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Flocking Control of a Group of Agents Using a Fuzzy-Logic-Based Attractive/Repulsive Function

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Abstract

In this study, a novel procedure is presented for control and analysis of a group of autonomous agents with point mass dynamics achieving flocking motion by using a fuzzy-logic-based attractive/repulsive function. Two cooperative control laws are proposed for a group of autonomous agents to achieve flocking formations related to two different centers (mass center and geometric center) of the flock. The first one is designed for flocking motion guided at mass center and the other for geometric center. A virtual agent is introduced to represent a group objective for tracking purposes. Smooth graph Laplacian is introduced to overcome the difficulties in theoretical analysis. A new fuzzy-logic-based attractive/repulsive function is proposed for separation and cohesion control among agents. The theoretical results are presented to indicate the stability (separation, collision avoidance and velocity matching) of the control systems. Finally, simulation example is demonstrated to validate the theoretical results.

Keywords: Flocking, Cooperative Control, Multi-Agent System, Fuzzy Logic

1. Introduction

A special behavior of large number of interacting dynamic agents called "flocking" has attracted many researchers from diverse fields of scientific and engineering disciplines. Examples of this behavior in the nature include flocks of birds, schools of fish, herds of animals, and colonies of bacteria.

In 1986, Reynolds introduced three heuristic rules that leads to the creation of the first computer animation model of flocking [1]. It should be noticed that these rules are also known as cohesion, separation, and alignment rules respectively in the literature. Similar problems have become a major thrust in systems and control theory, in the context of cooperative control, distributed control of multiple vehicles and formation control. A research field which is tightly related to the theme of this paper is that of consensus seeking of autonomous multi-agent. In this case, multi-agent achieve consensus if their associated state variables converge to a common value [2-5]. In the meantime, an important progression has been achieved on synchronous and/or asynchronous swarm stability analysis [6-8]. Pioneering works [9-11] on flocking motion of particle systems have properly explained the heuristic rules embedded in Reynolds model. One of the important works in [9-11] is the design of attractive/repulsive potential function. In [12], Gu and Hu proposed a flocking control algorithm for fixed and switching network of multi-agent respectively, in which the attractive/repulsive potential was designed using fuzzy logic. Stability is analyzed using the classical Lyapunov theory in fixed network and non-smooth analysis in dynamical network, respectively.

In this study, the flocking behaviors of multi-agent systems with point mass dynamics and dynamical network topology are investigated. The major difference or contribution compared with previous works, for example [9-12], can be outlined as follows. First of all, the new results are based on more general particle model and the flocking motion is centered at different centers, e.g., mass center and geometric center. Secondly, two new cooperative control laws are proposed such that desired collective behaviors (separation, collision avoidance and velocity matching) can be achieved. Finally, smooth graph Laplacian and smooth attractive/repulsive potential based on fuzzy logic are proposed to overcome the difficulties in theoretical analysis and for separation and cohesion control between agents, respectively. In contrast to [12], owing to the design of smooth attractive/repulsive potential based on fuzzy logic and application of smooth graph Laplacian, stability analysis both in fixed and dynamical networks can easily conducted using classical Lyapunov theory.

The rest of the paper is organized as follows. In Section

2, the problems are formulated based on algebraic graph theory, preliminaries about smooth collective potential function and fuzzy control function are provided. In Section 3, two flocking control laws based on fuzzy logic are proposed. Stability analysis is given in Section 4. Simulation results are provided in Section 5. Finally, concluding remarks are made in Section 6.

2. Problem Formulation and Preliminaries

2.1. System Dynamics

Consider a group of N agents (or particles) moving in an n-dimensional Euclidean space, each has point mass dynamics described by

$$\begin{cases} \dot{x}_i = v_i \\ m_i \dot{v}_i = u_i - k_i v_i, i = 1, 2, \ldots, N \end{cases} \quad (1)$$

where $x_i = (x_i^1, x_i^2, \ldots, x_i^n)^T \in \mathbb{R}^n$ is the position vector of agent i, $v_i = (v_i^1, v_i^2, \ldots, v_i^n)^T \in \mathbb{R}^n$ is its velocity vector, $m_i > 0$ is its mass, $u_i = (u_i^1, u_i^2, \ldots, u_i^n)^T \in \mathbb{R}^n$ is the control input acting on agent i, $k_i > 0$ is the velocity damping gain, and $-k_i v_i$ is the velocity damping term.

For flocking motion of a group of agents, the control objectives are to design flocking control laws such that:

a) The distances $\| x_j - x_i \|$ between any two neighbor agents are asymptotically convergent to a desired constant value d;

b) The velocity vectors v_i reach consensus, *i.e.*, $v_1 = v_2 = \ldots = v_N = v_c = v_r$, where v_c is the velocity vector of the center of a group of agents and v_r is the velocity vector of a virtual agent;

c) No collision between agents occurs during the flocking.

The theoretical framework presented in this paper for creation of flocking behavior relies on a number of fundamental concepts in algebraic graph theory [13] that are described below.

A weighted undirected graph will be used to model the interaction topology among agents. An undirected graph G consists of a set of vertices $\mathrm{V} = \{1, 2, \ldots, N\}$ and a set of edges $\mathrm{E} = V \times V$, where an edge is an unordered pair of distinct vertices in V. In graph G, the i th node represents agent i and a edge denoted as e_{ij} or (i, j) represents an information exchange link between agent i and j. The adjacency matrix $\mathrm{A(G)} = [a_{ij}]$ of a graph G is a matrix with nonzero elements satisfying the property $a_{ij} \neq 0 \Leftrightarrow (i, j) \in \mathrm{E}$. Throughout the paper, for simplicity of notation, we assume $a_{ii} = 0$ for all i (or the graphs have no loops). The graph is called weighted whenever the elements of its adjacency matrix are other than just $0-1$ elements. Here, weighted undirected graph is used in this paper. The degree matrix of G is a diagonal matrix $\Delta(\mathrm{G})$ with diagonal elements $\Sigma_{j=1}^N a_{ij}$ that are row-sums of $\mathrm{A(G)}$. The graph Laplacian is defined as $\mathrm{L(G)} = \Delta(\mathrm{G}) - \mathrm{A(G)}$. The Laplacian matrix $\mathrm{L(G)}$ always has a right eigenvector $1_N = (1, 1, \ldots, 1)^T$ associated with eigenvalue $l_1 = 0$. A graph G is called undirected if and only if the adjacency matrix $\mathrm{A(G)}$ is symmetric. The set of neighbors of node i is defined by

$$\mathrm{N}_i = \{ j \in \mathrm{V} : a_{ij} \neq 0 \} = \{ j \in \mathrm{V} : (i, j) \in \mathrm{E} \} \quad (2)$$

In fixed network topology, agent i can range or communication with a fixed set of neighbors. Therefore, the set N_i is time invariant. However, in dynamical or switching network topology, the set of neighbors of agent i is time-varying due to limited communication.

2.2. Smooth Collective Potential Function

Smooth collective potential function is originally proposed in [11]. The following is a brief introduction about it. For more detailed information, the reader is referred to [11].

In order to construct smooth collective potential function, a map $\|\cdot\|_s : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ is defined as

$$\| z \|_s = \frac{1}{e} \left(\sqrt{1 + e \| z \|^2} - 1 \right)$$

with a parameter $e > 0$. Note that $\| z \|_s$ is differentiable everywhere, but $\| z \|$ is not differentiable at $z = 0$.

Smooth adjacency matrix elements are constructed by using a scalar function $r_h(z)$ that smoothly varies between 0 and 1. One possible choice is as follows:

$$r_h(z) = \begin{cases} 1 & z \in [0, h) \\ \frac{1}{2}\left[1 + \cos\left(p \frac{z - h}{1 - h} \right) \right] & z \in [h, 1] \\ 0 & otherwise \end{cases} \quad (3)$$

where $h \in (0, 1)$. Using this function, a position-dependent adjacency matrix $\mathrm{A}(x)$ can be defined as $\mathrm{A}(x) = [a_{ij}(x)]$ with

$$a_{ij}(x) = r_h(\| x_j - x_i \|_s / r_s) \in [0, 1], j \neq i \quad (4)$$

and position-dependent Laplacian matrix as $\mathrm{L}(x) = \Delta(\mathrm{A}(x)) - \mathrm{A}(x)$, where $x = (x_1^T, x_2^T, \ldots, x_N^T)^T$, $r_s = \| r \|_s$,

$r > 0$ denote the interaction range between two agents.

The set of neighbors of agent i is defined by

$$N_i(x) = \{ j \in V : \| x_j - x_i \| < r \}.$$

By the definition of $\| z \|_s$, the control objective (1) can be expressed as following algebraic constraint:

$$\| x_j - x_i \|_s = d_s, \forall j \in N_i(x) \quad (5)$$

where $d_s = \| d \|_s$.

Given a interaction range $r > 0$, a neighboring graph $G(x)$ can be specified by V and the set of edges $E(x) = \{(i, j) \in V \times V : \| x_j - x_i \| < r, j \neq i\}$, that clearly depends on x.

A smooth collective potential function has the form:

$$V_1(x) = \frac{1}{2} \sum_i \sum_{j \neq i} \psi (\| x_j - x_i \|_s)$$

where $\psi(z)$ is a smooth pairwise attractive/repulsive potential with a finite cut-off at $z = r_s$ and a global minimum at $z = d_s$. In order to construct a smooth potential function $\psi(z)$, denote $f(z) = \nabla_z \psi(z)$ and define this function as:

$$f(z) = \nabla_z \psi(z) = \rho_h(z / r_s) \phi(z) \quad (6)$$

where $\phi(z)$ is some function to be designed. Obviously, function $f(z)$ should vanish for all $z \geq r_s$.

In the next section, the function $f(z)$ is implemented using fuzzy logic.

2.3. Preliminaries of Fuzzy Control Function

To the best of our knowledge, for flocking control, [12] is the first paper in which attractive/repulsive function is designed using fuzzy logic. In this section, we provide a brief introduction about fuzzy control function [12].

A set of fuzzy logic rules performs a mapping from an input $z \in R^P$ to a deterministic control $g(z)$, *i.e.*, fuzzy control function. For the *k*th dimension state $x_i^k (k = 1, 2, \ldots, n; i = 1, 2, \ldots, N)$, agent i uses states (x_i, x_j) to build a P-dimension vector $z = (z_1, z_2, \ldots, z_P)$ as fuzzy input. The corresponding fuzzy input set is $F_1, F_2, \ldots, F_P$. A fuzzy rule between agent i and agent j can be expressed as:

$$R^l : \text{IF } z_1 \text{ is } F_I^l \text{ AND, …, AND } z_P \text{ is } F_P^l,$$

$$\text{THEN } g_{ij}^{k,l} = q_O^l + \sum_{P=1}^{P} q_P^l z_P$$

where $k = 1, 2, \ldots, n; l = 1, 2, \ldots, L$, L is the number of fuzzy rules.

Use the Gaussian function to define the membership function of fuzzy set F_p^l: $\mu_{F_p^l} = \exp[-\frac{z_p - a_p^l}{2(\sigma_p^l)^2}]$, where $a_p^l, \sigma_p^l (p = 1, 2, \ldots, P)$ are the mean and variance, respectively. The activation degree of rule R^l is calculated by product operation:

$$\chi^l = \prod_{p=1}^{P} \exp[-\sum_{p=1}^{P} \frac{z_p - a_p^l}{2(\sigma_p^l)^2}].$$

The crisp output $g_{ij}^k(z), k = 1, 2, \ldots, n,$ is calculated by center of area method:

$$g_{ij}^k(z) = \sum_{l=1}^{L} \chi^l g_{ij}^{k,l} \Big/ \sum_{l=1}^{L} \chi^l.$$

3. Flocking Motion Guided at Mass Center and/or Geometric Center

In this section, two cooperative control algorithms are developed for flocking guided at mass center and geometric center respectively. In flocking motion, each agent applies a control input that consists of four terms:

$$u_i = u_i^f + u_i^c + u_i^g + k_i v_i \quad (7)$$

where $u_i^f = -\nabla_{x_i} V_1(x)$ is a gradient-based term and will be designed using fuzzy logic, u_i^c is a velocity consensus/alignment term, u_i^g is a navigation term due to a group objective and $k_i v_i$ is the velocity damping term.

Similar to [9-11], the velocity consensus/alignment term u_i^c is in the form

$$u_i^c = \sum_{j \in N_i(x)} a_{ij}(x)(v_j - v_i) \quad (8)$$

and

$$u_i^c = \sum_{j \in N_i(x)} m_i a_{ij}(x)(v_j - v_i) \quad (9)$$

for flocking motion guided at mass center and geometric center, respectively. The navigation term u_i^g is designed in the following form

$$u_i^g = -k_1 m_i (x_i - x_r) - k_2 m_i (v_i - v_r) + \frac{m_i}{m_r} f(x_r, v_r), k_1, k_2 > 0. \quad (10)$$

The pair $(x_r, v_r) \in \mathbb{R}^n \times \mathbb{R}^n$ is the desired state vector of the group center (mass center or geometric center). The desired state of the group center can be described by

$$\begin{cases} \dot{x}_r = v_r \\ m_r \dot{v}_r = f(x_r, v_r) \end{cases} \quad (11)$$

3.1. Fuzzy Attractive/Repulsive Control

For state vector x_i, the fuzzy input z consists of $\| x_j - x_i \|_s - d_s$, $j \in N(x)$ and $a_{ij}(x) n_{ij}^k$, $j \in N(x)$, where $n_{ij}^k = \frac{x_j^k - x_i^k}{\sqrt{1+\varepsilon \| x_j - x_i \|^2}}$, *i.e.*, $P = 2$ and $z = (z_1, z_2) = (\| x_j - x_i \|_s - d_s, a_{ij}(x) n_{ij}^k)$. The fuzzy output is defined as

$$g_{ij}^{k,l} = q_2^l a_{ij}(x) n_{ij}^k, k = 1, 2, \ldots, n.$$

which implies $q_0^l = q_1^l = 0$. A fuzzy rule R^l is then defined as:

$$\text{IF} \left\| x_j - x_i \right\|_s - d_s \text{ is } F_I^l \text{THEN } g_{ij}^{k,l} = q_2^l a_{ij}(x) n_{ij}^k$$

Therefore,

$$g_{ij}^k(z) = a_{ij}(x) \frac{\sum_{l=1}^{L} \xi^l q_2^l}{\sum_{m=1}^{M} \xi^l} n_{ij}^k.$$

The fuzzy output vector between agent i and j is

$$g_{ij}(z) = (g_{ij}^1, g_{ij}^2, \ldots, g_{ij}^n)^{\mathrm{T}} = a_{ij}(x) \frac{\sum_{l=1}^{L} \xi^l q_2^l}{\sum_{m=1}^{M} \xi^l} n_{ij},$$

where $n_{ij} = (n_{ij}^1, n_{ij}^2, \ldots, n_{ij}^n)^{\mathrm{T}} = \frac{x_j - x_i}{\sqrt{1+\varepsilon \| x_j - x_i \|^2}}$.

Denote the gradient of the attractive/repulsive potential $\psi(\| x_j - x_i \|_s)$ as:

$$\nabla_{z_1} \psi(\| x_j - x_i \|_s) = a_{ij}(x) \frac{\sum_{l=1}^{L} \xi^l q_2^l}{\sum_{l=1}^{L} \xi^l}$$

and denote $\varphi(\| x_j - x_i \|_s) = \frac{\sum_{l=1}^{L} \xi^l q_2^l}{\sum_{l=1}^{L} \xi^l}$, we have

$$\begin{aligned} g_{ij}(z) &= a_{ij}(x) \varphi(\| x_j - x_i \|_s) n_{ij} \\ &= \nabla_{z_1} \psi(\| x_j - x_i \|_s) n_{ij} \\ &= -\nabla_{z_1} \psi(\| x_j - x_i \|) \nabla_{x_i} z_1 \\ &= -\nabla_{x_i} \psi(\| x_j - x_i \|_s) \end{aligned} \tag{12}$$

Therefore, gradient-based control term can be designed as

$$\begin{aligned} u_i^f &= -\sum_{j \in N_i(x)} \nabla_{x_i} \psi(\| x_j - x_i \|_s) = \sum_{j \in N_i(x)} g_{ij}(z) \\ &= \sum_{j \in N_i(x)} a_{ij}(x) \varphi(\| x_j - x_i \|_s) n_{ij} \end{aligned} \tag{1}$$

3.2. Flocking Control guided at Group Centers

Consider the multi-agent motion relative to the group center x_c (the mass center x_{mc} or geometric center x_{gc}). The position and velocity vectors of agent i relative to x_c is denoted by $\tilde{x}_i = x_i - x_c$ and $\tilde{v}_i = v_i - v_c$, the collective state vectors of all agents relative to x_c by $\tilde{x} = x - 1_N \otimes x_c$ and $\tilde{v} = v - 1_N \otimes v_c$, where $x_c = x_{mc}$ or x_{gc}, $v_c = \dot{x}_c$, $\otimes$ is kronecker product.

The mass center of all agents is defined as

$$x_{mc} = \sum_{i=1}^{N} m_i x_i \Big/ \sum_{i=1}^{N} m_i \tag{14}$$

Note that

$$\begin{aligned} u_i^g &= -k_1 m_i (x_i - x_r) - k_2 m_i (v_i - v_r) + \frac{m_i}{m_r} f(x_r, v_r) \\ &= -k_1 m_i \tilde{x}_i - k_2 m_i \tilde{v}_i - k_1 m_i (x_{mc} - x_r) \\ &\quad - k_2 m_i (v_{mc} - v_r) + \frac{m_i}{m_r} f(x_r, v_r) \end{aligned} \tag{15}$$

and due to $\sum_{i=1}^{N} u_i^f = 0, \sum_{i=1}^{N} u_i^c = 0$, $\sum_{i=1}^{N} m_i \tilde{x}_i = 0$, $\sum_{i=1}^{N} m_i \tilde{v}_i = 0$, we have

$$\begin{aligned} \dot{v}_{mc} &= \sum_{i=1}^{N} m_i \dot{v}_i \Big/ \sum_{i=1}^{N} m_i \\ &= -k_1 (x_{mc} - x_r) - k_2 (v_{mc} - v_r) + \frac{1}{m_r} f(x_r, v_r) \end{aligned}$$

Then, the dynamic of mass center is given by

$$\begin{cases} \dot{x}_{mc} = v_{mc} \\ \dot{v}_{mc} = -k_1 (x_{mc} - x_r) - k_2 (v_{mc} - v_r) + \frac{1}{m_r} f(x_r, v_r) \end{cases} \tag{16}$$

Denote $e_{mc} = (x_{mc}^T - x_r^T, v_{mc}^T - v_r^T)^T$, the relative dynamic of center of mass is given by

$$\dot{e}_{mc} = \tilde{K} e_{mc} \tag{17}$$

where $\tilde{K} = K \otimes I_n, K = \begin{pmatrix} 0 & 1 \\ -k_1 & -k_2 \end{pmatrix}$.

We can choose proper control parameters $k_1 > 0$ and $k_2 > 0$ such that matrix K is Hurwitz stable, and from Lyapunov theory, for given positive matrix $Q \in \mathbb{R}^{2\times2}$, there exists a positive definite matrix $P \in \mathbb{R}^{2\times2}$, such that:

$$K^T P + PK = -Q. \tag{18}$$

For creation of flocking motion relative to mass center, we propose following control laws:

$$
\begin{aligned}
u_i &= u_i^{mc} \\
&= \sum_{j\in N_i(x)} a_{ij}(x)\varphi(\| x_j - x_i \|_s) n_{ij} \\
&\quad + \sum_{j\in N_i(x)} a_{ij}(x)(v_j - v_i) k_1 m_i (x_i - x_r) \\
&\quad - k_2 m_i (v_i - v_r) + \frac{m_i}{m_r} f(x_r, v_r) + k_i v_i
\end{aligned} \tag{19}
$$

where $\varphi(\| x_j - x_i \|_s) = \sum_{m=1}^{M} x_{ij}^m q_i^{m,2} \Big/ \sum_{m=1}^{M} x_{ij}^m$.

The geometric center of all agents is defined as

$$x_{gc} = \mathrm{ave}(x) = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{20}$$

Similarly, for geometric center, we propose following control law:

$$
\begin{aligned}
u_i &= u_i^{gc} \\
&= \sum_{j\in N_i(x)} a_{ij}(x) m_i \varphi(\| x_j - x_i \|_s) n_{ij} \\
&\quad + \sum_{j\in N_i(x)} a_{ij}(x) m_i (v_j - v_i) \\
&\quad - k_1 m_i (x_i - x_r) - k_2 m_i (v_i - v_r) + \frac{m_i}{m_r} f(x_r, v_r) + k_i v_i
\end{aligned} \tag{21}
$$

4. Analysis of Stability

In this section, we present our main results for flocking in multi-agent networks with dynamical topology, and conduct stability analysis based on classical Lyapunov theory and LaSalle's invariance principle. In [12], owing to the discontinuity of collective potential function in the case of dynamical topology, stability analysis is done using classical Lyapunov theory in fixed networks and nonsmooth analysis theory, which is difficult to understand for engineers in real applications, in dynamical networks, respectively. In our paper, due to the design of smooth collective potential function, in both cases of fixed network and dynamical network, stability analysis can be conducted based on classical Lyapunov theory and LaSalle's invariance principle.

For the collective motion relative to mass center x_{mc}, define energy function

$$V(x,v) = V_1(x) + V_2(x) + V_3(v) + V_4(x,v) \tag{22}$$

where,

$$
\begin{aligned}
V_1(x) &= \frac{1}{2}\sum_i \sum_{j\neq i} \psi(\| x_j - x_i \|_s) \\
&= \frac{1}{2}\sum_i \sum_{j\neq i} \psi(\| \tilde{x}_j - \tilde{x}_i \|_s),
\end{aligned}
$$

$$V_2(x) = \frac{1}{2} k_1 \sum_{i=1}^{N} m_i \tilde{x}_i^T \tilde{x}_i, \quad V_3(v) = \frac{1}{2}\sum_{i=1}^{N} m_i \tilde{v}_i^T \tilde{v}_i,$$

$$V_4(x,v) = e_{mc}^T (P \otimes I_n) e_{mc},$$

$k_1 > 0$ is the parameter of the navigation term.

Applying control law (19) to system (1), following theorem is held to explain the emergence of flocking behaviors of agents guided at center of mass x_{mc}.

Theorem 1 Consider a group of agents applying control law (19) with proper selected parameters $k_1, k_2 > 0$ satisfying (18) to system (1). Assume that the initial value $V(x(0), v(0))$ is finite. Then, the following statements hold.

(i) The solution of system (1) asymptotically converges to an equilibrium point (x^*, v_{mc}) where x^* a local minima of is $V_1(x) + V_2(x)$.

(ii) The velocities of all agents asymptotically converge to the velocity of mass center v_{mc}, and the velocity of mass center v_{mc} asymptotically converge to the desired velocity v_r, *i.e.*, $v_i \to v_{mc}, i = 1, 2, \ldots, N, v_{mc} \to v_r$.

(iii) No collision between agents occurs during the flocking.

Proof: First of all, we explain the fact that the energy function $V(x,v)$ is positive definite.

Obviously, $V_2(x), V_3(v)$ and $V_4(x,v)$ is positive. Therefore, the positive definiteness of energy function $V(x,v)$ is equivalent to that of $V_1(x)$. By similar analysis to theorem 1 of [12], $\psi(\| \tilde{x}_j - \tilde{x}_i \|_s)$ can be designed positive definite by using proper fuzzy rules, *i.e.*, $V_1(x)$ can be designed positive definite.

Secondly, the derivative of the energy function $V(x,v)$ is seminegative definite.

$$\dot{V}_1(x) = \sum_{i=1}^{N} \sum_{j\in N_i(x)} v_i^T \nabla_{x_i} \psi(\| x_j - x_i \|_s) = -\sum_{i=1}^{N} v_i^T u_i^f \tag{23}$$

$$\dot{V}_2(x) = k_1 \sum_{i=1}^{N} m_i \tilde{v}_i^T \tilde{x}_i \tag{24}$$

Due to $\sum_{i=1}^{N} m_i \tilde{v}_i = 0$, we have

$$\dot{V}_3(v) = -\dot{V}_1(x) - \tilde{v}^T (L \otimes I_n) \tilde{v} + \sum_{i=1}^{N} \tilde{v}_i^T u_i^g .$$

From (15) and note that

$$
\begin{aligned}
&\sum_{i=1}^{N} \tilde{v}_i^T [-k_1 m_i (x_{mc} - x_r) - k_2 m_i (v_{mc} - v_r) \\
&\quad + \frac{m_i}{m_r} f(x_r, v_r)] = 0,
\end{aligned}
$$

We have

$$\sum_{i=1}^{N} \tilde{v}_i^T u_i^g = -k_1 \sum_{i=1}^{N} m_i \tilde{v}_i^T \tilde{x}_i - k_2 \sum_{i=1}^{N} m_i \tilde{v}_i^T \tilde{v}_i .$$

Then,

$$\dot{V}_3(v) = -\dot{V}_1(x) - \dot{x}^T (L \otimes I_n)\dot{x} - \dot{V}_2(x) - k_2 \dot{x}^T (M \otimes I_n)\dot{x} \quad (25)$$

where $M = diag(m_1, m_2, \mathbf{L}, m_N)$.

From (17), we have

$$\dot{V}_4(x,v) = -e_{mc}^T (Q \otimes I_n) e_{mc} \quad (26)$$

From (23) to (26), we have

$$\begin{aligned}\dot{V}(x,v) &= V_1(x) + V_2(x) + V_3(v) + V_4(x,v) \\ &= -\dot{x}^T (L \otimes I_n)\dot{x} - k_2 \dot{x}^T (M \otimes I_n)\dot{x} \\ &\quad - e_{mc}^T (Q \otimes I_n) e_{mc} \le 0.\end{aligned} \quad (27)$$

Part (i) and part (ii) follow from LaSalle's invariance principle. As $\dot{V}(x,v)$ is seminegative definite, given $\Omega_c = \{(x,v) : V(x,v) \le c\}$, Ω_c is an invariant set. From $V(x,v) \le c$, we have $\| e_{mc} \|^2 \le c / l_{\min}(P \otimes I_n)$. Therefore, both $\| x_{mc} - x_r \|$ and $\| v_{mc} - v_r \|$ are bounded. Given the desired state (x_r, v_r) is bounded and from the definition of x_{mc} and v_{mc}, we know (x,v) is bounded.

From LaSalle's invariance principle, all states starting in Ω_c converge to the largest invariant set $S = \{(x,v) \in \Omega_c : \dot{V}(x,v) = 0\}$. Hence, all states converge to the largest invariant set $S = \{(x,v) \in \Omega_c : v_i = v_{mc} = v_r, x_{mc} = x_r\}$ asymptotically, *i.e.*, all agent velocities v_i converge to the velocity of center of mass, v_{mc}, and the position and velocity vectors of center of mass, x_{mc}, v_{mc}, converge to the desired states, x_r, v_r, asymptotically.

Furthermore, in stable state, $V(x,v) \to V_1(x) + V_2(x)$. There is a equilibrium point at (x^*, v_{mc}) where x^* is a local minima of $V_1(x) + V_2(x)$.

Finally, we prove part (iii) by contradiction. Assume there exists a time $t = t_1 > 0$ when two distinct agents k, l collide, *i.e.*, $x_k(t_1) = x_l(t_1)$. For all $t > 0$, we have

$$\begin{aligned} V_1(x(t)) &= \frac{1}{2}\sum_i \sum_{j \ne i} y\,(\| x_j - x_i \|_s) \\ &= y\,(\| x_k(t) - x_l(t) \|_s) \\ &\quad + \frac{1}{2}\sum_{i \in V \setminus \{k,l\}} \sum_{j \in V \setminus \{i,k,l\}} y\,(\| x_j - x_i \|_s) \\ &\ge y\,(\| x_k(t) - x_l(t) \|_s) \end{aligned}$$

At $t = t_1$, defining $y(0)$ larger than c leads to $V_1(x(t_1)) \ge c$, which is in contradiction with the invariant set Ω_c. Therefore, no two agents collide at any time $t \ge 0$.

Remark: From theorem 1, control objective a) and b) are achieved, but the geometric characterization of local minima x^* of $V_1(x) + V_2(x)$ is not possible satisfying the algebraic constraint (5). In [11], the authors pose two conjectures that establish the close relationship between geometric and graph theoretic properties of any local minima of $V_1(x) + V_2(x)$ and features of flocks. Based on the conjectures in [11], we can conclude that the local minima x^* of $V_1(x) + V_2(x)$ is a so-called *quasi- a -lattice* [11], *i.e.*, $-d \le \| x_j - x_i \|_s - d \le d$, $0 \le d = d_s, \forall (i,j) \in E(x)$. Obviously, it is very close to the conformation satisfying algebraic constraint (5).

When control laws (21) applied to system (1), similar theorem is established to explain the emergence of flocking behavior of agents guided at geometric center.

For the collective motion relative to geometric center x_{gc}, define energy function

$$U(x,v) = V_1(x) + U_2(x) + U_3(v) + V_4(x,v)$$

Where

$$V_1(x) = \frac{1}{2}\sum_i \sum_{j \ne i} y\,(\| x_j - x_i \|_s), U_2(x) = \frac{1}{2} k_1 \sum_{i=1}^{N} \tilde{x}_i^T \tilde{x}_i,$$

$U_3(v) = \frac{1}{2}\sum_{i=1}^{N} \tilde{v}_i^T \tilde{v}_i, V_4(x,v) = e_{gc}^T (P \otimes I_n) e_{gc}$, $k_1 > 0$ is the parameter of the navigation term, $e_{gc} = (x_{gc}^T - x_r^T, v_{gc}^T - v_r^T)^T$.

We have following theorem.

Theorem 2 Consider a group of agents applying control law (21) with proper selected parameters $k_1, k_2 > 0$ satisfying (18) to system (1). Assume that the initial value $U(x(0), v(0))$ is finite. Then, the following statements hold.

1) The solution of system (1) asymptotically converges to an equilibrium point (x^*, v_{gc}) where x^* a local minima of is $V_1(x) + U_2(x)$.

2) The velocities of all agents asymptotically converge to the velocity of geometric center v_{gc}, and the velocity of geometric center v_{gc} asymptotically converge to the desired velocity v_r, *i.e.*, $v_i \to v_{gc}$, $i = 1, 2, \mathbf{L}, N$, $v_{gc} \to v_r$.

3) No collision between agents occurs during the flocking.

Proof: The proof of this theorem is similar to that of theorem 1.

5. Simulation

In this section, mass center guided flocking motion is simulated in 2-dimentional space. The following parameters were fixed throughout the simulation: $d = 2, r = 1.2d, e = 0.5$, and $h = 0.3$ for $r_h(z)$. Eight fuzzy sets are designed for the fuzzy control input $\| x_j - x_i \| - d_s$. They are LN, N, SN, Z, SP, P, LP, and PP as shown in **Figure 1**. Eight fuzzy rules are designed as follows:

IF $\| x_j - x_i \|_s - d_s$ is LN THEN

$$g_{ij}^{k,1} = -90 r_h(\| x_j - x_i \|_s / r_s) n_{ij}^k ;$$

IF $\| x_j - x_i \|_s - d_s$ is N THEN

$$g_{ij}^{k,2} = -80 r_h(\| x_j - x_i \|_s / r_s) n_{ij}^k ;$$

IF $\| x_j - x_i \|_s - d_s$ is SN THEN

$$g_{ij}^{k,3} = -50 r_h(\| x_j - x_i \|_s / r_s) n_{ij}^k ;$$

IF $\| x_j - x_i \|_s - d_s$ is Z THEN $g_{ij}^{k,4} = 0$;

IF $\| x_j - x_i \|_s - d_s$ is SP THEN

$$g_{ij}^{k,5} = 0.5 r_h(\| x_j - x_i \|_s / r_s) n_{ij}^k ;$$

IF $\| x_j - x_i \|_s - d_s$ is P THEN

$$g_{ij}^{k,6} = r_h(\| x_j - x_i \|_s / r_s) n_{ij}^k ;$$

IF $\| x_j - x_i \|_s - d_s$ is LP THEN

$$g_{ij}^{k,7} = 1.5 r_h(\| x_j - x_i \|_s / r_s) n_{ij}^k ;$$

IF $\| x_j - x_i \|_s - d_s$ is PP THEN

$$g_{ij}^{k,8} = 2 r_h(\| x_j - x_i \|_s / r_s) n_{ij}^k .$$

Control parameters k_1, k_2 are selected to be $k_1 = k_2 = 1$. The two eigenvalues of matrix $K = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$ are $-05000 + 08660i$ and $-05000 - 08660i$. Therefore, matrix K is Hurwitzstable. Let $Q = I_2$, using Matlab command *lyap*(*K,Q*), we obtain positive definite matrix $P = \begin{pmatrix} 1.5 & -0.5 \\ -0.5 & 1 \end{pmatrix}$.

Figure 1. Fuzzy membership function.

Simulation is calculated within 20 seconds time by using Matlab Simulink. In addition, the position of each agent is marked with a right triangle sign.

Figures 2 to **5** show the simulation results within 2-D flocking using control law (19) for 50 agents. **Figures 2** to **4** show snapshots of 2-D flocking at time $t = 0, 4.3495$, and 20 (sec). The initial position and initial velocity coordinates were uniformly chosen in the random domain of $[0,3] \times [0,3]$ and $[0,1] \times [0,1]$, respectively. The mass of each agent was also uniformly chosen in a random domain of [0.5, 1.5]. A steady configuration was formed

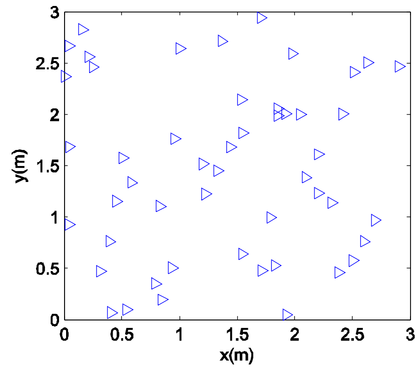

Figure 2. Initial positions of 50 agents.

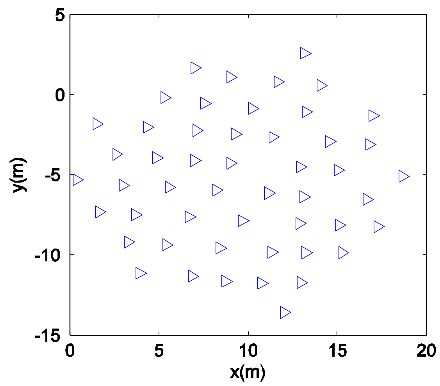

Figure 3. Configuration of 50 agents at t = 4.3495 (sec).

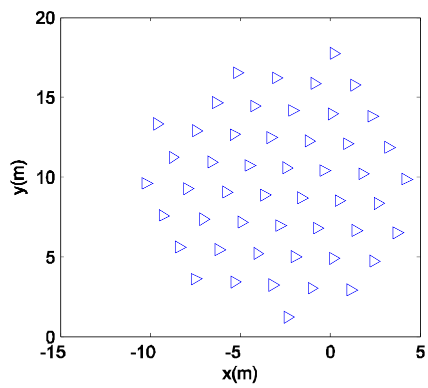

Figure 4. Final configuration of 50 agents at t = 20 (sec).

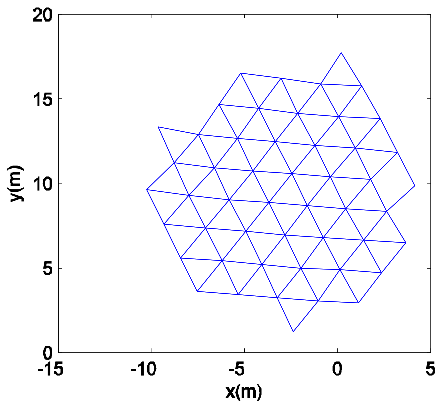

Figure 5. Position-dependent neighboring graph at t = 20 (sec).

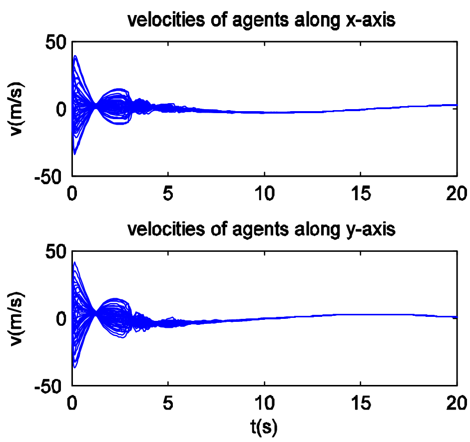

Figure 6. Velocities of 50 agents along x-axis and y-axis respectively.

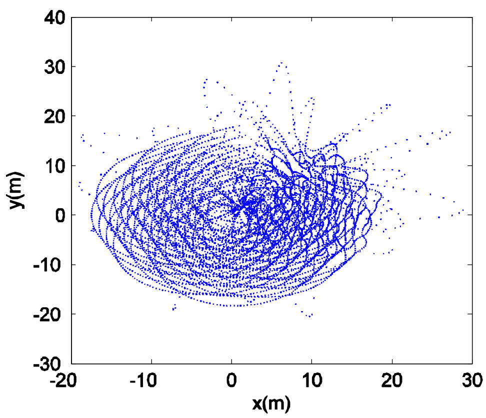

Figure 7. Trajectories of all agents within 20 (sec) time.

as shown in **Figure 4** and maintained thereafter. A virtual agent

$$x_r(t) = (10\sin(03t), 10\cos(03t))^T,$$
$$v_r(t) = (3\cos(03t), -3\sin(03t))^T$$

was used for this example. For highly disconnected neighboring graph $\mathrm{G}(x)$ in initial state, **Figure 5** shows the connected neighboring graph $\mathrm{G}(x)$ corresponding to the final configuration. **Figure 6** shows velocity matching is achieved along x-axis and y-axis respectively. **Figure 7** shows the trajectories of all agents within 20(sec) simulation time and the cohesive behaviors. The simulation demonstration with control law (21) was similar to that conducted by control laws (19), and therefore is not necessarily repeated here.

6. Conclusions

This paper establishes a theoretical framework for design and analysis of flocking control algorithms using a fuzzy-logic-based attractive/repulsive potential function for multiple agent networks with dynamical topology. Two cooperative control laws have been proposed for a group of autonomous agents to achieve flocking motion relative to different centers (mass center and geometric center). A virtual agent is introduced to represent a group objective for tracking purposes. Smooth Laplacian and smooth fuzzy-logic-based attractive/repulsive potential are proposed to overcome the difficulties in stability analysis. Simulation results validated the theoretical results.

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