International Journal of Astronomy and Astrophysics, 2016, 6, 99-104
Published Online March 2016 in SciRes. http://www.scirp.org/journal/ijaa
http://dx.doi.org/10.4236/ijaa.2016.61007

Einstein Rosen Mesonic Perfect Fluid Cosmological Model with Time Dependent Λ-Term

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Received 12 October 2015; accepted 25 March 2016; published 29 March 2016

Abstract

Mesonic perfect fluid solutions are found in general relativity with the aid of Einstein's Rosen cylindrically symmetric space time. A static vacuum model and a non-static cosmological model corresponding to perfect fluid are investigated. The cosmological term Λ is found to be a decreasing function of time which is supported by the result found from recent type Ia Supernovae observations. The various physical and geometrical features of the model are discussed.

Keywords

General Relativity, Perfect Fluid, Time-Dependent Term Λ

1. Introduction

Theory of general relativity (Einstein 1916) has served as basis for the study of cosmological models of universe. The cosmological term Λ has been introduced in 1917 by Einstein to modify his own equation of general relativity. Now this Λ-term remains a focal point of interest in modern theories. In 1930s distinguished cosmologists, A. S. Eddington and Abbey Georges Lemaitre felt that introduction of Λ-term has attractive features in cosmology and models, so it should be discussed deeply. Moreover models with cosmological time-dependent term-Λ are becoming popular as they help to solve the cosmological constant problem in natural way. The generalized Einstein's theory of gravitation with time-dependent G and Λ has been proposed by Lau [1]. The possibility of variable G and Λ in Einstein's theory has also been studied by Dersarkissian [2].

To study the nature of scalar field without mass parameters interacting with perfect fluid in Einstein's Rosen space time is a subject of interest due to its significant role in the description of the universe at the early stages of evolution. Patel [3] obtained the static and nonstatic plane symmetric solutions of the field equations in pres-

How to cite this paper: Mete, V.G. and Elkar, V.D. (2016) Einstein Rosen Mesonic Perfect Fluid Cosmological Model with Time Dependent Λ-Term. *International Journal of Astronomy and Astrophysics*, **6**, 99-104.
http://dx.doi.org/10.4236/ijaa.2016.61007

ence of zero mass scalar field. Singh and Deo [4] considered Robertson-Walker metric and investigated the problem of zero mass scalar field.

Recently many authors like Tsagas and Maartens [5], Sahni and Starboinsky [6], Peeble [7], Padmanabhan [8], Vishwakarma [9]-[14], Pradhan *et al.* [15] [16], Sahu and Panigrahi [17], Sahu and Mohapatra [18] motivate us to study the cosmological models involved with Λ-term. Mohanty *et al.* [19] obtained a class of exact solutions of Einstein's field equations with attractive massive scalar field in LRS Bianchi type-I space time. Mohanty and Mishra [20] have studied the feasibility of Bianchi type-VIII and IX space time with a time-dependent gauge function and a matter field in the term of perfect fluid. Mishra [21] has constructed the non-static plane symmetric Zeldovich fluid model with a time-dependent gauge function.

Very recently Adhav *et al.* [22] have studied cylindrically symmetric Einstein Rosen cosmological model with wet dark fluid (WDF) in general relativity. Katore *et al.* [23] have investigated cylindrically symmetric Einstein Rosen space time with bulk viscosity and zero mass scalar field in Lyra geometry. Bivudutta Mishra *et al.* [24] have studied the perfect fluid distribution in the scale invariant theory of gravitation. Katore *et al.* [25] have investigated Einstein Rosen inflationary universe in presence of massless scalar field with flat potential.

In this paper we consider the cylindrically symmetric space time in mesonic perfect fluid with time-dependent Λ-term in general theory of relativity. A static vacuum model and a non-static cosmological model are presented and studied in detail.

2. The Metric and Field Equation

We consider the nonstatic cylindrically symmetric Einstein Rosen metric

$$\mathrm{d}s^2 = \mathrm{e}^{2\alpha-2\beta}\left(\mathrm{d}t^2-\mathrm{d}r^2\right)-r^2\mathrm{e}^{-2\beta}\mathrm{d}\phi^2-\mathrm{e}^{2\beta}\mathrm{d}z^2 \tag{1}$$

where α and β are both the functions of r and t only.

We denote the coordinates r,ϕ,z, and t as x^1,x^2,x^3,x^4 respectively.

The Einstein's field equations with the cosmological term Λ are given by

$$R_{ij}-\frac{1}{2}Rg_{ij}+\Lambda g_{ij}=-8\pi\left(T_{ij}^{p}+T_{ij}^{m}\right) \tag{2}$$

where

$$T_{ij}^{p}=\left(p+\rho\right)u_iu_j-pg_{ij} \tag{3}$$

$$g_{ij}u^iu^j=1 \tag{4}$$

and

$$T_{ij}^{m}=v_iv_j-\frac{1}{2}g_{ij}v_kv^k \tag{5}$$

are respectively the energy momentum tensors for the perfect fluid and massless scalar field. The massless scalar field satisfies the Klein-Gordan wave equation

$$g_{ij}v_{;ij}=0. \tag{6}$$

Here ρ,p,u,v and Λ are respectively the energy density, pressure, four velocity vector of the fluid, scalar mesonic field and cosmological constant. Hereafter the semicolon (;) denotes covariant differentiation.

Using commoving coordinate system, the set of field Equation (2) for the metric (1) reduces to the following forms

$$\frac{1}{\mathrm{e}^{2\alpha-2\beta}}\left(\beta_1^2+\beta_4^2-\frac{\alpha_1}{r}\right)-\Lambda=-8\pi\left[p+\frac{1}{2}\left(\frac{v_1^2}{\mathrm{e}^{2\alpha-2\beta}}+\frac{v_4^2}{\mathrm{e}^{2\alpha-2\beta}}\right)\right] \tag{7}$$

$$\frac{1}{\mathrm{e}^{2\alpha-2\beta}}\left(\alpha_{44}-\alpha_{11}-\beta_1^2+\beta_4^2\right)-\Lambda=-8\pi\left[p+\frac{1}{2}\left(\frac{-v_1^2}{\mathrm{e}^{2\alpha-2\beta}}+\frac{v_4^2}{\mathrm{e}^{2\alpha-2\beta}}\right)\right] \tag{8}$$

$$\frac{1}{e^{2\alpha-2\beta}}\left(2\beta_{11}-2\beta_{44}+\frac{2\beta_1}{r}-\alpha_{11}+\alpha_{44}-\beta_1^2+\beta_4^2\right)-\Lambda=-8\pi\left[p+\frac{1}{2}\left(\frac{-v_1^2}{e^{2\alpha-2\beta}}+\frac{v_4^2}{e^{2\alpha-2\beta}}\right)\right] \tag{9}$$

$$\frac{1}{e^{2\alpha-2\beta}}\left(\beta_1^2+\beta_4^2-\frac{\alpha_1}{r}\right)+\Lambda=-8\pi\left[\rho+\frac{1}{2}\left(\frac{v_1^2}{e^{2\alpha-2\beta}}+\frac{v_4^2}{e^{2\alpha-2\beta}}\right)\right] \tag{10}$$

and

$$2\beta_1\beta_4-\frac{\alpha_4}{r}=-8\pi v_1 v_4. \tag{11}$$

The Klein-Gordon Equation (6) for the metric (1) yields

$$v_{44}-\frac{v_1}{r}-v_{11}=0 \tag{12}$$

Equations (7)-(12) are highly nonlinear partial differential equations and hence it is very difficult to solve them, as there exists no standard method to derive their solution.

Here we consider two particular physical important cases:

1) static vacuum model and 2) non-static cosmological model.

Further to avoid the mathematical complexities, we consider scalar field v to be the functions of *t* only and cosmological constant Λ is depending on time *t*.

2.1. Static Vacuum Model

In this case we consider $p=\rho=0$ and α,β are functions of *r* only.

Therefore, in this case the field Equations (7)-(12) reduces the following set of equations

$$\beta_1^2-\frac{\alpha_1}{r}=0 \tag{13}$$

$$\alpha_{11}+\beta_1^2=0 \tag{14}$$

$$2\beta_{11}+\frac{2\beta_1}{r}-\alpha_{11}-\beta_1^2=0 \tag{15}$$

$$v_{44}=0. \tag{16}$$

The solutions of the field equations are given by

$$\alpha=c_1\log r+c_2,\ \beta=c_3\log r+c_4 \tag{17}$$

where c_i, $i=1,2,3,4$ are integrating constants.

After a suitable choice of coordinates, Einstein-Rosen cylindrically symmetric metric (1) can be written as

$$ds^2=r^{2(A-B)}\left(dt^2-dr^2\right)-r^{2(1-B)}d\phi^2-r^{2B}dz^2. \tag{18}$$

2.2. Non-Static Cosmological Model

Here we consider α,β are functions of *t* only. In this case the field Equations (7)-(12) reduces the following set of equations

$$\frac{1}{e^{2\alpha-2\beta}}\left(\beta_4^2\right)-\Lambda=-8\pi\left[p+\frac{1}{2}\left(\frac{v_4^2}{e^{2\alpha-2\beta}}\right)\right] \tag{19}$$

$$\frac{1}{e^{2\alpha-2\beta}}\left(\alpha_{44}+\beta_4^2\right)-\Lambda=-8\pi\left[p+\frac{1}{2}\left(\frac{v_4^2}{e^{2\alpha-2\beta}}\right)\right] \tag{20}$$

$$\frac{1}{e^{2\alpha-2\beta}}\left(-2\beta_{44}+\alpha_{44}+\beta_4^2\right)-\Lambda=-8\pi\left[p+\frac{1}{2}\left(\frac{v_4^2}{e^{2\alpha-2\beta}}\right)\right] \tag{21}$$

$$\frac{1}{e^{2\alpha-2\beta}}\left(\beta_4^2\right)+\Lambda=-8\pi\left[\rho+\frac{1}{2}\left(\frac{v_4^2}{e^{2\alpha-2\beta}}\right)\right] \tag{22}$$

$$\frac{\alpha_4}{r}=0 \tag{23}$$

and

$$v_{44}=0. \tag{24}$$

The exact solution of this equation is given by

$$\alpha=k_1,\ \beta=k_2t+k_3,\ v=k_4t+k_5 \tag{25}$$

where $k_i'^s, i=1,2,3,4,5$ are integrating constants.

Now using the equation of state

$$p=\gamma\rho,\ 0\le\gamma\le1 \tag{26}$$

we obtain the physical quantities

$$\rho=\frac{-1}{8\pi(1+\gamma)}\left[\frac{2m^2}{e^{2(mt+n)}}+\frac{8\pi l^2}{e^{2(mt+n)}}\right] \tag{27}$$

and

$$\Lambda=\frac{(1-\gamma)}{(1+\gamma)}\left(\frac{m^2}{e^{2(mt+n)}}+\frac{4\pi l^2}{e^{2(mt+n)}}\right) \tag{28}$$

where $m=k_2, n=k_1-k_3, v_4=l$ are constants.

After a suitable choice of coordinates and constants, Einstein-Rosen cylindrically symmetric metric (1) becomes

$$ds^2=e^{-2T}\left(dT^2-dr^2\right)-r^2e^{-2T}d\phi^2-e^{2T}dz^2\ . \tag{29}$$

2.3. Physical Model

Here we discuss three models corresponding to $\gamma=0,1,\frac{1}{3}$

Case-I: When $\gamma=0$ (dust Distribution)

From Equation (26), we obtain

$$p=0 \tag{30}$$

Therefore in this case the energy density and cosmological constant takes the form

$$\rho=\frac{-1}{8\pi}\left[\frac{2m^2}{e^{2(mt+n)}}+\frac{8\pi l^2}{e^{2(mt+n)}}\right] \tag{31}$$

$$\Lambda=\left(\frac{m^2}{e^{2(mt+n)}}+\frac{4\pi l^2}{e^{2(mt+n)}}\right) \tag{32}$$

Case-II: When $\gamma=1$

In this case the energy density and cosmological constant are equal *i.e.* $p=\rho$ and takes the form

From Equation (26), we obtain

$$p=\rho=\frac{-1}{16\pi}\left[\frac{2m^2}{e^{2(mt+n)}}+\frac{8\pi l^2}{e^{2(mt+n)}}\right] \tag{33}$$

$$\Lambda=0. \tag{34}$$

Case-III: When $\gamma = \frac{1}{3}$

In this case from Equation (26), we obtain the energy density and cosmological constant in the form

$$\rho = 3p = \frac{-3}{32\pi}\left[\frac{2m^2}{e^{2(mt+n)}} + \frac{8\pi l^2}{e^{2(mt+n)}}\right] \tag{35}$$

$$\Lambda = \frac{1}{2}\left(\frac{m^2}{e^{2(mt+n)}} + \frac{4\pi l^2}{e^{2(mt+n)}}\right). \tag{36}$$

From Equations (32) and (36) we observe that the cosmological constant term Λ is a decreasing function of time whereas $\Lambda = 0$ when energy density and pressure are in equilibrium.

3. Some Physical and Kinematical Properties

Here we study Physical and Kinematical properties of the cosmological model given by Equation (29). For the model (29) the expressions for the spatial volume V, scalar expansion θ, shear scalar σ and deceleration parameter q are

$$\text{Spatial volume} \quad V = r e^{-2T} \tag{37}$$

$$\text{Scalar expansion} \quad \theta = -2e^{-T} \tag{38}$$

$$\text{Shear scalar} \quad \sigma^2 = \frac{1}{6}e^{-2T} \tag{39}$$

$$\text{Deceleration parameter} \quad q = -1. \tag{40}$$

The spatial volume V tend to zero as T tends to ∞, the scalar expansion is negative thus the universe is contracting. The positive value of deceleration parameter q indicates that the model decelerates in the stander way. But in the present observation the model inflates because the deceleration parameter q is negative. $\frac{\sigma}{\theta^2} \neq 0$ as $T \to \infty$ *i.e.* the model is anisotropic and does not approach isotropy.

4. Conclusion

We have studied Einstein Rosen cylindrically symmetric static vacuum model and non-static cosmological model with mesonic perfect fluid with time-dependent cosmological constant term Λ in general relativity. We have discussed three physical models corresponding to values of γ, *i.e.* $\gamma = 0, 1, \frac{1}{3}$. It is observed that non-static cosmological model is nonsingular; contracting and deceleration parameter indicates inflation. The time-dependent cosmological term Λ is decreasing function of time and it approaches to small positive value at late time.

Acknowledgements

V. D. Elkar is thankful to the University Grants Commission, New Delhi, India for providing fellowship under F.I.P.

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