

Class of Charged Fluid Balls in General Relativity

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How to cite this paper: Sah, A. and Chandra, P. (2016) Class of Charged Fluid Balls in General Relativity. *International Journal of Astronomy and Astrophysics*, 6, 494-511. <http://dx.doi.org/10.4236/ijaa.2016.64038>

Received: November 21, 2016

Accepted: December 26, 2016

Published: December 29, 2016

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Abstract

In the present study, we have obtained a new analytical solution of combined Einstein-Maxwell field equations describing the interior field of a ball having static spherically symmetric isotropic charged fluid within it. The charge and electric field intensity are zero at the center and monotonically increasing towards the boundary of the fluid ball. Besides these, adiabatic index is also increasing towards the boundary and becomes infinite on it. All other physical quantities such as pressure, density, adiabatic speed of sound, charge density, adiabatic index are monotonically decreasing towards the surface. Causality condition is obeyed at the center of ball. In the limiting case of vanishingly small charge, the solution degenerates into Schwarzschild uniform density solution for electrically neutral fluid. The solution joins smoothly to the Reissner-Nordstrom solution over the boundary. We have constructed a neutron star model by assuming the surface density $\rho = 2 \times 10^{17} \text{ kgm}^{-3}$. The mass of the neutron star comes $1.58 M_{\odot}$ with radius 14.574 km.

Keywords

Exact Solution, Einstein's Field Equations, Charged Fluid Ball, Compact Star, General Relativity

1. Introduction

An analysis of the Reissner-Nordstrom metric shows that a spherically symmetric distribution of charged dust may avoid the catastrophic gravitational collapse, a seemingly unavoidable feature of Schwarzschild's [1] geometry exterior to an electrically neutral fluid sphere of mass bigger than certain critical limit. As in evidence we have Bonnor's model [2] [3] of the equilibrium ball of charged dust in contrast to the Oppenheimer-Snyder [4] continually contracting ball of electrically neutral dust. Though Bonnor's model has been found to be unstable to small radial perturbations and also to a change in the total charge content of the system, it initiates a general interest in the study of the

implications of Einstein-Maxwell field equations with reference to the general relativistic prediction of gravitational collapse. On the other hand, it is generally considered that a black hole may carry non-zero net charge, apart from its mass and angular momentum. Such an assumption may require the collapse of stellar masses of charged matter. It has been suggested by Shvartsman [5] that on account of interaction between a star and its surroundings, it is possible that stellar systems carrying electric charge may exist in nature. It is therefore not surprising that in recent years the problem of finding non-singular, physically meaningful solution of Einstein-Maxwell field equations for static ball of charged coherent perfect has received wide attention. The inclusion of charge seems to affect the stability of the system—the stability of Schwarzschild's uniform density sphere increases by the introduction of net surface charge. It has been shown that the stability is more profound if the same amount of charge be distributed uniformly throughout within the sphere.

The search for the exact solutions is of continuous interest to researcher. Buchdahl [6] proposed a famous bound on the mass radius ratio of relativistic fluid spheres which is an important contribution in order to study the stability of the fluid spheres. Delgaty-Lake [7] studied all the then existing solutions and established that Adler [8], Heintzmann [9], etc. do not satisfy all the well behaved conditions and also pointed out that only nine solutions are well behaved; out of which seven in curvature coordinates (Tolman [10], Finch and Skea [11], Patvardhav and Vaidya [12], Mehra [13], Kuchowicz [14], Matese and Whitman [15], Durgapal's two solutions [16]) and only two solutions (Nariai [17], Goldman [18]) in isotropic coordinates. Ivanov [19] [20] [21] [22] [23], Neeraj Pant [24], Maurya and Gupta [25], Pant *et al.* [26] [27], Pant and Sah [28] [29], Tewari, Charan and Chandra [30], Sah, Chandra and Charan [31] studied the existing well behaved solutions of Einstein's field equations. Some pioneer work in Relativity is given by Herrera *et al.* [32]-[37], Tewari and Charan [38] [39] [40] [41]. Nduka [42] [43], Whitman and Burch [44], Tikekar [45], Ivanov [46], Ray *et al.* [47], Stettner [48], Krori and Barua [49], Ray and Das [50], Pant and Negi [51], Florides [52], Dionysiou [53], Pant *et al.* [54] etc. gave the well behaved solution for charged fluid sphere. Pant *et al.* [55], Pant and Tewari [56], Fuloria *et al.* [57] gave charge analogue of Heintzmann, Adler, Durgapal's relativistic exact solution respectively. Gupta and Maurya [58] gave charge analogue of Durgapal and Fuloria superdense star. Bijalwan and Gupta [59], Gupta and Kumar [60] gave charge analogue of Schwarzschild's interior solution.

In this paper, we present a new solution of Einstein-Maxwell field equations in spherically symmetric coordinates which are well behaved solutions charge analogous solution of Sah and Chandra [61]. In our present study the paper consists of nine sections. In Section 2, Einstein's field equations for charged fluid sphere in canonical coordinates are given. In Section 3, gravitational binding energy of a charged fluid sphere is given. Section 4 consists of boundary conditions for well behaved solutions. New class of solution of Einstein's field equations for a charged fluid sphere in canonical coordinates is given in Section 5. Section 6 stipulates the properties of this new class of solution of Einstein-Maxwell field equations. In Section 7 the matching conditions of interior

metric of the charged fluid with the exterior metric are given. For better illustration of our physically accepted solution, the relevant physical quantities are presented by tables and figures in Section 8. Finally, some concluding remarks have been made in Section 9.

2. Field Equations for a Charged Fluid Sphere in Canonical Coordinates

The Einstein-Maxwell field equations in general relativity are given by

$$R_{\nu}^{\mu} - \frac{1}{2}Rg_{\nu}^{\mu} = -\frac{8\pi G}{c^4}T_{\nu}^{\mu} \tag{1}$$

where R_{ν}^{μ} , R , g_{ν}^{μ} and T_{ν}^{μ} , are Ricci mixed tensor, scalar curvature, metric tensor and the energy momentum tensor for fluid sphere respectively. T_{ν}^{μ} is conserved quantity such that

$$T_{\nu;\mu}^{\mu} = 0 \tag{2}$$

The energy momentum tensor for a charged fluid sphere is defined as

$$T_{\nu}^{\mu} = M_{\nu}^{\mu} + E_{\nu}^{\mu} \tag{3}$$

Here M_{ν}^{μ} is the part of the energy momentum tensor due to matter distribution of the system and, for a perfect fluid distribution, it is given by

$$M_{\nu}^{\mu} = (\rho c^2 + p)u^{\mu}u_{\nu} - pg_{\nu}^{\mu} \tag{4}$$

where ρ and p are the density and isotropic pressure of the fluid element measured locally in its proper reference frame. The density ρ gives total matter energy in proper volume V as

$$c^2M_p = \int_V \rho c^2 dV \tag{5}$$

u^{μ} is element's time-like four-velocity vector such that

$$g_{\nu}^{\mu}u^{\mu}u^{\nu} = 1 \tag{6}$$

E_{ν}^{μ} is the part of energy momentum tensor due to electromagnetic character of matter within the fluid sphere and is defined by

$$E_{\nu}^{\mu} = \frac{1}{4\pi} \left[-F^{\mu\lambda}F_{\nu\lambda} + \frac{1}{4}\delta_{\nu}^{\mu}F^{\lambda l}F_{\lambda l} \right] \tag{7}$$

where the electromagnetic tensor $F^{\mu\nu}$ satisfies Mexwells equations

$$F_{\mu\nu,\lambda} + F_{\nu\lambda,\mu} + F_{\lambda\nu,\mu} = 0 \tag{8}$$

$$\frac{\partial}{\partial x^{\nu}} \left\{ (-g)^{\frac{1}{2}} F^{\mu\nu} \right\} = 4\pi (-g)^{\frac{1}{2}} J^{\mu} \tag{9}$$

J^{μ} is the 4-current density for a fluid of null charge conductivity and a conserved quantity such that

$$J_{;\nu}^{\mu} = 0 \tag{10}$$

and is given by

$$J^\mu = \sigma u^\mu \quad (11)$$

σ being the charge density of the fluid element and gives the total charge contained in proper volume V as

$$Q = \int_V \sigma dV \quad (12)$$

The total non gravitational energy in a proper volume V is given by

$$E_p = \int_V T_0^0 dV \quad (13)$$

We consider a static spherically symmetric charged perfect fluid distribution. The interior space-time metric for spherically symmetric fluid distribution in canonical coordinate is given by

$$ds^2 = A^2 dt^2 - B^2 dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (14)$$

where A and B are functions of r only.

The electrostatic field is described by the only non-singular components F^{10} , $F^{01} (= -F^{10})$ of $F^{\mu\nu}$.

In view of (8), (9) and (10) we obtain

$$F^{01} = \frac{Qc}{r^2 AB} \quad (15)$$

where Q stands for the total charge contained within the sphere of radius r and is given as

$$Q = \int_0^r 4\pi r^2 B \sigma dr \quad (16)$$

In view of the metric (14) and energy momentum tensor (3), the field Equation (1) gives

$$\frac{8\pi G}{c^4} p = \frac{1}{B^2} \left(\frac{2A'}{rA} + \frac{1}{r^2} \right) - \frac{1}{r^2} + \frac{GQ^2}{c^4 r^4} \quad (17)$$

$$\frac{8\pi G}{c^4} p = \frac{1}{B^2} \left(\frac{A''}{A} + \frac{A'}{rA} - \frac{B'}{rB} - \frac{A'B'}{AB} \right) - \frac{GQ^2}{c^4 r^4} \quad (18)$$

$$\frac{8\pi G}{c^2} \rho = \frac{1}{B^2} \left(\frac{2B'}{rB} - \frac{1}{r^2} \right) + \frac{1}{r^2} - \frac{GQ^2}{c^4 r^4} \quad (19)$$

In view of Equation (17) and Equation (18), pressure isotropy gives

$$\frac{2GQ^2}{c^4 r^4} = \frac{1}{B^2} \left(\frac{A''}{A} - \frac{A'}{rA} - \frac{A'B'}{AB} - \frac{B'}{rB} - \frac{1}{r^2} \right) + \frac{1}{r^2} \quad (20)$$

The charge conservation Equation (10) is identically satisfied whereas (2) for energy-momentum gives rise to the following surviving equation

$$p' = -(\rho c^2 + p) \frac{A'}{A} + \frac{QQ'}{4\pi r^4} \quad (21)$$

which is clearly contained in the field Equations (17) to (20).

We have three equations to determine five unknown functions A , B , p , ρ and σ . Thus, we find a two degree of arbitrariness in the general relativistic problem of the

electrically charged fluid ball, and to obtain a solution we can always choose one of these functions arbitrarily and correlate other to this by certain relation. The volume field surrounding the charged sphere is described by the Reissner-Nordstrom field

$$ds^2 = \left(1 - \frac{2GM}{c^2 r} + \frac{G\epsilon^2}{c^4 r^2}\right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 r} + \frac{G\epsilon^2}{c^4 r^2}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (22)$$

$$F^{01} = -F^{10} = \frac{\epsilon}{r^2} \quad (23)$$

where M and E are constants. We observe that whereas in Schwarzschild's field that total energy Mc^2 is constant, in case of Reissner-Nordstrom field the total energy $\left(Mc^2 - \frac{\epsilon^2}{r}\right)$ increases as r increases. At large distances the Reissner-Nordstrom field

approximates to Schwarzschild's field. The junction of interior and exterior field over the boundary $r = r_\Sigma$ of the sphere is governed by the junction conditions due to Darmois (1927), viz. the continuity of the first and second fundamental forms across the boundary, which imply the continuity of $g_{\mu\nu}$ and the fluid pressure across the boundary. For the junction of electromagnetic field it is sufficient to consider the continuity of $F^{\mu\nu}$ (and not its first derivatives) across the boundary. In view of (16) and (23) the continuity of $F^{\mu\nu}$ gives

$$\epsilon = Q_\Sigma$$

Thus the constant ϵ measures the total charge contained within the ball.

Also we have

$$\frac{1}{B^2} = 1 - \frac{2G}{c^2} \frac{m(r)}{r} - \frac{2G}{c^4} \frac{\mu(r)}{r} \quad (24)$$

where

$$m(r) = \int 4\pi\rho r^2 dr \quad (25)$$

and

$$\mu(r) = \int \frac{Q^2}{2r^2} dr = \int 4\pi r^2 \frac{Q^2}{8\pi r^4} dr \quad (26)$$

The continuity of $g^{\mu\nu}$ over the boundary $r = r_\Sigma$ demands

$$m(r_\Sigma) + \frac{\mu(r_\Sigma)}{c^2} = M - \frac{1}{2c^2} \frac{\epsilon^2}{r_\Sigma} \quad (27)$$

Thus, in general, the constant M can not be identified with Euclidean mass of the sphere as against the case of uncharged sphere in which case $M = m(r_\Sigma)$. In view of (27) we have $M > m(r_\Sigma)$ we observe what the distribution of total energy within the charged sphere as measured by an electrically neutral test particle close to the boundary of the star is similar to that within a sphere of electrically neutral perfect fluid. Equation (27) can be rewritten as,

$$Mc^2 = \int_0^{r_\Sigma} \rho c^2 \cdot 4\pi r^2 dr + \int_0^{r_\Sigma} \frac{Q^2}{8\pi r^4} \cdot 4\pi r^2 dr + \int_\infty^{r_\Sigma} \frac{Q^2}{8\pi r^4} \cdot 4\pi r^2 dr \quad (28)$$

Thus three physical quantities contribute to Mc^2 viz. 1) the total matter energy within the ball distributed as if the geometry is Euclidean, 2) the total electromagnetic energy within the ball as if the geometry is Euclidean 3) the total electromagnetic energy distributed over the entire exterior space-time as if the geometry there too is euclidean.

The gravitational redshift of massive spherically symmetric ball is

$$1 + Z = \frac{1}{g_{00}^2} \quad (29)$$

which gives central (Z_0) and surface (Z_Σ) gravitational redshifts

$$Z_0 = \frac{c}{A} - 1 \quad (30)$$

and

$$Z_\Sigma = \left(1 + \frac{rB'}{B}\right)^{-1} - 1 \quad (31)$$

3. Gravitational Binding Energy of a Charged Sphere

In view of (11) the total non gravitational energy of a charged sphere is given

$$E_p = \int_0^{r_\Sigma} \left(\rho c^2 + \frac{Q^2}{8\pi r^4} \right) \cdot 4\pi r^2 B dr \quad (32)$$

Also the total energy of the ball measured by an electrically neutral test particle close to the boundary is given by

$$E_\Sigma = Mc^2 - \frac{1}{2} \frac{\epsilon^2}{r_\Sigma} \quad (33)$$

Clearly the difference $E_p - E_\Sigma$ is the expression for the gravitational energy of the charged fluid sphere as measured by an observer close to the boundary. The negative of this quantity is the gravitational binding energy of the system. In Newtonian limit we obtain from (24), (32) and (33)

$$E_p - E_\Sigma = \int_0^{r_\Sigma} \frac{4\pi G}{c^4} r \left(\rho c^2 + \frac{Q^2}{8\pi r^4} \right) [m(r)c^2 + \mu(r)] dr \quad (34)$$

For a particle at large distances from the object total energy E_∞ approximates to Mc^2 . The difference $\frac{\epsilon^2}{2r_\Sigma}$ between the two energy measurements is the classical expression for the energy of vacuum electrostatic field surrounding a sphere of charge ϵ and radius r_Σ . In the Reissner-Nordstrom field energy is distributed in the entire space time. As such the set of field Equations (17) to (19) can be solved under two given relations or assumptions. Physically speaking, one of them has to be the equation of state for fluid and another a law for the distribution of charge with the sphere. The non-singular solution due to Naduka [42] [43] and that due to Whitman and Burch [44] follow the charge distribution given by $\sigma = \text{constant}$. B^{-1} .

4. Boundary Conditions for Well Behaved Solution

For well behaved nature of the solution in isotropic coordinates, the following conditions should be satisfied:

1) The solution should be free from geometrical and physical singularities. Metric potentials A and B must be non-zero positive finite for free from geometrical singularities while central pressure, central density, should be positive and finite or $\rho_0 > 0$ and $p_0 > 0$ for free from physical singularities.

2) The solution should have maximum positive values of pressure and density at the center and monotonically decreasing towards the surface of fluid object *i.e.*

i) $\left(\frac{dp}{dr}\right)_0 = 0$ and $\left(\frac{d^2p}{dr^2}\right)_0 < 0$ such that the pressure gradient, $\frac{dp}{dr}$ is negative for $0 \leq r \leq r_\Sigma$.

ii) $\left(\frac{d\rho}{dr}\right)_0 = 0$ and $\left(\frac{d^2\rho}{dr^2}\right)_0 < 0$ such that the density gradient, $\frac{d\rho}{dr}$ is negative for $0 \leq r \leq r_\Sigma$.

3) At boundary pressure, p must vanish.

4) The pressure, p , and density ρ should be positive.

5) Solution should have positive value of pressure-density ratio which must be less than 1 (weak energy condition) and less than $\frac{1}{3}$ (strong energy condition) throughout within the fluid object and monotonically decreasing as well (Pant and Negi [51]).

6) The casualty condition must be satisfied for this velocity of sound should be less than that of light throughout the model *i.e.* $0 \leq \sqrt{\frac{dp}{c^2 d\rho}} < 1$. The velocity of sound should be monotonically decreasing towards the surface and increasing with the increase of density *i.e.* $\frac{d}{dr}\left(\frac{dp}{d\rho}\right) < 0$ or $\left(\frac{d^2p}{d\rho^2}\right) > 0$. In this context it is worth mentioning that the equation of state at ultra-high distribution has the property that the sound speed is decreasing outwards.

7) For realistic matter, the adiabatic index $\gamma > 1$ *i.e.* $\frac{p}{\rho} < \frac{dp}{d\rho}$, everywhere within the ball.

8) The red shift at the center z_0 and at the boundary should be positive, finite and monotonically decreasing in nature with the increase of r .

Under these conditions, we have to assume the one of the gravitational potential component in such a way that the field Equation (1) can be integrated and solution should be well behaved.

5. New Class of Well Behaved Solution

We present the following general analytic solution of the field Equations (17) to (20).

$$A = b + d(1 - ar^2)^{n+1} \quad (35)$$

$$B = (1 - ar^2)^n \quad (36)$$

The isotropic pressures, matter-energy density, charge, charge density and red shift of charged fluid ball are given by

$$\frac{8\pi G}{c^4} p = \left\{ \frac{-4a(n+1)d}{(1-ar^2)^n [b+d(1-ar^2)^{n+1}]} + \frac{1+(2n-1)ar^2 - (1-ar^2)^{2n+1}}{2r^2(1-ar^2)^{2n+1}} \right\} \quad (37)$$

$$\frac{8\pi G}{c^2} \rho = \left\{ \frac{(1-ar^2)^{2n+1} - (10n-1)ar^2 - 1}{2r^2(1-ar^2)^{2n+1}} \right\} \quad (38)$$

$$\frac{2GQ^2}{c^4 r^4} = \frac{(1-ar^2)^{2n+1} + (2n+1)ar^2 - 1}{r^2(1-ar^2)^{2n+1}} \quad (39)$$

$$\frac{8\pi\sqrt{G}\sigma}{c^2} = \frac{\sqrt{2} \left\{ (1-ar^2)^{2n+2} + (4n^2-1)a^2r^4 + 2(n+1)ar^2 - 1 \right\}}{r^2(1-ar^2)^{\frac{4n+3}{2}} \left\{ (1-ar^2)^{2n+1} + (2n+1)ar^2 - 1 \right\}^{\frac{1}{2}}} \quad (40)$$

$$Z = \frac{(c-b) - d(1-ar^2)^{n-1}}{b - d(1-ar^2)^{n-1}} \quad (41)$$

Here a , b , and d are arbitrary constants.

In order to construct a new relativistic model, we assume $n = -2$ then we have

$$A = b + d(1-ar^2)^{-1} \quad (42)$$

$$B = (1-ar^2)^{-2} \quad (43)$$

$$\frac{8\pi G}{c^4} p = \frac{a}{2} \left\{ \frac{8d(1-ar^2)^3}{b(1-ar^2)+d} - 8 + 18ar^2 - 16a^2r^4 + 5a^3r^6 \right\} \quad (44)$$

$$\frac{8\pi G}{c^2} \rho = \frac{a}{2} (24 - 66ar^2 + 64a^2r^4 - 21a^3r^6) \quad (45)$$

$$\frac{2GQ^2}{c^4} = a^2r^6 (6 - 8ar^2 + 3a^2r^4) \quad (46)$$

$$\frac{8\pi\sqrt{G}\sigma}{c^2} = a \frac{(1-ar^2)^2 (36 - 64ar^2 + 30a^2r^4)}{(12 - 16ar^2 + 6a^2r^4)^{\frac{1}{2}}} \quad (47)$$

$$Z = \frac{(c-b)(1-ar^2) - d}{b(1-ar^2) + d} \quad (48)$$

Here a , b , and d are arbitrary constants.

The variation in pressure, density, charge, charge density and red shift with radial distance are given as

$$\frac{8\pi G}{c^4} p' = -a^2 r \left[\frac{8d(1-ar^2)^2 \{2b(1-ar^2) + 3d\}}{\{b(1-ar^2) + d\}^2} - 18 + 32ar^2 - 15a^2r^4 \right] \tag{49}$$

$$\frac{8\pi G}{c^2} \rho' = -a^2 r (66 - 128ar^2 + 63a^2r^4) \tag{50}$$

$$\frac{\sqrt{G} Q'}{c^2} = \frac{ar^2(18 - 32ar^2 + 15a^2r^4)}{(12 - 16ar^2 + 6a^2r^4)^{\frac{1}{2}}} \tag{51}$$

$$Z' = \frac{-2cadr}{\{b(1-ar^2) + d\}^2} \tag{52}$$

6. Properties of the Solution

For real values of metric potentials A and B , $ar^2 < 1 \forall r$. **Figure 1** shows that the metric potentials A and B are positive at the center which are slightly and monotonically increasing with r for suitable choice of constants a, b , and d . For the positive central value of $A(b+d) > 0$,

The central value of p , ρ and σ are given as,

$$\frac{8\pi G}{c^4} p_0 = -\frac{(4ab)}{(b+d)} \tag{53}$$

$$\frac{8\pi G}{c^2} \rho_0 = 12a \tag{54}$$

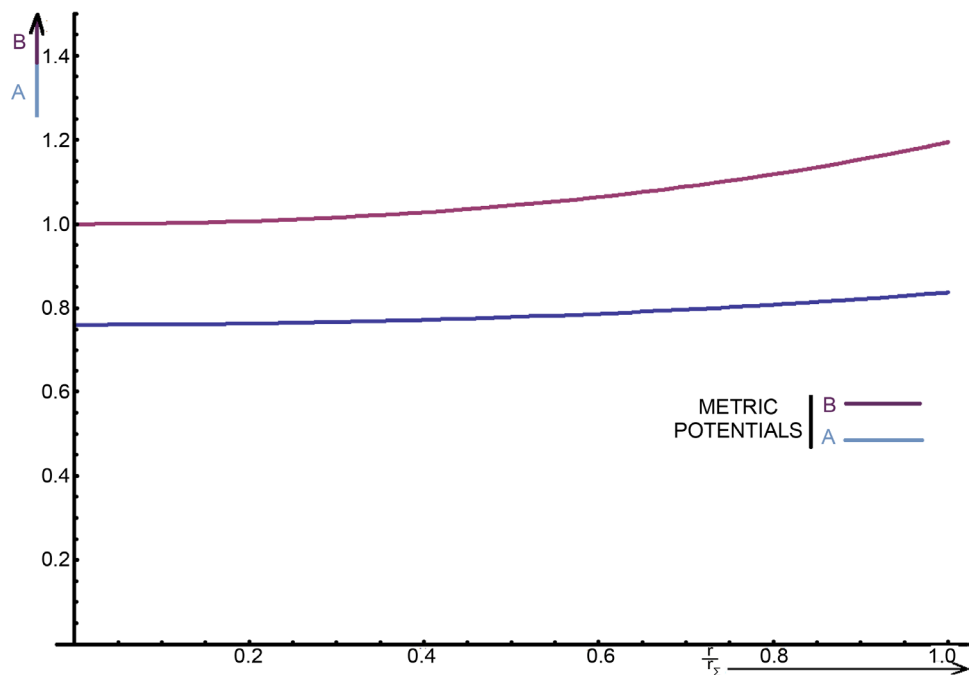


Figure 1. Variation of metric potentials with $\frac{r}{r_0}$.

$$Q_0 = 0 \quad (55)$$

$$\frac{8\pi\sqrt{G}}{c^2}\sigma_0 = 6\sqrt{3}a \quad (56)$$

$$Z_0 = \frac{c-b-d}{b+d} \quad (57)$$

It is clear from Equation (53) to Equation (57) that for positive central values of physical quantities p_0 , ρ_0 , Q_0 , σ_0 and Z_0 are positive if $a > 0$, $b+d < c$ and $-1 < \frac{b}{d} < 0$. In view of Equations (49) and (50), the variation in the pressure and density with the distance from the center of fluid ball are identically zero at the center.

At the center of fluid ball the second order derivatives of pressure and density with respect to radial distance from the center of fluid ball are

$$\frac{8\pi G}{c^4}(p'')_0 = 2a^2 \frac{(9b^2 + 10bd - 3d^2)}{(b+d)^2} \quad (58)$$

The pressure is maximum at the center if $(p'')_0 < 0$ i.e. $\frac{-5-\sqrt{52}}{9} < \frac{b}{d} < \frac{-5+\sqrt{52}}{9}$.

$$\frac{8\pi G}{c^4}(\rho'')_0 = -66a^2 \quad (59)$$

The density is maximum at the center for all constants as $(\rho'')_0 < 0$.

The central equation of state

$$\left(\frac{p}{c^2\rho}\right)_0 = \alpha = -\frac{b}{3(b+d)} \quad (60)$$

α must satisfies the condition $0 < \alpha \leq 1$ which demands $b < 0$ and $-1 < \frac{b}{d} < 0$.

The central value of $\frac{dp}{c^2 d\rho}$ is given by

$$\left(\frac{dp}{c^2 d\rho}\right)_0 = -\frac{(9b^2 + 10bd - 3d^2)}{33(b+d)^2} \quad (61)$$

The causality condition at the center $0 < \left(\frac{dp}{c^2 d\rho}\right)_0 \leq 1$ gives

$$\frac{-5-\sqrt{52}}{9} < \frac{b}{d} < \frac{-5+\sqrt{52}}{9}.$$

It is found that ρ and σ fall monotonically from their maximum positive values at the center up to non negative values at the boundary (Figure 2) and p falls monotonically from it's maximum positive values at the center up to zero value at the boundary while charge increases from zero at the center to maximum positive value at the boundary (Figure 3) for different values of the parameters a , b , d satisfying

$$a > 0, \quad b < 0, \quad d > 0, \quad c > b+d > 0 \quad \text{and} \quad -1 < \frac{b}{d} < 0 \quad (62)$$

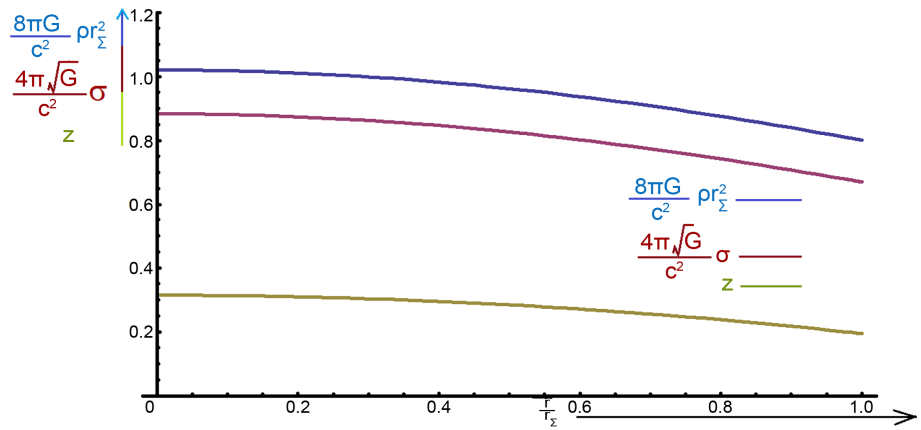


Figure 2. Variation of energy density, charge density and red shift with $\frac{r}{r_\Sigma}$.

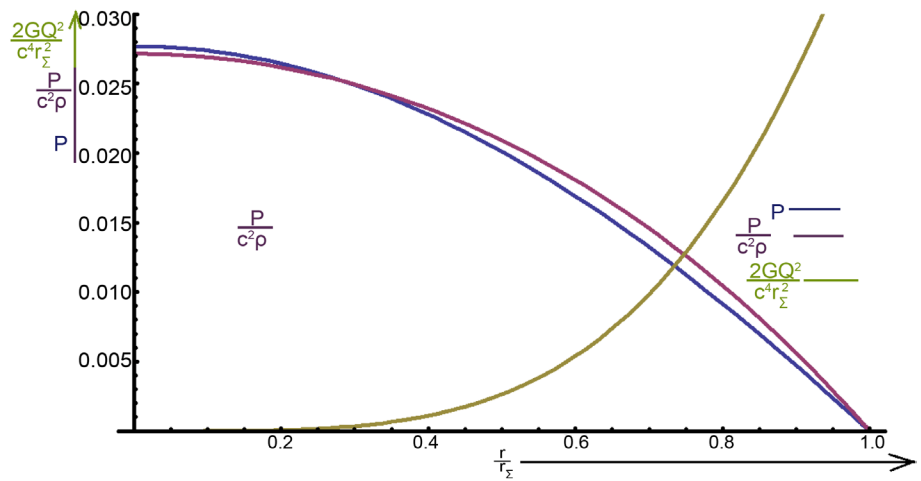


Figure 3. Variation of pressure, pressure density ratio and charge with $\frac{r}{r_\Sigma}$.

Figure 4 shows that speed of sound is less than speed of light *i.e.* $0 < \frac{dp}{c^2 d\rho} \leq 1$ and the ratio of speeds of sound and light $\sqrt{\frac{dp}{c^2 d\rho}}$ falls monotonically from center to the boundary of the fluid ball.

7. Matching Conditions of Boundary

The solution so obtained are to be matched over the pressure free boundary of fluid sphere smoothly with the Reissner-Nordstrom metric:

$$ds^2 = \left(1 - \frac{2GM}{c^2 r} + \frac{G\epsilon^2}{c^4 r^2}\right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 r} + \frac{G\epsilon^2}{c^4 r^2}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (63)$$

which requires the continuity of A^2 and B^2 across the boundary $r = r_\Sigma$ and $(p)_{r_\Sigma} = 0$; $Q_{r_\Sigma} = \epsilon$. Thus

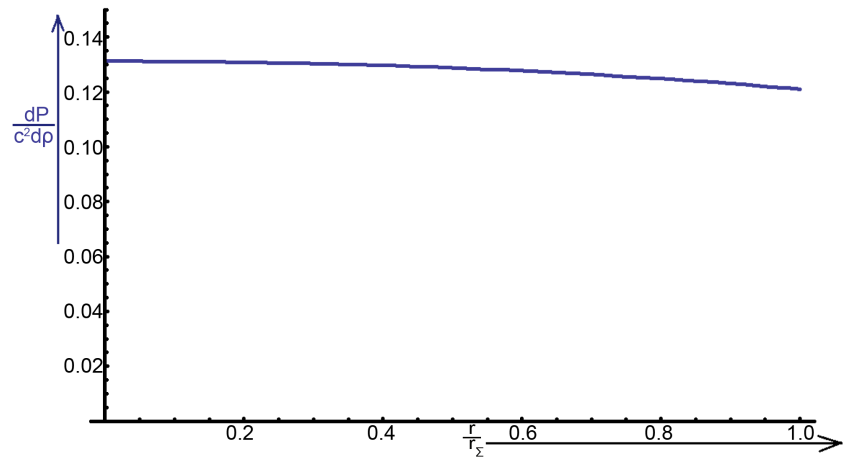


Figure 4. Variation of adiabatic velocity of sound with $\frac{r}{r_s}$.

$$a = \frac{1}{r_s^2} \left(1 - X^{\frac{1}{4}} \right); \quad 0 < X < 1 \tag{64}$$

$$b = \frac{\left(3X^{\frac{3}{4}} - X^{\frac{1}{2}} - X^{\frac{1}{4}} - 1 \right) c}{8X^{\frac{1}{4}}} \tag{65}$$

$$d = \frac{c}{8} \left(5X^{\frac{3}{4}} + X^{\frac{1}{2}} + X^{\frac{1}{4}} + 1 \right) \tag{66}$$

where $X = 1 - 2S_p + \frac{GQ_s^2}{c^4 r_s^2}$; $S_p = \frac{GM}{c^2 r_s}$, *Schwarzschild parameter*.

8. Tables of Numerical Values of Physical Quantities and Their Graphs

In view of Equations (64) to (66) the values of X , b , d and $\frac{b}{d}$ are 0.701, $-0.062c$, $0.0823c$ and -0.0753 respectively and the value of $ar_s^2 = 0.085$ for $S_p = 0.16$ and surface density $2 \times 10^{14} \text{ gmcm}^{-3}$. For better illustration of our physically accepted solution, the relevant physical quantities are presented by means of **Table 1**, **Table 2** and **Figures 1-4** for these constants.

In order to construct a super dense star model, we prescribe the surface density of the star as $2 \times 10^{17} \text{ kgm}^{-3}$ and the values of constants a , b and d can be evaluated for different values of S_p for describing the well behaved solutions. Thus a compact star model can be constructed by finding mass and radius corresponding to assumed surface density. The variation in the mass and radius with schwarzschild parameters for our model of compact star is tabulated in **Table 3** and the variation of surface charge, Surface Density, Schwarzschild Parameter and X with ar_s^2 showing different models is shown in **Figure 5**.

Table 1. $\frac{A}{c}$, B , ρ , p , and $\frac{p}{c^2\rho}$ for $ar_\Sigma^2 = 0.085$.

<i>S.N.</i>	$\frac{r}{r_\Sigma}$	$\frac{A}{c}$	B	$\frac{8\pi}{c^2}G\rho r_\Sigma^2$	$\frac{8\pi}{c^4}Gpr_\Sigma^2$	$\frac{p}{c^2\rho}$
1	0.0	0.76085	1.00000	1.02000	0.02768	0.02715
2	0.1	0.76155	1.00170	1.01762	0.02737	0.02690
3	0.2	0.76365	1.00683	1.01048	0.02644	0.02617
4	0.3	0.76719	1.01547	0.99862	0.02490	0.02493
5	0.4	0.77219	1.02776	0.98210	0.02277	0.02319
6	0.5	0.77871	1.04389	0.96099	0.02009	0.02091
7	0.6	0.78682	1.06413	0.93541	0.01690	0.01806
8	0.7	0.79661	1.08881	0.90546	0.01323	0.01462
9	0.8	0.80818	1.11837	0.87129	0.00915	0.01050
10	0.9	0.82169	1.15335	0.83303	0.00471	0.00565
11	1.0	0.83729	1.19442	0.79085	0.00000	0.00000

Table 2. $\frac{dp}{c^2d\rho}$, $\frac{\rho dp}{p d\rho}$, σ , Q , and Z for $ar_\Sigma^2 = 0.218$.

<i>S.N.</i>	$\frac{r}{r_\Sigma}$	$\frac{1}{c^2} \frac{dp}{d\rho}$	$\frac{\rho}{p} \frac{dp}{d\rho}$	$\frac{8\pi\sqrt{G}\sigma}{c^2}$	$\frac{GQ^2}{c^4r^2}$	Z
1	0.0	0.13119	4.83330	0.88334	0.00000	0.31432
2	0.1	0.13099	4.86945	0.88107	0.00021	0.31311
3	0.2	0.13037	4.98243	0.87412	0.00085	0.30948
4	0.3	0.12935	5.18736	0.86295	0.00188	0.30345
5	0.4	0.12791	5.51540	0.84794	0.00331	0.29501
6	0.5	0.12605	6.02752	0.82972	0.00512	0.28416
7	0.6	0.12376	6.84985	0.80905	0.00728	0.27093
8	0.7	0.12104	8.28097	0.78683	0.00979	0.25532
9	0.8	0.11788	11.22241	0.76402	0.01265	0.23733
10	0.9	0.11427	20.20590	0.74168	0.01582	0.21700
11	1.0	0.11020	∞	0.72088	0.01943	0.19433

9. Conclusion

We have given a new solution for spherically symmetric isotropic charged fluid ball. It has been observed that the physical parameters pressure, density, adiabatic speed of sound and redshift are positive at the centre and within the limit of realistic state equation and monotonically decreasing and the causality condition is obeyed throughout the fluid ball. The charge and electric field intensity are zero at the center and monotonically increasing towards the intervening surface. Thus, the solution is well behaved for all values of Schwarzschild parameter S_p within the charged fluid ball. Our solution is useful to construct the models of compact star like Strange star family,

Table 3. Variation in ar_{Σ}^2 , $\frac{GQ^2}{c^4 r_{\Sigma}^2}$, S_p , X , $\frac{8\pi}{c^2} G\rho r_{\Sigma}^2$, r_{Σ} (inKm) and $\frac{M}{M_{\odot}}$ with Schwarzschild parameter S_p .

$S.N.$	ar_{Σ}^2	$\frac{GQ^2}{c^4 r_{\Sigma}^2}$	S_p	X	$\frac{8\pi}{c^2} G\rho r_{\Sigma}^2$	r_{Σ} (inKm)	$\frac{M}{M_{\odot}}$
1	0.080	0.017	0.150	0.7164	0.765	14.330	1.451
2	0.086	0.019	0.158	0.6978	0.810	14.750	1.573
3	0.089	0.021	0.167	0.6887	0.831	14.940	1.684
4	0.092	0.022	0.171	0.6797	0.851	15.118	1.745
5	0.096	0.024	0.178	0.6678	0.873	15.312	1.840
6	0.101	0.026	0.186	0.6531	0.904	15.582	1.956
7	0.106	0.029	0.195	0.6387	0.936	15.855	2.087
8	0.110	0.031	0.202	0.6274	0.961	16.065	2.190
9	0.115	0.034	0.209	0.6134	0.988	16.289	2.290
10	0.120	0.036	0.218	0.5996	1.017	16.527	2.432
11	0.126	0.040	0.228	0.5835	1.048	16.777	2.582
12	0.130	0.042	0.235	0.5728	1.070	16.952	2.689
13	0.134	0.045	0.242	0.5624	1.091	17.118	2.796
14	0.140	0.049	0.251	0.5470	1.119	17.336	2.937
15	0.145	0.052	0.259	0.5344	1.141	17.505	3.060

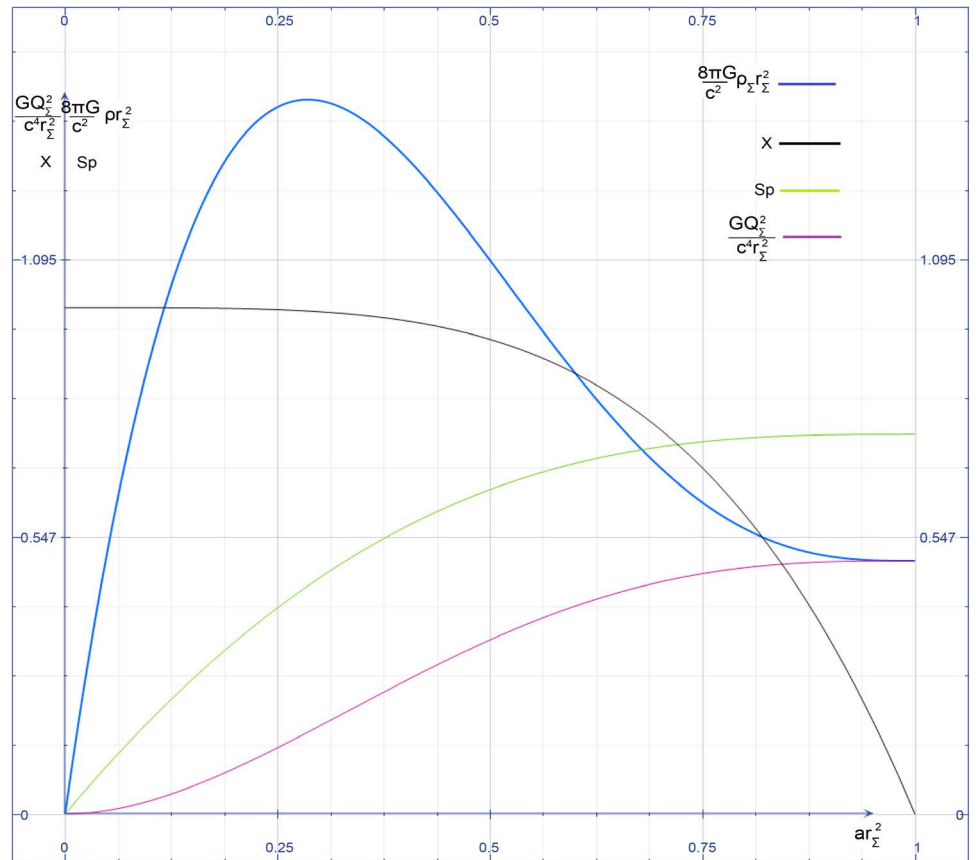


Figure 5. Variation of surface charge, surface density, Schwarzschild parameter and X with ar_{Σ}^2 showing different models.

Neutron star and many more. We have discussed a model of massive neutron star having mass $1.58 M_{\odot}$ and radius 14.66 km with surface density $2 \times 10^{17} \text{ Kg m}^{-3}$ and central density $2.6 \times 10^{17} \text{ kg m}^{-3}$. The central pressure of neutron star is $6.2 \times 10^{32} \text{ N/m}^2$ while the surface pressure of the star is zero. The electric field intensity at the center is zero and at the surface it comes $2.37 \times 10^{31} \text{ C}^2 \text{ m}^{-2}$. **Table 3** shows that we can construct different models for neutron star having mass lies between $1.45 M_{\odot}$ and $3.06 M_{\odot}$ for different values of S_p , ranging from 0.15 to 0.259. The solution reduces to Schwarzschild interior solution for $n = -1/2$ for electrically neutral fluid ball.

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