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Constraints on Neutrino Masses from Baryon Acoustic Oscillation Measurements

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Abstract

From 21 independent Baryon Acoustic Oscillation (BAO) measurements we obtain the following sum of masses of active Dirac or Majorana neutrinos: $\sum m_\nu = 0.711 - 0.335 \cdot \delta h + 0.050 \cdot \delta b \pm 0.063\ \text{eV}$, where $\delta h \equiv (h - 0.678)/0.009$ and $\delta b \equiv \left(\Omega_b h^2 - 0.02226\right)/0.00023$. This result may be combined with independent measurements that constrain the parameters $\sum m_\nu$, h, and $\Omega_b h^2$. For $\delta h = \pm 1$ and $\delta b = \pm 1$, we obtain $m_\nu < 0.43\ \text{eV}$ at 95% confidence.

Keywords

Neutrino Mass, Baryon Accoustic Oscillations, Cosmology

1. Introduction

We extend the analysis presented in "Study of baryon acoustic oscillations with SDSS DR13 data and measurements of Ω_k and $\Omega_{\text{DE}}(a)$" [1] to include neutrino masses. The present analysis has three steps: 1) we calculate the distance of propagation r_s, in units of c/H_0, referred to the present time, of sound waves in the photon-electron-baryon plasma until decoupling by numerical integration of Equation (16) and Equation (17) of Ref. [1]; 2) we fit the Friedmann equation of evolution of the universe to 21 independent Baryon Acoustic Oscillation (BAO) distance measurements listed in [1] used as uncalibrated standard rulers and obtain the length d of these rulers, in units of c/H_0, referred to the present time; and 3) we set

$$r_s = d \tag{1}$$

to constrain the sum of neutrino masses $\sum m_\nu$. c is the speed of light, and $H_0 \equiv 100h\ \text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ is the present day Hubble expansion parameter.

2. Constraints on Neutrino Masses

The main body of this article assumes: 1) flat space, *i.e.* $\Omega_k = 0$, and 2) constant dark energy density relative to the critical density, *i.e.* Ω_{DE} independent of the expansion parameter *a*. These constraints are in agreement with all observations to date [1] [2]. Results without these constraints are presented in **Appendix 1**. Results with partial data sets are presented in **Appendix 2**.

To be specific we consider three active neutrino flavors with three eigenstates with nearly the same mass m_ν, so $\sum m_\nu = 3m_\nu$. This is a useful scenario to consider since our current limits on m_ν^2 are much larger than the mass-squared-differences Δm^2 and Δm_{21}^2 obtained from neutrino oscillations [2]. These neutrinos become non-relativistic at a neutrino temperature $T_\nu = m_\nu/3.15$ or a photon temperature $T = m_\nu (11/4)^{1/3}/3.15$. The corresponding expansion parameter is $a_\nu = T_0/T = 5.28\times10^{-4}(1\,\mathrm{eV}/m_\nu)$.

The matter density relative to the present critical density is Ω_m/a^3 for $a > a_\nu$. Ω_m includes the density $\Omega_\nu = h^{-2}\sum m_\nu/94\,\mathrm{eV}$ of Dirac or Majorana neutrinos that are non-relativistic today. Note that for Dirac neutrinos we are considering the scenario in which right-handed neutrinos and left-handed anti-neutrinos are sterile and never achieved thermal equilibrium. Our results can be amended for other specific scenarios. For $a < a_\nu$ we take the matter density to be $(\Omega_m - \Omega_\nu)/a^3$. The radiation density is $\Omega_\gamma N_{\mathrm{eq}}/(2a^4)$ for $a < a_\nu$, where $N_{\mathrm{eq}} = 3.36$ for three flavors of Dirac (mostly) left-handed neutrinos and right-handed anti-neutrinos. We also take $N_{\mathrm{eq}} = 3.36$ for three active flavors of Majorana left-handed and right-handed neutrinos. For $a > a_\nu$, we take the radiation density to be $(\Omega_\gamma N_{eq}/2 - a_\nu\Omega_\nu)/a^4 = \Omega_\gamma/a^4$. The present density of photons relative to the critical density is $\Omega_\gamma = 2.473\times10^{-5}h^{-2}$ [2].

The data used to obtain *d* are 18 independent BAO distance measurements with Sloan Digital Sky Survey (SDSS) data release DR13 galaxies in the redshift range $z = 0.1$ to 0.7 [3] [4] [5] summarized in Table 3 of [1], two BAO distance measurements in the Lyman-alpha forest (Ly*α*) at $z = 2.36$ (cross-correlation [6]) and $z = 2.34$ (auto-correlation [7]) summarized in Section 6 of [1], and the Cosmic Microwave Background (CMB) correlation angle $\theta_{\mathrm{MC}} = 0.010410 \pm 0.000005$ [2] [8], used as an uncalibrated standard ruler. Note that the correlation angle θ_{MC} is also determined by BAO. These 21 independent BAO measurements and full details of the fitting method are presented in [1].

As a reference we take

$$h = 0.678 \pm 0.009, \quad \Omega_b h^2 = 0.02226 \pm 0.00023 \tag{2}$$

(at 68% confidence) from "Planck TT + low P + lensing" data (that does not contain BAO information) [2]. Ω_b is the present density of baryons relative to the critical density.

Due to correlations and non-linearities we obtain our final result (Equation (9) below) with a global fit. The following equations are included to illustrate the dependence of r_s and *d* on the cosmological parameters *h*, $\Omega_b h^2$ and $\sum m_\nu$

in limited ranges of interest. Integrating the comoving sound speed of the photon-baryon-electron plasma until $a_{\text{dec}} = 1/(1+z_{\text{dec}})$ with $z_{\text{dec}} = 1089.9 \pm 0.4$ [2] we obtain

$$r_s \approx 0.0339 \times A \times \left(\frac{0.28}{\Omega_m}\right)^{0.24} \tag{3}$$

with

$$A \approx 0.990 + 0.007 \cdot \delta h - 0.001 \cdot \delta b + 0.020 \cdot \frac{\sum m_\nu}{1\,\text{eV}}, \tag{4}$$

where

$$\delta h \equiv (h - 0.678)/0.009, \tag{5}$$

$$\delta b \equiv \left(\Omega_b h^2 - 0.02226\right)/0.00023. \tag{6}$$

To obtain d we minimize the χ^2 with 21 terms, corresponding to the 21 BAO observables, with respect to Ω_{DE} and d, and obtain $\Omega_{\text{DE}} = 0.718 \pm 0.003$ and

$$d \approx 0.0340 \pm 0.0002, \tag{7}$$

with χ^2 per degree of freedom 19.8/19, and correlation coefficient 0.989 between Ω_{DE} and d (this high correlation coefficient is due to the high precision of θ_{MC}). Setting $r_s = d$ we obtain

$$\sum m_\nu \approx 0.73 - 0.35 \cdot \delta h + 0.05 \cdot \delta b \pm 0.15\,\text{eV}. \tag{8}$$

A more precise result is obtained with a global fit by minimizing the χ^2 with 21 terms varying Ω_{DE} and $\sum m_\nu$ directly. We obtain $\Omega_{\text{DE}} = 0.7175 \pm 0.0023$ and

$$\sum m_\nu = 0.711 - 0.335 \cdot \delta h + 0.050 \cdot \delta b \pm 0.063\,\text{eV}, \tag{9}$$

with $\chi^2/\text{d.f.} = 19.9/19$, and correlation coefficient 0.924 between Ω_{DE} and $\sum m_\nu$. This is our main result. Equation (9) is obtained from BAO measurements alone, and is written in a way that can be combined with independent constraints on the cosmological parameters $\sum m_\nu$, h and $\Omega_b h^2$, such as measurements of the power spectrum of density fluctuations $P(k)$, the CMB, and direct measurements of the Hubble parameter.

Setting $\delta h = \pm 1$ and $\delta b = \pm 1$ we obtain the following upper bound on the mass of active neutrinos $m_\nu = \frac{1}{3} \sum m_\nu$:

$$m_\nu < 0.43\,\text{eV at 95\% confidence.} \tag{10}$$

References

[1] Hoeneisen, B. (2017) Study of Baryon Acoustic Oscillations with SDSS DR13 Data and Measurements of Ω_k and $\Omega_{\text{DE}}(a)$. *International Journal of Astronomy and Astrophysics*, **7**, 11-27. https://doi.org/10.4236/ijaa.2017.71002

[2] Patrignani, C., *et al.* (2016) Review of Particle Physics. *Chinese Physics C*, **40**, Article ID: 100001.

[3] Albareti, F.D., *et al.* (2016) The Thirteenth Data Release of the Sloan Digital Sky Survey: First Spectroscopic Data from the SDSS-IV Survey Mapping Nearby Galaxies at Apache Point Observatory. SDSS Collaboration, arXiv:1608.02013.

[4] Dawson, K.S., Schlegel, D.J., Ahn, C.P., *et al.* (2013) The Baryon Oscillation Spectroscopic Survey of SDSS-III. *Astronomical Journal*, **145**, 10. https://doi.org/10.1088/0004-6256/145/1/10

[5] Dawson, K.S., Kneib, J.-P., Percival, W.J., *et al.* (2016) The SDSS-IV Extended Baryon Oscillation Spectroscopic Survey: Overview and Early Data. *Astronomical Journal*, **151**, 44. https://doi.org/10.3847/0004-6256/151/2/44

[6] Font-Ribera, A., *et al.* (2014) Quasar-Lyman α Forest Cross-Correlation from BOSS DR11: Baryon Acoustic Oscillations. arXiv:1311.1767. https://doi.org/10.1088/1475-7516/2014/05/027

[7] Delubac, T., *et al.* (2014) Baryon Acoustic Oscillations in the Lyα Forest of BOSS DR11 Quasars. arXiv:1404.1801v2.

[8] Planck Collaboration (2015) Results XIII, Astron. & Astrophys. Submitted, arXiv:1502.01589v2.

Appendix

Appendix 1. Removing constraints

Freeing Ω_k and keeping Ω_{DE} constant we obtain $\Omega_k = -0.003 \pm 0.006$, $\Omega_{\mathrm{DE}} + 2.2\Omega_k = 0.719 \pm 0.003$, and

$$\sum m_\nu = 0.623 - 0.334 \cdot \delta h + 0.050 \cdot \delta b \pm 0.191\,\mathrm{eV}, \tag{11}$$

with $\chi^2/\mathrm{d.f.} = 19.6/18$.

Fixing $\Omega_k = 0$ and letting $\Omega_{\mathrm{DE}}(a) = \Omega_{\mathrm{DE}} \cdot \{1 + w_a \cdot (1-a)\}$ we obtain $\Omega_{\mathrm{DE}} = 0.716 \pm 0.004$, $w_a = 0.064 \pm 0.148$, and

$$\sum m_\nu = 0.603 - 0.349 \cdot \delta h + 0.052 \cdot \delta b \pm 0.257\ \mathrm{eV}, \tag{12}$$

with $\chi^2/\mathrm{d.f.} = 19.7/18$.

Freeing Ω_k and letting $\Omega_{\mathrm{DE}}(a) = \Omega_{\mathrm{DE}} \cdot \{1 + w_a \cdot (1-a)\}$ we obtain $\Omega_k = -0.008 \pm 0.004$, $\Omega_{\mathrm{DE}} + 2.2\Omega_k = 0.718 \pm 0.004$, $w_a = 0.227 \pm 0.069$, and

$$0 < \sum m_\nu = -0.388 - 0.350 \cdot \delta h + 0.050 \cdot \delta b \pm 0.830\ \mathrm{eV}, \tag{13}$$

with $\chi^2/\mathrm{d.f.} = 17.8/17$.

Appendix 2. Removing data.

In this Appendix we apply the constraints $\Omega_k = 0$ and Ω_{DE} constant. Removing the measurement of θ_{MC} we obtain $\Omega_{\mathrm{DE}} = 0.722 \pm 0.011$ and

$$\sum m_\nu = 0.579 - 0.333 \cdot \delta h + 0.049 \cdot \delta b \pm 0.285\ \mathrm{eV}, \tag{14}$$

with $\chi^2/\mathrm{d.f.} = 19.7/18$.

Removing the measurement of θ_{MC} and the two Lyα measurements we obtain $\Omega_{\mathrm{DE}} = 0.716 \pm 0.014$ and

$$\sum m_\nu = 0.743 - 0.330 \cdot \delta h + 0.049 \cdot \delta b \pm 0.366\ \mathrm{eV}, \tag{15}$$

with $\chi^2/\mathrm{d.f.} = 11.2/16$.

Keeping only the measurement of θ_{MC} we need to fix Ω_{DE} in order to get zero degrees of freedom and have a unique solution. The best way to fix Ω_{DE} is with BAO measurements, and that is the purpose of the present study.