are used for channel estimation.

To transform data from frequency domain into time domain, Inverse Fast Fourier Transform (IFFT) block is employed

Elimination of Inter Symbol Interference (ISI) [10] is done by inserting a guard interval, in front of transmitted symbol, which is the copy of the last  $M_g$  samples of each symbol. Then the signal passes through the frequency selective fading channels, in 4 paths, and white gaussian noise is added to the signal.

At receiver all of the procedures are done reversely; guard interval is removed and signal is sent to FFT block. But before demodulation, channel is estimated by extracting pilot tones and applied to received data to extract output data, also in each antenna the 2 received data stream is averaged before sending to demodulation block.

## 3. Channel Estimation

### 3.1. Comb Type Pilot Based Initial Channel Estimation

Two transmitter antennas send data streams, so that  $K$  subcarriers are assigned as pilot tones in  $M$  subcarriers which are known at receiver. They are inserted in each symbol, in a distance of  $M_s < M/M_g$ , emerged of sampling theory. While,  $M_s$  denotes the distance between subcarriers. The received signal can be expressed as  $Y = XH + W$ , where  $X$  and  $W$  show transmitted signal and additive white gaussian noise respectively. Complex channel response, including  $L$  coefficients, is shown by  $H = [h_1, \dots, h_L]$ .

At receiver, each antenna receives two data streams, in the other word four vectors of received data is available. Channel estimation is performed by LS method to train sequences, by extracting pilots at receiver which is shown by  $Y^P$  and pre-known transmitted pilots as  $X^P$ :

$$H_{LS}(m) = \left( (X^P(m))^H X^P(m) \right)^{-1} (X^P(m))^H Y^P(m) \tag{1}$$

In comb type pilot arrangement [5], interpolating is necessary to extend the estimation to other subcarriers. There are different kinds of interpolators but linear is adequate one in this research. The channel in  $(k \times M/K + m)$ th subcarrier,  $0 < m < M/K$ ,  $0 < k < K$ , obtaining linear interpolator, is expressed as:

$$\hat{H} \left( k \times \frac{M}{K} + m \right) = \hat{H}(p(k)) + m \times \frac{\hat{H}(p(k+1)) - \hat{H}(p(k))}{M/K} \tag{2}$$

Channel estimation is applied in adjacent pilot subcarriers to evaluate channel in subcarriers between them. As the system is  $2 \times 2$ , 4 channel estimation vectors will be gained.

### 3.2. Improved Channel Estimation, Applying PCA Method

PCA is a non-parametric method to re-express data, using basis vectors, applying simple linear algebra. PCA provides a way to reduce complex data to a lower dimension. Selecting Orthogonal directions for principal components are solutions to predict original data.

PCA is widely applied in pattern recognition, image processing, moreover it is a proper classifier, but it isn't common in channel estimation. Here, the proposed approach of using this method in channel estimation is described. Generally PCA can be summarized in 3 steps.

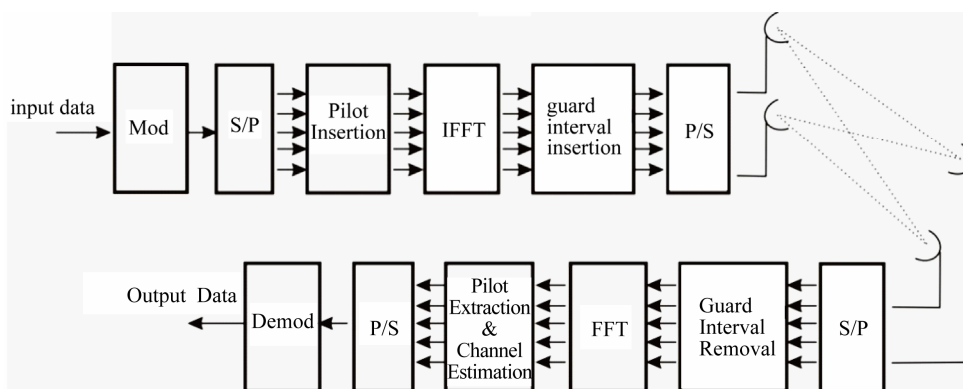


Figure 1. Schematic of MIMO-OFDM system.

First Step: Data is arranged in a  $M \times N$  matrix.

This paper is aiming to improve the estimated data which is channel information. The mentioned matrix can be built by them. Therefore a  $M \times 4$  matrix is made of 4,  $M \times 1$  channel vector and each column of the channel matrix is considered as a dimension:

$$\hat{h} = [\hat{h}_1, \hat{h}_2, \dots, \hat{h}_M] \quad (3)$$

Second Step: Normalizing data by subtracting the mean of data.

At first the mean is calculated and normalization is implemented by following equations:

$$\begin{aligned} \mu &= \frac{1}{M} \sum_{i=1}^M h_i \\ \hat{h} &= \hat{h} - \mu \end{aligned} \quad (4)$$

After the subtracting mean which is the average across each dimension, the data set will have the mean of zero.

Third Step: The most important step is calculating the Singular Value Decomposition (SVD) or eigenvectors of the covariance, which shows the distribution of data. Covariance matrix is obtained by (5):

$$C = \hat{h}\hat{h}^H \quad (5)$$

Since the data is 4 dimensional, a  $4 \times 4$  covariance matrix is calculated. By determining eigenvalues and eigenvectors of square covariance matrix, useful information about data is gained.

SVD is a way to analyze data and demonstrates covariance, using eigenvectors  $[e_1, e_2, \dots, e_N]$  and scalar eigenvalues  $[\lambda_1, \lambda_2, \dots, \lambda_N]$ . Also the vectors of  $[he_1, he_2, \dots, he_N]$ , form an orthogonal basis, so that the vector  $he_k$  has the length of  $\sqrt{\lambda_k}$ .

$$\begin{aligned} Ce &= \lambda e \\ \frac{1}{N} \left( \sum_{i=1}^N \hat{h}_i \hat{h}_i^H \right) e &= \lambda e \end{aligned} \quad (6)$$

Leads to:

$$\lambda = \frac{1}{N} \sum_{i=1}^N (e^H \hat{h}_i)^2 \quad (7)$$

Finally the channel estimation transforming by a projection operation is expressed by (8):

$$\hat{h}_p = (\hat{h} - \mu)e \quad (8)$$

The eigenvectors with the highest eigenvalues is the principle component of data set. Therefore the eigenvectors are ordered by eigenvalues, highest to lowest, which highest vector will be used for channel projection. Additionally the eigenvector with lower standard deviation is more appropriate for channel projection.

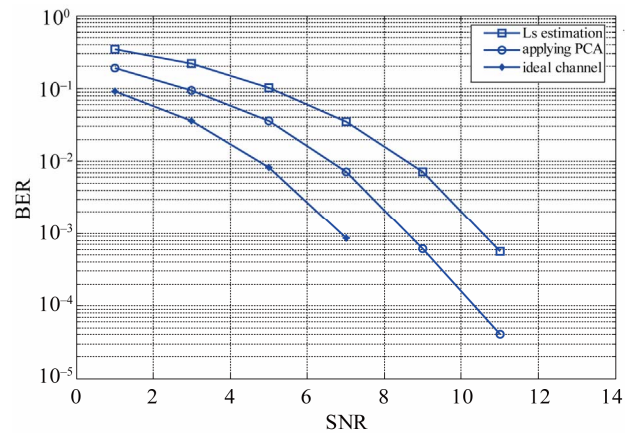
## 4. Results

Parameters of MIMO-OFDM system, used in simulation, are summarized in the **Table 1** [8]. A  $2 \times 2$  system, including 2 transmitter antennas and 2 receiver antennas is simulated in MATLAB software. Guard interval is chosen greater than delay spread to eliminate ISI. There are 4 multipath fading channels using jakes spectrum type. Specific channel parameters such as delay spread, Doppler frequency, and tap power, are extracted from Stanford University Interim channel model [11], which were measured for fixed broadband wireless applications.

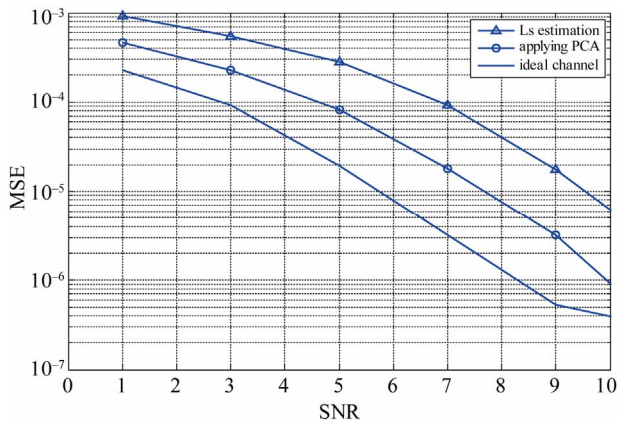
Transferred pilots, inserted among data based on comb type, are used to estimate channel. As previously described in section 3, pilots are extracted and channel is estimated, using LS, then linear interpolation is applied to indicate channel in all subcarriers. Finally PCA is used to project data onto direction of eigenvectors, which minimizes the projection error and keeps some properties such as width and length, as they are significantly important in channel estimation. The channel estimation improvement, using PCA is shown in **Figure 2**, in Bit Error Rate (BER) performance and **Figure 3** in Mean square Error (MSE), compared with ideal channel, while fading effects were ignored, but the impact of white noise was considered. These figures show the channel

**Table 1. channel parameter, used in simulation [8].**

parameter	specifications
Number of transmitter	2
Number of receiver	2
FFT size	1024
Guard interval	256
Pilot Interval	5
Bandwidth	1.75 MHz
Data modulation	QPSK
Pilots modulation	BPSK
Channel type	Rayleigh fading, SUI model



**Figure 2. BER performance of channel estimation comparing with ideal channel.**



**Figure 3.** MSE performance of channel estimation comparing with ideal channel.

estimation improvement, so that the system works in lower Signal to Noise Ratio (SNR) by using advantages of PCA method. Therefore it is unnecessary to use complex algorithms to enhance estimation quality. Additionally, the benefits of MIMO systems are manipulated by contributing all channels in estimation.

## 5. Conclusions

Acquiring accurate information at receiver depends on channel estimation quality. Therefore in this paper, channel estimation improvement was considered in a  $2 \times 2$  MIMO-OFDM system. This kind of system gets advantages of both MIMO and OFDM techniques. It has high channel capacity and also ISI and Inter-Channel Interference (ICI) are removed due to converting frequency selective fading channel to flat fading subchannels. In channel estimation section, channel was estimated initially by LS method, using training sequences which were sent in some subcarriers with equal distances emerge of sampling theory. After evaluating channel in pilot subcarriers, linear interpolator was used to estimate channel in all subcarriers. In the final step, to improve channel estimation, PCA method was chosen to project data onto directions of eigenvalues and reduced complex data to lower dimension. Simulation results show im-

provement of channel estimation in BER and MSE performance.

## 6. References

- [1] E. Telatar, "Capacity of Multi-Antenna Gaussian Channels," *European Transactions on Telecommunication*, Vol. 10, No. 6, 1999, pp. 585-595. [doi:10.1002/ett.4460100604](https://doi.org/10.1002/ett.4460100604)
- [2] J. G. Proakis, "Digital Communications," 4th Edition, McGraw-Hill, New York, 2001, pp. 709-800.
- [3] S. Roy and C. Y. Li, "A Subspace Blind Channel Estimation Method for OFDM Systems without Cyclic Prefix," *IEEE Transactions on Wireless Communications*, Vol. 1, No. 4, 2002, pp. 572-579. [doi:10.1109/TWC.2002.804160](https://doi.org/10.1109/TWC.2002.804160)
- [4] Y. Li, "Simplified Channel Estimation for OFDM Systems with Multiple Transmit Antennas," *IEEE Transactions on Wireless Communications*, Vol. 1, No. 1, 2002, pp. 67-75. [doi:10.1109/7693.975446](https://doi.org/10.1109/7693.975446)
- [5] M.-H. Hsieh and C.-H. Wei, "Channel Estimation for OFDM Systems Based on Comb-Type Pilot Arrangement in Frequency Selective Fading Channels," *IEEE Transactions on Consumer Electronics*, Vol. 44, No. 1, 1998, pp. 217-225. [doi:10.1109/30.663750](https://doi.org/10.1109/30.663750)
- [6] H. Minn and N. Al-Dhahir, "Optimal Training Signals for MIMO OFDM Channel Estimation," *IEEE Transactions on Wireless Communications*, Vol. 5, No. 5, 2006, pp. 1158-1168. [doi:10.1109/TWC.2006.1633369](https://doi.org/10.1109/TWC.2006.1633369)
- [7] S. Haykin, "Neural Networks," Prentice-Hall, Upper Saddle River, 1994, pp. 363-370.
- [8] M. Nasser and H. Bakhshi, "Iterative Channel Estimation Algorithm in Multiple Input Multiple Output Orthogonal Frequency Division Multiplexing Systems," *Journal of Computer Science*, Vol. 6, No. 2, 2010, pp. 224-228. [doi:10.3844/jcssp.2010.224.228](https://doi.org/10.3844/jcssp.2010.224.228)
- [9] A. Oppenheim and R. Schaffer, "Discrete Time Signal," 2nd Edition, Prentice-Hall, Upper Saddle River, 1999, p. 146.
- [10] R. V. Nee and R. Prasad, "OFDM for Wireless Multimedia Communications," Artech House, London, 2000, pp. 33-50.
- [11] V. Erceg, K. V. S. Hari, M. S. Smith and D. S. Baum, "Channel Models for Fixed Wireless Applications," IEEE 802.16.3 Task Group Contributions, 2001.