

Consensus of a kind of Dynamical Agents in Network with Time Delays

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Abstract

This paper investigates the collective behavior of a class of dynamic agents with time delay in transmission networks. It is assumed that the agents are Lyapunov stable distributed on a plane and their location coordinates are measured by some remote sensors with certain error and transmitted to their neighbors. The control protocol is designed on the transmitted information by a linear decentralized law. The coordination of dynamical agents is shown under the condition that the error is small enough. Numerical simulations demonstrate that our theoretical results are valid.

Keywords: Multi-Agent System, Time Delay, Consensus Protocol

1. Introduction

Distributed coordination of network of dynamic agents has attracted a great attention in recent years. Modeling and exploring these coordinated dynamic agents have become an important issue in physics, biophysics, systems biology, applied mathematics, mechanics, computer science and control theory [1-10]. How and when coordinated dynamic agents achieve aggregation is one of the interesting topics in the research area. Such problem may also be described as a consensus control problem.

To describe the collective behavior of agents in a large scale network, the agent in the network usually is modeled by a very simple mathematical model, which is an approximation of real objects. Saber and Murray [3,4] proposed a systematical framework of consensus problems in networks of dynamic agents. In their work the dynamics of the agent is modelled by a simple scalar continuous-time integrator $\dot{x} = u$, the convergence analysis is provided in different types of the network topologies. Following the work of [2,3], G. Xie and L. Wang [8] study the case where the dynamics of each agent is second order. In their work, they show that by means of a simple linear control protocol based on the structure of the graph, the dynamical agents will eventually achieve aggregation, *i.e.*, all agents will gradually move into a fixed position of the line, meanwhile their velocities converge to zero.

In networks of the dynamic agents, time delays and sensor error may arise naturally, *e.g.*, because of the moving of the agents, the congestion of the communication channels and the finite transmission speed due to the physical characteristics of the medium transmitting the information. The different protocols with time delays have been investigated [2,4]. And the sensor error or communicated errors are also to be considered in the consensus protocol [9]. It is shown that the collective behavior of dynamical agent will depend on the communicated error and the algebraic characterization of the communicated network topology.

In this paper, we study the consensus of multiagent systems where the dynamics of each agent is second order. The agents may represent the vehicles or mobile robots spread over a wild area and they communicate by means of some remote sensors with certain error and time delay. When the agents are moving in a plane, the consensus conditions will depend on the time delay, communicated error and the algebraic characterization of network topology, as well as the dynamical behavior of agents.

This paper is organized as follows. In Section 2, we recall some properties on graph theory and give the problem formulation. In Section 3 the main results of this paper are given and some simulation results are presented in Section 4. Final section is a conclusion.

2. Preliminaries

By $G = (V, E, A)$, we denote an undirected graph with an weighted adjacency matrix $A = [a_{ij}]$, where $V = \{p_1, \dots, p_M\}$ is the set of nodes, $E \subseteq V \times V$ is the set of edges. The node indexes belong to a finite index set $\underline{M} \triangleq \{1, \dots, M\}$. An edge of G is denoted by $e_{ij} = (p_j, p_i)$. The adjacency elements a_{ij} are defined in following way: $e_{ij} \in E \Leftrightarrow a_{ij} > 0$. Moreover, we assume $a_{ii} = 0$ for all $i \in \underline{M}$. The set of neighbors of node p_i is denoted by $N_i = \{p_j \in V \mid (p_i, p_j) \in E\}$.

A diagonal matrix $D = \text{diag}\{d_1, \dots, d_M\}$ is a degree matrix of G with $d_i = \sum_{j=1}^M a_{ij}$. Then the Laplacian of the weighted graph G is defined as $L = D - A$. A graph is called connected if there exists a path between any two distinct vertices of the graph.

An important fact of L is that all the row sums are zero, therefore, $1_M = (1, \dots, 1)^T$ is an eigenvector of L associated with zero eigenvalue. Moreover, the graph G is connected if and only if its Laplacian L is satisfied $\text{rank}(L) = M - 1$ and all eigenvalues of L are of positive real numbers except that only one eigenvalue is zero [2].

Consider a network of dynamical agents defined by a graph $G = (V, E, A)$. The node set V consists of dynamical agents $P_i, i \in \underline{M}$. Let $x_i = (x_{i1}, x_{i2})^T \in \mathbb{R}^2$ be the coordinate of dynamical agent P_i , then the dynamics of p_i are identical and described as follows.

$$\begin{aligned} \dot{x}_i &= v_i \\ m_i \dot{v}_i &= K v_i + u_i \\ y_i &= F \begin{pmatrix} x_i \\ v_i \end{pmatrix} \end{aligned} \tag{1}$$

where x_i is indicates the location vector of agent p_i in the plane, $v_i = (v_{i1}, v_{i2})^T$ represents its velocity vector of the i th agent, m_i is its mass and $K = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$ is a dynamical feedback matrix of the agent. F is an observation matrix of the agent by some remote sensor. In what follows we simply assume that $m_i = 1$ and $p_i = x_i$. Let $F = [C \ 0]$ which means that the location information of the i th agent is only measured by some remote sensor and is transmitted to its neighbors through the network. The matrix C is assumed to be the form $C = \begin{bmatrix} 1 & \sigma_i \\ -\sigma_i & 1 \end{bmatrix}$. The parameter σ_i indicates that the

network communicated error or the coordinates used for sensor could be different from that of the agents.

For the dynamic agent (1) in network we have following assumption.

Assumption 1. The dynamics (1) is Lyapunov stable when it disconnected with its neighbors, meaning that the dynamical agent as an autonomous will gradually stop by moving a finite distance for any non-zero initial velocity.

We shall give the conditions, under which the network of dynamical agents (1) achieve asymptotical consensus meaning that there exists a fixed position (equilibrium) $x^* \in \mathbb{R}^2$ such that for $i \in \underline{M}$

$$\begin{aligned} \lim_{t \rightarrow \infty} x_i(t) &= x^* \\ \lim_{t \rightarrow \infty} v_i(t) &= 0_{2 \times 1} \end{aligned} \tag{2}$$

Due to time-delay in communicated network, the control protocol of the dynamical agent p_i is a neighbor-based linear control law in the form that

$$u_i = \sum_{p_j \in N_i} a_{ij} (y_j(t - \tau_{ij}(t)) - y_i(t - \tau_{ij}(t))) \tag{3}$$

where N_i is the set of neighbors of agent p_i and a_{ij} are adjacency elements of A . The $\tau_{ij}(t) \geq 0$, denoting the communication transmission time-delay from agent p_j to agent p_i . In this paper, we discuss the consensus of dynamical agent under the condition of the communicated error and the time delay. We focus on the simplest possible case where the time-delays in all channels are equal to τ and the communicated errors are equal to σ .

Remark 1. If we choose $\sigma = 0$ and $k_{11} = k_{22} = k$, $k_{12} = k_{21} = 0$, then the two-dimension agent systems (1) with the control protocol (3) can be decoupled into two identical linear systems and it was discussed by some literatures with or without time delays(such as [8,9]).

3. Consensus of Dynamic Agents

We denote the initial locations and the initial velocities of the agents by

$$x(0) = (x_1^T(0) \dots x_M^T(0))^T, \quad v(0) = (v_1^T(0) \dots v_M^T(0))^T$$

respectively. Under control protocol (3) with $\tau_{ij}(t) = \tau$, the dynamical equation of agent p_i is written by

$$\dot{\xi}_i(t) = A \xi_i(t) + B \sum_{p_j \in N_i} a_{ij} (\xi_j(t - \tau) - \xi_i(t - \tau))$$

where $\xi_i(t) = (x_i^T(t), v_i^T(t))^T, i \in \underline{M}$

$$A = \begin{bmatrix} 0_{2 \times 2} & I_{2 \times 2} \\ 0_{2 \times 2} & K \end{bmatrix}, \quad B = \begin{bmatrix} 0_{2 \times 2} & 0_{2 \times 2} \\ I_{2 \times 2} & 0_{2 \times 2} \end{bmatrix}. \tag{4}$$

Furthermore, let $\xi(t) = (\xi_1^T(t), \dots, \xi_M^T(t))^T$, then the dynamic network is of the following form

$$\dot{\xi}(t) = Y_1 \xi(t) - Y_2 \xi(t - \tau) \tag{5}$$

where

$$Y_1 = I_M \otimes A, \quad Y_2 = L \otimes B \tag{6}$$

And L is the Laplacian associated with the connected graph G . Because (5) is a standard linear time-delay dynamical systems, its stability analysis is equivalent to analyzing the eigenvalues and eigenvectors of matrix $\Upsilon = \Upsilon_1 - e^\tau \Upsilon_2$.

Lemma 1. The matrix $\Upsilon = \Upsilon_1 - e^\tau \Upsilon_2$ has two eigenvectors associated with zero eigenvalue. Let α , β be the left and right eigenvectors (denoted by matrices) of matrix Υ associated with zero eigenvalue, respectively. Then

$$\alpha^T = \frac{1}{\sqrt{M}} 1_M \otimes \begin{bmatrix} K^T \\ -I_{2 \times 2} \end{bmatrix}, \quad \beta = \frac{1}{\sqrt{M}} 1_M \otimes \begin{bmatrix} K^{-1} \\ 0_{2 \times 2} \end{bmatrix} \quad (7)$$

and $\alpha\beta = I_{2 \times 2}$, where $1_M = (1, \dots, 1)^T \in R^M$.

Proof It is well known that (refer to [2,11]) the graph $G = (V, E, A)$ is connected if and only if its Laplacian satisfies that $\text{rank}(L) = M - 1$. Moreover, $1_M = (1, \dots, 1)^T$ is an eigenvector of L associated with zero eigenvalue. Then, there is only one zero eigenvalue of L , all the other ones are positive and real. By the definition of (6) and $\Upsilon = \Upsilon_1 - e^\tau \Upsilon_2$, one has

$$\begin{aligned} \alpha \cdot \Upsilon &= \left(\frac{1}{\sqrt{M}} 1_M^T \otimes [K \quad -I_{2 \times 2}] \right) \cdot (\Upsilon_1 - e^\tau \Upsilon_2) \\ &= \left(\frac{1}{\sqrt{M}} 1_M^T \otimes [K \quad -I_{2 \times 2}] \begin{bmatrix} 0_{2 \times 2} & I_{2 \times 2} \\ 0_{2 \times 2} & K \end{bmatrix} \right) \\ &\quad - \left(\frac{1}{\sqrt{M}} 1_M^T e^\tau L \otimes -e^\tau L \otimes [K \quad -I_{2 \times 2}] \begin{bmatrix} 0_{2 \times 2} & 0_{2 \times 2} \\ I_{2 \times 2} & 0_{2 \times 2} \end{bmatrix} \right) \\ &= 0_{4M \times 2}^T \end{aligned}$$

Thus, $\alpha^T = \frac{1}{\sqrt{M}} 1_M \otimes \begin{bmatrix} K^T \\ -I_{2 \times 2} \end{bmatrix}$ represents the two left eigenvectors of Υ .

Similarly, it is easy to check $\Upsilon \cdot \beta = 0_{4M \times 2}$, which implies that $\beta = \frac{1}{\sqrt{M}} 1_M \otimes \begin{bmatrix} K^{-1} \\ 0_{2 \times 2} \end{bmatrix}$ represents two left-eigenvectors of Υ . And it holds that $\alpha \cdot \beta = I_{2 \times 2}$.

Theorem 1 If the dynamical feedback matrix K in dynamical agent (1) satisfies Assumption 1, $\sigma_i = \sigma$ in the C of (1) and the time delays $\tau_{ij}(t) = \tau$ in (3) satisfies

$$\sigma_{\min} < \sigma < \sigma_{\max} \quad (8)$$

with

$$\sigma_{\min} = \frac{abc - \sqrt{d}}{2e^\tau \lambda (a^2 + b^2)}, \quad \sigma_{\max} = \frac{abc + \sqrt{d}}{2e^\tau \lambda (a^2 + b^2)} \quad (9)$$

where $a = k_{21} - k_{12}$, $b = -k_{11} - k_{22}$, $c = k_{11}k_{22} - k_{12}k_{21}$, $d = (abc)^2 + 4\lambda e^\tau (a^2 + b^2)b^2c$ and $\lambda = \lambda_M$ denotes the biggest eigenvalue of matrix L , then all of the eigen-

values of $\Upsilon = \Upsilon_1 - e^\tau \Upsilon_2$ defined in (6) are of negative real parts except for only two zero eigenvalues.

Proof As the graph G is connected, one denotes the eigenvalues of L by $0 = \lambda_1 < \lambda_2 \leq \dots \leq \lambda_M = \lambda$. There exists an orthogonal matrix W such that

$$W^T L W = \text{diag} \{ \lambda_1, \lambda_2, \dots, \lambda_M \}.$$

One can verify the following formulae.

$$\begin{aligned} &(W^T \otimes I_{4 \times 4}) \Upsilon (W \otimes I_{4 \times 4}) \\ &= (W^T \otimes I_{4 \times 4}) (I_M \otimes A - e^\tau L \otimes B) (W \otimes I_{4 \times 4}) \\ &= \begin{bmatrix} A - \lambda_1 e^\tau B & 0_{4 \times 4} & \dots & 0_{4 \times 4} \\ 0_{4 \times 4} & A - \lambda_2 e^\tau B & \dots & 0_{4 \times 4} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{4 \times 4} & 0_{4 \times 4} & \dots & A - \lambda_M e^\tau B \end{bmatrix} \end{aligned}$$

Then the dynamical behavior of the network (1) is characterized by the eigenvalues of $A - \lambda_i e^\tau B$, $i \in \underline{M}$.

First we discuss the block with $\lambda_1 = 0$. By Assumption 1, one has

$$k_{11} + k_{22} < 0, \quad k_{12}k_{21} < k_{11}k_{22} \quad (10)$$

and $\text{rank}(A - \lambda_1 e^\tau B) = 2$, its four eigenvalues satisfy

$$s_1 = s_2 = 0, \quad \text{Re}(s_3) < 0, \quad \text{Re}(s_4) < 0$$

For $\lambda_i > 0$, one has $A - \lambda_i e^\tau B = \begin{bmatrix} 0_{2 \times 2} & I_2 \\ -\lambda_i e^\tau C & k \end{bmatrix}$. As

$\text{rank}(C) = 2$, then $\text{rank}(A - \lambda_i e^\tau B) = 4$. Therefore, $\Upsilon = \Upsilon_1 - e^\tau \Upsilon_2$ has only two zero eigenvalues. Consider the characteristic polynomial of $A - \lambda_i e^\tau B$ for $i \in \underline{M}$.

$$\begin{aligned} \pi_{A - \lambda_i B}(s) &= \det(sI - (A - \lambda_i e^\tau B)) \\ &= \begin{vmatrix} s & 0 & -1 & 0 \\ 0 & s & 0 & -1 \\ \lambda_i e^\tau & \delta \lambda_i e^\tau & s - k_{11} & -k_{12} \\ -\delta \lambda_i e^\tau & \lambda_i e^\tau & -k_{21} & s - k_{22} \end{vmatrix} \\ &= s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4 \end{aligned}$$

where

$$\begin{aligned} a_1 &= -k_{11} - k_{22}, \quad a_2 = k_{11}k_{22} - k_{12}k_{21} + 2\lambda_i e^\tau, \\ a_3 &= e^\tau \lambda_i [\delta(k_{21} - k_{12}) - (k_{11} + k_{22})], \\ a_4 &= e^{2\tau} \lambda_i^2 (1 + \delta^2) \end{aligned} \quad (11)$$

Construct the Routh array of $\pi_{A - \lambda_i B}(s)$

$$\begin{array}{c|ccc} s^4 & 1 & a_2 & a_4 \\ s^3 & a_1 & a_3 & 0 \\ s^2 & b_1 & b_2 & 0 \\ s^1 & c_1 & 0 & \\ s^0 & d & 1 & \end{array}$$

with $b_1 = \frac{a_1 a_2 - a_3}{a_1}$, $b_2 = d_1 = a_4 = e^{2\tau} \lambda_i^2 (1 - \delta^2)$,

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1} = \frac{a_1 a_2 a_3 - a_3^2 - a_1^2 a_4}{a_1 b_1}.$$

By the *Routh–Hurwith* criterion, for stability it is necessary that $a_1 > 0, b_1 > 0, c_1 > 0, d_1 > 0$. Therefore, the dynamical network is stable if and only if the following inequalities hold

$$\begin{cases} a_1 > 0 \\ a_4 > 0 \\ a_1 a_2 - a_3 > 0 \\ a_1 a_2 a_3 - a_3^2 - a_1^2 a_4 > 0 \end{cases} \quad (12)$$

By (10) and (11), it is obvious that the first and second inequalities in (12) hold, and one may easily verify that the third inequality in (12) holds only if the fourth inequalities holds. Then

$$\begin{aligned} & a_1 a_2 a_3 - a_3^2 - a_1^2 a_4 \\ &= -\{(k_{11} + k_{22}) \cdot (k_{11} k_{22} - k_{12} k_{21}) \cdot [\delta(k_{21} - k_{12}) \\ &\quad - (k_{11} + k_{22})]\} e^\tau \lambda_i + e^{2\tau} \lambda_i^2 \delta^2 [(k_{11} + k_{22})^2 + (k_{21} - k_{12})^2] \end{aligned}$$

So the fourth inequality in (12) can be rewritten as the following form.

$$b^2 c + abc \sigma_i - e^\tau \lambda_i (a^2 + b^2) \sigma_i^2 > 0 \quad (13)$$

where $a = k_{21} - k_{12}$, $b = -k_{11} - k_{22}$, $c = k_{11} k_{22} - k_{12} k_{21}$. From it, one may obtain

$$\frac{abc - \sqrt{d_i}}{2e^\tau \lambda_i (a^2 + b^2)} < \sigma_i < \frac{abc + \sqrt{d_i}}{2e^\tau \lambda_i (a^2 + b^2)}$$

where $d_i = (abc)^2 + 4\lambda_i e^\tau (a^2 + b^2) b^2 c$. Under the case, the matrix $A - \lambda_i e^\tau B$ is *Hurwith*. Then it is obvious that the matrices $A - \lambda_i e^\tau B$ are *Hurwith* for all $i = 2, \dots, M$ under the condition of (8) and (9). Therefore, all of the eigenvalues of the matrix $Y = Y_1 - e^\tau Y_2$ defined in (6) are of negative real parts except for only two zero eigenvalues.

Theorem 2 Under conditions of Theorem 1, the control protocol (3) globally and asymptotically achieves the collective behavior of the dynamic agents.

Proof Under conditions of Theorem 1, all of the eigenvalues of the matrix $Y = Y_1 - e^\tau Y_2$ defined in (6) are of negative real parts except for only two zero eigenvalues. By $\Psi := (\theta_1, \theta_2, \dots, \theta_{4M-1}, \theta_{4M}) \in R^{4M \times 4M}$ one denotes right-eigenvectors of Y associated with eigenvalues $\gamma_1, \gamma_2, \dots, \gamma_{2M}$ respectively.

It holds that $Y\Psi = \Psi\Lambda$, $\Lambda = \text{diag}\{0_{2 \times 2}, J_1, \dots, J_M\}$, J_1 denotes the Jordan form of two order associated with the eigenvalues γ_1 and γ_2 . J_i denotes the Jordan form of four order associated with the eigenvalues

$\gamma_{4i-3}, \gamma_{4i-2}, \gamma_{4i-1}$ and γ_{4i} for all $i = 2, \dots, M$.

Let $\Psi^{-1} = \tilde{\Psi} := (\tilde{\theta}_1^T, \tilde{\theta}_2^T, \dots, \tilde{\theta}_{4M-1}^T, \tilde{\theta}_{4M}^T)^T \in R^{4M \times 4M}$, where $\tilde{\theta}_i; i \in 4M$ are $4M$ row left-eigenvectors of Y correspondingly. Then it holds that $\tilde{\Psi} \cdot Y = \tilde{\Psi} \Lambda$.

As $0 > \text{Re}(\gamma_3) \geq \text{Re}(\gamma_4) \geq \dots \geq \text{Re}(\gamma_{4M-1}) \geq \text{Re}(\gamma_{4M})$,

one has $\lim_{t \rightarrow \infty} e^{Yt} = \begin{bmatrix} I_{2 \times 2} & 0_{2 \times (4M-2)} \\ 0_{(4M-2) \times 2} & 0_{(4M-2) \times (4M-2)} \end{bmatrix}$ and

$$\lim_{t \rightarrow \infty} \exp(Yt) = \Psi \begin{bmatrix} I_{2 \times 2} & 0_{2 \times (4M-2)} \\ 0_{(4M-2) \times 2} & 0_{(4M-2) \times (4M-2)} \end{bmatrix} \tilde{\Psi}$$

Let $\alpha = (\theta_1 \ \theta_2)$ and $\beta = \begin{pmatrix} \tilde{\theta}_1 \\ \tilde{\theta}_2 \end{pmatrix}$, one has $\lim_{t \rightarrow \infty} \exp(\gamma t)$

$= \beta \alpha$.

Due to the fact that $\Psi \cdot \Psi^{-1} = \Psi \cdot \tilde{\Psi} = I_{4M \times 4M}$, α and β satisfy the property $\alpha \beta = I_{2 \times 2}$.

$$\begin{aligned} \lim_{t \rightarrow \infty} \xi(t) &= \lim_{t \rightarrow \infty} \exp(Yt) \xi(0) = \beta \alpha \xi(0) \\ &= \frac{1}{M} \left(\mathbf{1}_M \otimes \begin{bmatrix} k^{-1} \\ 0_{2 \times 2} \end{bmatrix} \right) \cdot \left(\mathbf{1}_M^T \otimes [k \ -I_2] \right) \xi(0) \\ &= \frac{1}{M} \mathbf{1}_M \mathbf{1}_M^T \otimes \begin{bmatrix} I_2 & -k^{-1} \\ 0_{2 \times 2} & 0_{2 \times 2} \end{bmatrix} \cdot \xi(0) \end{aligned}$$

Therefore, $\lim_{t \rightarrow \infty} x_i(t) = \frac{1}{M} \left\{ \sum_{j=1}^M [x_j(0) - k^{-1} v_j(0)] \right\}$ and

it is obvious that

$$\lim_{t \rightarrow \infty} v_i(t) = 0, \quad i \in \{1, 2, \dots, M\}$$

This implies the protocol (3) globally asymptotically achieves the collective behavior of the dynamic agents.

Corollary 1 The dynamical feedback matrix K is chosen to be $k_{12} = k_{21} = k_1$ and $k_{11} = k_{22} = k_2$, then the control protocol (3) globally and asymptotically achieves the collective behavior of the dynamic agents if the time delays τ satisfy $\tau < \ln \frac{k_2 - k_1}{\lambda \delta^2}$.

Proof It is easy to verify according to the proof of Theorem 1 and Theorem 2. Under Assumption 1, one has $k_{12} k_{21} < k_{11} k_{22}$. Thus, one finds that $c > 0$ and it further implies that $\sigma_{\min} < 0$ and $\sigma_{\max} > 0$ in (9) for bounded time delay. Thus we have the following result.

Corollary 2 The dynamical agents achieve collective behavior if the network communicated error σ is sufficient small for the bounded time delays.

4. Simulations

Numerical simulations will be given to illustrate the theoretical results obtained in the previous section. Con-

sider five dynamic agents under network described in **Figure 1**.

We can obtain the Laplacian matrix L of the graph G of **Figure 1** and its eigenvalues are $\lambda_1 = 0$, $\lambda_2 = \lambda_3 = \frac{1}{2}(7 - \sqrt{13})$, $\lambda_4 = \lambda_5 = \frac{1}{2}(7 + \sqrt{13})$, $\lambda_6 = 4$.

We consider that the dynamic agent (1) in the network has $k = \begin{bmatrix} -1 & 0.1 \\ 0.2 & -1 \end{bmatrix}$, $\sigma = 0.1$ in the observation matrix C and the time delay $\tau = 0.1$ in (3). Thus, it is *Lyapunov* stable and satisfies Assumption 1. One can get $a = 0.1$, $b = 2$, $c = 0.98$, $\sqrt{\Delta} = 12.1324$ and the $\sigma = 0.1$ belongs to the range of parameters, *i.e.*,

$$\sigma_{\min} = -0.2125 < \sigma < 0.2764 = \delta_{\max}$$

When a control protocol (3) is applied into the agents in network, the collective behavior of dynamic agents takes place according to our result. **Figures 2-3** give simulation results of the collective behavior of the agents while their velocities.

5. Conclusion

In this paper, we discuss the collective behavior of dynamical agents in network which associated with a graph

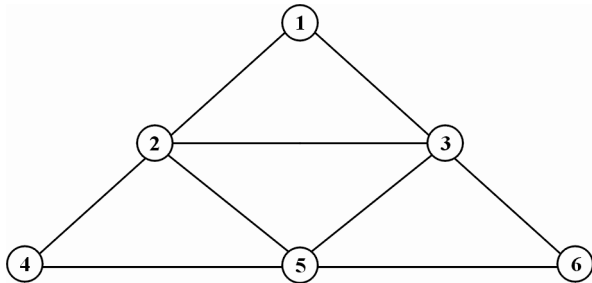


Figure 1. An undirected graph G with $M = 6$ nodes.

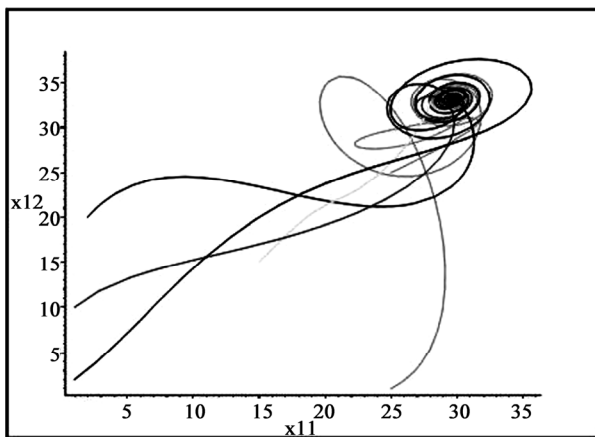


Figure 2. State trajectories of the agents in G .

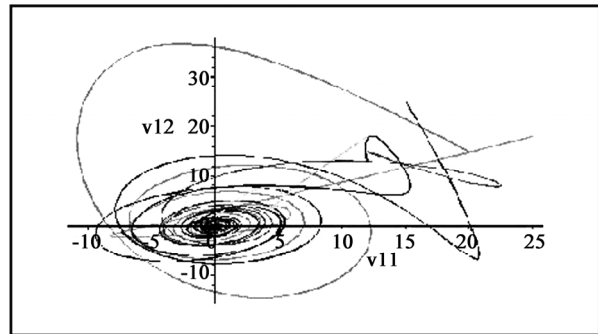


Figure 3. Velocity trajectories of the agents in G .

G . It is assumed that the agents are Lyapunov stable distributed on a plane and their location coordinates are measured by some remote sensor with certain error and transmitted to its neighbors. Based on the transmitted information with time-delay, the control protocol is designed as a linear decentralized law. The coordination of dynamical agents is shown under the condition that the error is small enough. There is a tradeoff between robustness of a protocol to time-delays and the sensor error.

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