

Cosmology Should Directly Use the Doppler's Formula to Calculate the Red Shift of Ia Supernova

—The Proofs That Metric Red Shift Is Inapplicable and Dark Energy Does Not Exist

Xiaochun Mei, Ping Yu

Institute of Innovative Physics, Fuzhou, China
Email: ycwlyjs@yeah.net, Yupingpingyu@yahoo.com

Received March 28, 2013; revised April 29, 2013; accepted May 7, 2013

Copyright © 2013 Xiaochun Mei, Ping Yu. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

ABSTRACT

The Doppler formula should be used directly to calculate red shift of Cosmology. The first is gravity, the second is the Doppler's effect and the third is the Compton scattering. The red shift of cosmology is considered to be caused by the receding motions of celestial bodies, of which essence is the Doppler's effect. However, the basic formula used to calculate the relationship between red shift and distance for Ia supernova in cosmology is $z+1 = R(t_0)/R(t_1)$ which is based on the R-W metric and related to the scalar factor $R(t)$. This is different from the Doppler formula which is related to speed factor $\dot{R}(t)$. Because the R-W metric is only a mathematical structure of space, the metric red shift is not an independent law of physics, this inconsistency is not allowed in physics. It is proved strictly in this paper that the formula of metric red shift is only the result of the first order approximation. If higher order approximations are considered, we can obtain a restrict condition $\dot{R}(t) = \text{constant}$. It indicates that if the formula of metric red shift holds, it can only be suitable to describe the spatial uniform expansion, unsuitable for the practical universal process with acceleration. The further study reveals that the R-W metric violates the invariability principle of light's speed in vacuum. The time delay caused by relative velocity between light's source and observer is neglected. So the R-W metric violates the special relativity and is not suitable to be used as the basic space-time frame in cosmology. The formula used to calculate the relationship between red shift and distance of Ia supernova is wrong and subsequently the deduced results about dark energy and the accelerating universe are incredible. Cosmology should use directly the Doppler formula to calculate red shift. It is proved that based on the Doppler's formula and by the method of numerical calculation, the relation of red shift and distance of Ia supernova can be explained well. The hypotheses of dark energy and the accelerating expansion of the universe are completely unnecessary in cosmology.

Keywords: Cosmology; Doppler Formula; R-W Metric; Hubble Law; Supernova; Dark Energy; Dark Material

1. Introduction

As we known that there are mainly three mechanisms leading to the red shift of spectrum. The first is gravity which relates to mass and gravity constant. The second is Doppler's effect which relates to velocity. The third is the Compton scattering which is related to the energy transformation of photon. According to the Hubble law, the spectrum red shift of extragalactic nebula was proportional to the distance between observer and luminous celestial body. The red shift of cosmology is considered to be the Doppler's effect. In 1998, cosmic observers found that the red shift of Ia supernova deviated from the linear relation of the Hubble law. By fitting the observation values with standard theory of cosmology, cosmolo-

gists declared that about 26% material in the universe was dark material and about 70% was dark energy. The universe seems to do accelerating expansion [1,2].

The red shift of cosmology is considered to be caused by the receding motions of celestial bodies, so its essence is the Doppler's effect related to speed. We should use Doppler's formula to calculate the red shift. However, it is strange that the basic formula used to calculate the relation of red shift and distance of Ia supernova is completely different from the Doppler formula. The basic formula used in cosmology is [3]

$$z+1 = \frac{v_1}{v_0} = \frac{R(t_0)}{R(t_1)} \quad (1)$$

The formula is based on the R-W metric in which ν_1 is the frequency of emitted light at time t_1 , ν_0 is the frequency of received light at time t_0 . The formula is related to scalar factor $R(t)$. But the Doppler's formula is related to speed factor $\dot{R}(t)$. It is proved in this paper that they can not be consistent. This inconsistency is not allowed in physics.

It is proved strictly that Formula (1) is only the result of first order approximation. When higher order approximations are taken into account, the restriction condition $\dot{R}(t) = \text{constan}$ will be obtained. Therefore, Formula (1) is only suitable to describe the process of spatial uniform expansion; it's unsuitable for the practical process of the universal expansion with acceleration. More strictly, if we do not consider approximation, both Formula (1) and the restriction relations do not exist. That is to say, we have no metric red shift actually.

In fact, the W-R metric is only a mathematical structure of space-time, rather than an independent law of physics. It is introduced to simplify the Einstein's equation of gravity field to obtain the Friedmann equation of cosmology. It is not absolutely necessary to use the R-W metric in cosmology. Using normal coordinates directly, we can also establish the equation of cosmology based on the Einstein's equation of gravity field. In this case, there is no the metric red shift formula (1). In this sense, the red shift of metric is not independent in physics. If it is correct, it cannot be contradicted with the Doppler's and gravity red shifts.

The further analysis indicates that the R-W metric violates the invariability principle of light's speed in vacuum when it is used to describe the uniform motion of light's source. The time contraction between observer and moving light's source is also neglected when the R-W metric is use to describe the spatial expansion. So the R-W metric is not one of special relativity and is unsuitable to be used as the basic space-time frame in cosmology. Especially, it is not suitable to be used to describe the high red shift of supernova in which the high speed expansion of the universe is involved.

In summary, the formula used to calculate the relation of red shift and distance of Ia supernova in cosmology has essential mistake. Therefore, dark energy and the accelerating universe are incredible. The Doppler formula should be used directly to calculate red shift of Cosmology. It is proved by the method of numerical calculation that if the Doppler's formula and the (revised) Newtonian formula of gravity are used, the high red shift of Ia supernova can be explained well. We do not need the hypothesis of dark energy and accelerating universe again.

Let's discuss the problem in the metric red shift formula. The inconsistency between the metric red shift and the Doppler's red shift will be discussed in appendix.

2. Unavailability of Metric Red Shift Formula

2.1. The Restriction Condition $\dot{R}(t) = \text{Constant}$ for the Metric Red Shift Formula

Standard cosmology uses the R-W metric to describe the universal space-time with form

$$ds^2 = c^2 dt^2 - R^2(t) \times \left(\frac{d\bar{r}^2}{1 - \kappa \bar{r}^2} + \bar{r}^2 d\theta^2 + \bar{r}^2 \sin^2 \theta d\varphi^2 \right) \quad (2)$$

Here $R(t)$ is scalar factor and κ is considered as the factor of spatial curvature. The co-moving coordinate is used in cosmology. The space is considered flat when $\kappa = 0$. When (2) is used to describe the expansive universe, luminous celestial bodies are considered to be fixed on the coordinate \bar{r}, θ, φ which do not changed with time. Let's repeat the process to obtain Formula (1) based on the R-W metric. Suppose that light moves along the radius direction $\theta = \varphi = 0$ with $ds = 0$, we get from (2)

$$\frac{cdt}{R(t)} = \frac{d\bar{r}}{\sqrt{1 - \kappa \bar{r}^2}} \quad (3)$$

Light source is fixed at point \bar{r} and \bar{r} does not change with time. But to light's motion, \bar{r} changes with time. Suppose that photon's coordinate is \bar{r}_1 at moment t_1 and photon arrives at the original point $\bar{r}_0 = 0$ at moment t_0 . The integral of (3) is [3]

$$\int_{t_1}^{t_0} \frac{cdt}{R(t)} = - \int_{\bar{r}_1}^0 \frac{d\bar{r}}{\sqrt{1 - \kappa \bar{r}^2}} = \sin n\bar{r}_1 = \begin{cases} \sin \bar{r}_1 & k = 1 \\ \bar{r}_1 & k = 0 \\ \sinh \bar{r}_1 & k = -1 \end{cases} \quad (4)$$

The negative sign indicates that light moves along the direction to decrease \bar{r} . Suppose that a light wave is emitted during the period of time from t_1 to $t_1 + \tau_1$ with period $\tau_1 = c/\nu_1$. Observer receives the light during the period of time from t_0 to $t_0 + \tau_0$ with period $\tau_0 = c/\nu_0$. According to the current understanding, because $t_0 + \tau_0$ and $t_1 + \tau_1$ are decided by the same \bar{r}_1 , we have

$$\int_{t_1}^{t_0} \frac{dt}{R(t)} = \int_{t_1 + \tau_1}^{t_0 + \tau_0} \frac{dt}{R(t)} \quad (5)$$

Because τ_0 and τ_1 are small, according to present theory, we obtain from (5)

$$\frac{\tau_0}{R(t_0)} = \frac{\tau_1}{R(t_1)} \quad (6)$$

Based on (6) and the definition of red shift, we obtain

(1). By considering (1) and the Friedmann equation of cosmology, as described in Section 3, we can obtain (1). We now prove that (6) is the result of the first order approximation. If higher order approximations are considered, we can obtain $\dot{R}(t_0) = \dot{R}(t_1)$ or $\dot{R}(t) = \text{constant}$. So the formula (1) is only suitable to the uniformly expansive universe, unsuitable to describe the universal process with acceleration. Let

$$f'(t) = \frac{1}{R(t)} \tag{7}$$

and take the integral of (5), we have

$$f(t_1) - f(t_0) = f(t_1 + \tau_1) - f(t_0 + \tau_0) \tag{8}$$

By developing (8) into the Taylor series, we obtain

$$\begin{aligned} f(t_0) - f(t_1) &= f(t_0) + f'(t_0)\tau_0 + \frac{1}{2!}f''(t_0)\tau_0^2 \\ &+ \frac{1}{3!}f'''(t_0)\tau_0^3 + \dots - f(t_1) - f'(t_1)\tau_1 \\ &- \frac{1}{2!}f''(t_1)\tau_1^2 - \frac{1}{3!}f'''(t_1)\tau_1^3 - \dots \end{aligned} \tag{9}$$

Because τ_0 and τ_1 are very small, if let the items with same orders are equal to each other, we have

$$f'(t_0)\tau_0 = f'(t_1)\tau_1 \tag{10}$$

$$f''(t_0)\tau_0^2 = f''(t_1)\tau_1^2 \tag{11}$$

$$f'''(t_0)\tau_0^3 = f'''(t_1)\tau_1^3 \tag{12}$$

$$f''''(t_0)\tau_0^4 = f''''(t_1)\tau_1^4 \dots \tag{13}$$

By considering (7), (10) is just (6). So the metric red shift formula (1) is actually the approximation of first order. (11) can be written as

$$\frac{\dot{R}(t_0)}{R^2(t_0)}\tau_0^2 = \frac{\dot{R}(t_1)}{R^2(t_1)}\tau_1^2 \tag{14}$$

Substituting (6) in (14), we obtain

$$\dot{R}(t_0) = \dot{R}(t_1) \tag{15}$$

Because t_0 and t_1 are arbitrary, (15) indicates that $\dot{R}(t) = \text{constant}$, *i.e.*, the expansion speed does not depend on time and we have $\ddot{R}(t_0) = \ddot{R}(t_1) = \ddot{R}(t) = 0$. From (12), we have

$$\left[-\frac{\ddot{R}(t_0)}{R^2(t_0)} + \frac{2\dot{R}^2(t_0)}{R^3(t_0)} \right] \tau_0^3 = \left[-\frac{\ddot{R}(t_1)}{R^2(t_1)} + \frac{2\dot{R}^2(t_1)}{R^3(t_1)} \right] \tau_1^3 \tag{16}$$

Substituting (6) and (15) in (16), we get

$$\frac{\tau_0^3}{R^3(t_0)} = \frac{\tau_1^3}{R^3(t_1)} \tag{17}$$

It is still (6). From (13), we get

$$\begin{aligned} &\left[-\frac{\ddot{R}(t_0)}{R^2(t_0)} + \frac{6\dot{R}(t_0)\dot{R}(t_0)}{R^3(t_0)} - \frac{6\dot{R}^3(t_0)}{R^4(t_0)} \right] \tau_0^4 \\ &= \left[-\frac{\ddot{R}(t_1)}{R^2(t_1)} + \frac{6\dot{R}(t_1)\dot{R}(t_1)}{R^3(t_1)} - \frac{6\dot{R}^3(t_1)}{R^4(t_1)} \right] \tau_1^4 \end{aligned} \tag{18}$$

Because of $\ddot{R}(t) = 0$, we have $\ddot{R}(t) = 0$. By considering (15), we still obtain (6). It can be known that no matter what high orders are considered, we always obtain these two relations. So the metric can only be written as

$$R(t) = a + bt \tag{19}$$

That is to say, if (6) holds, it can only be used to describe the uniform expansion of space. Because gravity exists in the universe, Formula (1) can not be used to describe the practical process of the universal expansion with acceleration.

More strictly, we only have Formula (9). Formulas (10)-(13) do not exist in mathematics actually. It means that it is impossible for us to have a new red shift mechanism based on the R-W metric, which is different from the Doppler's and gravity red shifts. The reason is that the R-W metric is only a mathematical structure of space-time, rather an independent physical law.

3.2. The Problem of the R-W Metric

Firstly, constant κ in (2) is considered as curvature factor and the metric is considered to describe flat space-time when $\kappa = 0$. This idea has been proved wrong that when scalar factor $R(t)$ is related to time [4]. That is to say that the R-W metric has no constant curvature in general situations. By using the formula of the Riemannian geometry and making strict calculation, the space-time curvatures of the R-W metric actually are

$$\begin{aligned} K_{01} = K_{02} = K_{03} &= -\frac{\ddot{R}}{R} \\ K_{12} = K_{13} = K_{23} &= \frac{\kappa - \dot{R}^2}{R^2} \end{aligned} \tag{20}$$

Here K_{0j} is the curvature of space-time closing part and K_{ij} is the curvature of pure spatial part. This result is completely different from the current understanding. Therefore, constant κ is not the factor of spatial curvature. Instead, it is a certain adjustable parameter. The R-W metric does not represent the flat space-time when $\kappa = 0$.

In fact, we can use more simple method to prove the result above. As we know in geometry, the principle to judge an arbitrary metric to be flat or not is that if we can find a coordinate to transform the metric to one with flat space-time, the original metric is flat essentially. If we cannot, the original metric is curved in essence. By using common coordinate system, the four dimensional

metric of flat space-time is

$$ds^2 = c^2 dt^2 - (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2) \quad (21)$$

By using the co-moving coordinate or the coordinate transformation $r = R(t)\bar{r}$ in (21), we obtain

$$ds^2 = c^2 \left[1 - \frac{\dot{R}^2(t)\bar{r}^2}{c^2} \right] dt^2 - 2R(t)\dot{R}(t)\bar{r}d\bar{r}dt - R^2(t)(d\bar{r}^2 + \bar{r}^2 d\theta^2 + \bar{r}^2 \sin^2 \theta d\varphi^2) \quad (22)$$

However, let $\kappa = 0$ in (2), we obtain

$$ds^2 = c^2 dt^2 - R^2(t)(d\bar{r}^2 + \bar{r}^2 d\theta^2 + \bar{r}^2 \sin^2 \theta d\varphi^2) \quad (23)$$

(22) is different from (23) unless $\dot{R}(t) = 0$. Because we can obtain (21) by using $r = R(t)\bar{r}$ in (22), so the space-time described by (22) is flat in essential. However, it is easy to see that when $R(t) \neq \text{constant}$ we can not find a coordinate transform to change (23) into (21), so (23) can not be the metric of flat space-time.

We can prove generally that if (23) described light's motion in flat space, it would violate the invariability principle of light's speed in vacuum. For the light source fixed on the reference fame, its coordinate \bar{r} does not change with time. But for the motion of light, the coordinate \bar{r} changes with time. Suppose that light moves along the direction of radius, we have $ds = 0$ and $d\theta = d\varphi = 0$. According to (23), we have

$$\frac{d\bar{r}}{dt} = \pm \frac{c}{R(t)} \quad (24)$$

The velocity of light relative to the observer rested at the original point of coordinate system is

$$V_c(t) = \frac{dr(t)}{dt} = \bar{r} \frac{d}{dt} R(t) + R(t) \frac{d\bar{r}}{dt} = \dot{R}(t)\bar{r} \pm c = V(t) \pm c \quad (25)$$

(25) indicates that light's velocity is related to the velocity of special expansion. Suppose that space expands uniformly, let $R(t) = a + bt$, according to (25), we have $V_c(t) = b\bar{r} \pm c$. At the moment when light is emitted out, (25) is just the Galileo's addition rule of light's velocity. When light moves towards observer, minus sign is taken and light's speed is less than its speed in vacuum. When the light moves apart from observer, plus sign is taken and light's speed is great than its speed in vacuum. Especially, because \bar{r} increases with time, enough long time later, light's speed may greatly exceed its speed in vacuum. This is not allowed in physics, so the R-W metric is not the metric of special relativity.

We can use two simple examples to show that if (23) describe the metric of flat space, it can not describe light's motion. Suppose that light's source is located at

point \bar{r}_0 and emits a bundle of light at time $t = 0$. This light arrives at observer located at position $\bar{r}(t)$ at time t . Suppose that space expands in a uniform speed which corresponds to the uniform motion of light's source. Let $R(t) = a + bt$ and take the integral of (24), we obtain

$$\bar{r}(t) - \bar{r}_0 = \pm \frac{c}{b} \ln \left(1 + \frac{b}{a} t \right) \quad (26)$$

Let $\bar{r}_0 = 1$, $a = 1$ and $b \approx 3 \times 10^8$, we get the speed of light's source $V(t) = b\bar{r}_0 \approx 3 \times 10^8$ m/s which is near light's speed. Suppose that observer is at the original point of coordinate system with $\bar{r}(t) = 0$, according to (26), we have $1 = \ln(1 + 3 \times 10^8 t)$ and $t = 5.73 \times 10^{-7}$ s. This result is obviously incorrect. According to special relativity, light's speed does not depend on the speed of light's source. Because the initial distance between observer and light's source is 1m, the time that light arrives at observer should be 3.33×10^{-7} s. The error is as great as 58%. In fact, charged particles move at near light's speed in high energy accelerators. And experiments show that radiation lights can not propagate in this way.

Suppose that the acceleration of spatial expansion is a constant and let $R(t) = a + bt + gt^2/2$. Take $\bar{r} = \bar{r}_0$ when $t = 0$. The integral of (24) is

$$\bar{r} - \bar{r}_0 = \frac{\pm c}{\sqrt{b^2 - 2ag}} \times \left[\ln \frac{gt + b - \sqrt{b^2 - 2ag}}{gt + b + \sqrt{b^2 - 2ag}} - \ln \frac{b - \sqrt{b^2 - 2ag}}{b + \sqrt{b^2 - 2ag}} \right] \quad (27)$$

Taking $a = 1$, $b = -1$ and $g = 0.3$, we have $R(0) = 1$ and $(b^2 - 2ag)^{1/2} = 0.633$. If $t = 8.16$, we have $R(8.16) = 2.83$. The speed of light's source is $V = (b + gt)\bar{r}_0 = 1.45\bar{r}_0$. Meanwhile, we have

$$gt + b - \sqrt{b^2 - 2ag} = 0 \quad \text{and} \quad b - \sqrt{b^2 - 2ag} = -1.63.$$

The first item in (27) becomes minus infinite and the second item is meaningless, so (27) can not yet describe the motion of light in this case. Therefore, in general, the R-W metric is unsuitable to describe light's motion.

This problem does not exist in (22). Suppose that light moves along the direction of radius, we have

$$c^2 \left[1 - \frac{\dot{R}^2(t)\bar{r}^2}{c^2} \right] dt^2 - 2R(t)\dot{R}(t)\bar{r}d\bar{r}dt - R^2(t)d\bar{r}^2 = 0 \quad (28)$$

$$\frac{d\bar{r}}{dt} = -\frac{\dot{R}(t)\bar{r}}{R(t)} \pm \frac{c}{R(t)} \quad (29)$$

By considering (29), light's velocity is

$$V_c = \frac{dr}{dt} = \frac{dR(t)\bar{r}}{dt} = \dot{R}(t)\bar{r} + R(t)\frac{d\bar{r}}{dt} = \pm c \quad (30)$$

It indicates that light's speed is unchanged in vacuum, so (22) is the metric of special relativity.

We have $g_{00} = 1$ in (23), so it is considered to have a uniform time for whole space. However, because of spatial expansion, the clock fixed at position \bar{r} has a speed relative to observer who is rested at the original point of coordinate system. According to special relativity, there is time delay between them, but (23) can not describe this relation. According to (22), we have

$$d\tau = dt\sqrt{1 - \frac{\dot{R}^2(t)\bar{r}^2}{c^2}} = dt\sqrt{1 - \frac{V^2}{c^2}} \quad (31)$$

This is just the delay of special relativity, so (22) is the metric of relativity in flat space-time, but (23) is not. Similarly, it is proved in mathematics that the four dimensional metric in which three dimensional space has constant curvature κ is

$$ds^2 = c^2 dt^2 - \left(\frac{dr^2}{1 - \kappa r^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \right) \quad (32)$$

By using co-moving coordinate in (32), we obtain

$$ds^2 = \left[c^2 - \frac{\dot{R}^2(t)\bar{r}^2}{1 - \kappa R^2(t)\bar{r}^2} \right] dt^2 - \frac{2\dot{R}(t)R(t)\bar{r}dtd\bar{r}}{1 - \kappa R^2(t)\bar{r}^2} - R^2(t) \left(\frac{d\bar{r}^2}{1 - \kappa R^2(t)\bar{r}^2} + \bar{r}^2 d\theta^2 + \bar{r}^2 \sin^2 \theta d\varphi^2 \right) \quad (33)$$

When $\kappa = 0$, (33) becomes (22), so (33) is correct metric which satisfies special relativity with special curvature κ . We have $g_{00} \neq 1$, $g_{01} \neq 0$ and $g_{11} = R(t)$ in (33). Its form is much more complicated than the R-W metric (2). When light moves along the direction of radius with $ds = 0$, we have

$$\left[c^2 - \frac{\dot{R}^2(t)\bar{r}^2}{1 - \kappa R^2(t)\bar{r}^2} \right] dt^2 - \frac{2\dot{R}(t)R(t)\bar{r}dtd\bar{r}}{1 - \kappa R^2(t)\bar{r}^2} - \frac{R^2(t)d\bar{r}^2}{1 - \kappa R^2(t)\bar{r}^2} = 0 \quad (34)$$

We obtain

$$\frac{d\bar{r}}{dt} = -\frac{\dot{R}(t)\bar{r}}{R(t)} \pm \frac{c\sqrt{1 - \kappa R^2(t)\bar{r}^2}}{R(t)} \quad (35)$$

If we use (29) and (35) to calculate the red shift of cosmology, *i.e.*, also so called metric red shift, the results are certainly related to velocity. However, we can not separate variables in (29) and (35), so we can not write them in the simple form of (3) and can not get the same

formulas similar to (1).

The R-W metric violates the invariability principle of light's speed in vacuum and neglects the time delay between observers and moving light's source caused by relative motion. So the red shift formula of metric based on the R-W metric can not be correct. It is improper to use the metric as the basic frame of space-time in cosmology. We should use the Doppler's formula to calculate the red shift directly.

2.3. Other Problems in the Formula for Calculating the Red Shift of Ia Supernova

Based on (1) in cosmology, the following formula is obtained to calculate the relation of red shift and distance of Ia supernova [5]

$$H_0 d_L = \frac{c(1+z)}{\sqrt{|\Omega_{k0}|}} \sin n \left[\sqrt{|\Omega_{k0}|} \times \int_0^z \frac{dz'}{\sqrt{(1+z')^2 (1 + \Omega_{m0} z') - z'(2+z')\Omega_\lambda}} \right] \quad (36)$$

Here d_L is luminosity distance. In light of the R-W metric, we have $d_L = (1+z)r = (1+z)R_0\bar{r}$. By fitting with practical observations, cosmologists deduced that dark energy was about 70% and non-baryon dark material was about 26% of the universal material. An accelerating universe is assumed.

We now discuss the problems existing in (36). The Friedmann equation of cosmology is

$$\left(\frac{\dot{R}}{R} \right)^2 + \frac{\kappa}{R^2} = \frac{8\pi G}{3} (\rho_m + \rho_\lambda) \quad (37)$$

Here ρ_m is the density of normal material, ρ_λ is the density corresponding to cosmic constant. At present time t_0 , we have $\rho_0 = \rho(t_0)$ and $H_0 = H(t_0)$, the Friedmann equation of cosmology can be written as

$$\frac{3H_0^2}{8\pi G} - (\rho_{m0} + \rho_\lambda) = -\frac{3\kappa}{8\pi GR_0^2} \quad (38)$$

Defining critical density ρ_c as

$$\rho_c = \frac{3H_0^2}{8\pi G} \quad (39)$$

Let $\Omega_{m0} = \rho_{m0}/\rho_c$, $\Omega_\lambda = \rho_\lambda/\rho_c$, $\Omega_0 = \Omega_{m0} + \Omega_\lambda$, we write (38) as

$$1 - \Omega_0 = -\frac{\kappa}{R_0^2 H_0^2} \quad (40)$$

Let $a(t) = R(t)/R(t_0)$, we have $a(t_0) = 1$ at present time. Because κ is a constant, we can write the Friedmann equation as

$$\dot{a}^2(t) - \frac{8\pi G}{3}(\rho_m + \rho_\lambda)a^2 = -\frac{\kappa}{R_0^2} = H_0^2 - \frac{8\pi G}{3}\rho_0 \quad (41)$$

By considering

$$\rho R^3 = \rho_0 R_0^3 \text{ or } \rho a^3 = \rho_0 \quad (42)$$

(41) can be written as

$$\left(\frac{da}{dt}\right)^2 = H_0^2 \left(1 - \Omega_{m0} + \frac{\Omega_{m0}}{a} + \Omega_\lambda a^2 - \Omega_\lambda\right) \quad (43)$$

We have $da/dt = \dot{a}$ or $dt = da/\dot{a}$, so (4) can be written as

$$\sin n\bar{r} = \frac{1}{R_0} \int_{1/(1+z)}^1 \frac{da}{a\dot{a}} \quad (44)$$

The upper limit of the integral is $a(t_0) = 1$ and lower limit is $a(t_1) = 1/(1+z)$. By intruding so-called energy effective curvature density [5]

$$\rho_k = -\frac{3\kappa}{8\pi GR^2} \quad (45)$$

We have $\Omega_k = \rho_k/\rho_c$ and $\Omega_{0k} = \rho_{0k}/\rho_c$. We write (40) as

$$1 - \Omega_0 = -\frac{\kappa}{R_0^2 H_0^2} = \Omega_{k0} \quad (46)$$

According to Document [5], (46) is written as

$$R_0 = \frac{1}{H_0 \sqrt{|\Omega_{k0}|}} \quad (47)$$

By introducing transformation $a = 1/(1+z)$ in (44) we obtained (36). However, (47) is obviously wrong. (46) contains constant κ , but (47) does not. According to (46), when $\kappa = 0$ we have $\Omega_{k0} = 0$, so R_0 is limited. But according to (47), when $\Omega_{k0} = 0$ we have $R_0 = \infty$. It means that R_0 is infinite at present time in flat space. But this is completely impossible. According to (46), correct result should be

$$R_0 = \frac{\sqrt{|\kappa/\Omega_k|}}{H_0} \quad (48)$$

In fact, it is unnecessary for us to introduce relation $\rho_k = -\kappa/(a^2 R^2)$. Whether or not space is flat depends on $\sin n\bar{r} = \bar{r}$, $\sin n\bar{r} = \sin \bar{r}$ or $\sin n\bar{r} = \sinh \bar{r}$. For flat space, by taking $\kappa = 0$ in (41) and substituting it in (44), and by considering $\Omega_{m0} + \Omega_\lambda = 1$ and $\sin n\bar{r} = \bar{r}$, we get

$$H_0 R_0 \bar{r} = \int_0^z \frac{dz'}{\sqrt{(1+z')^2 (1 + \Omega_{m0} z')}} \quad (49)$$

Because (47) can not hold, (36) can only be written as

$$d_L = R_0 (1+z) \sin n \left[\frac{1}{R_0 H_0} \times \int_0^z \frac{dz'}{\sqrt{(1+z')^2 (1 + \Omega_{m0} z') - z'(2+z')\Omega_\lambda}} \right] \quad (50)$$

When space is flat, we have $\kappa = 0$, $\Omega_{m0} + \Omega_\lambda = 1$, and $\sin n\bar{r} = \bar{r}$. So (50) becomes

$$H_0 R_0 \bar{r} = \int_0^z \frac{dz'}{\sqrt{(1+z')^2 (1 + \Omega_{m0} z') - z'(2+z')\Omega_\lambda}} \quad (51)$$

Comparing with (49), (51) has an additional item containing Ω_λ in the radical sign of integrated function. The reason is that when $\kappa = 0$, the right side of (43) has only two items with

$$\left(\frac{da}{dt}\right)^2 = H_0^2 \left(\frac{\Omega_{m0}}{a} + \Omega_\lambda a^2\right) \quad (52)$$

But (36) is calculated without considering this factor, so that additional three items are added. If space is curved for $\kappa = 1$, we still have $\Omega_{m0} + \Omega_\lambda = 1$ and can obtain

$$d_L = R_0 (1+z) \sin \left[\frac{1}{R_0 H_0} \int_0^z \frac{dz'}{\sqrt{(1+z')^3 \Omega_{m0} + \Omega_\lambda}} \right] \quad (53)$$

Similarly, for $\kappa = -1$, we have

$$d_L = R_0 (1+z) \sinh \left[\frac{1}{R_0 H_0} \int_0^z \frac{dz'}{\sqrt{(1+z')^3 \Omega_{m0} + \Omega_\lambda}} \right] \quad (54)$$

(53) and (54) are also different from (36), so (36) is wrong. Note that this is a mistake of mathematics, and has nothing to do with physics. So we can not use it to calculate the relation of red shift and distance of Ia supernova. More essential, if $\dot{R}(t) = \text{constant}$, we have $\dot{a}(t) = \text{constant}$. Substituting it in (44), we can not obtain (36). In this case, let $\dot{a} = A/R_0$, substitute it in (44), for $\kappa = 0$, we obtain $A\bar{r} = \ln(1+z)$ without practical significance.

It is proved below that if the Doppler's formula is used for calculation, the hypotheses of dark energy and the accelerated universe will become unnecessary.

3. Using the Doppler's Formula to Calculate the Red Shift of Ia Supernova

3.1. The Friedmann Equation Needs Relativity Revision

Standard cosmology is based on the principle of cosmology. The principle declares that the distribution of universal material is uniform and isotropic. So the ma-

terial density is only related to time and unrelated to spatial coordinates with the form $\rho(t)$. To match with the principle, the R-W metric is used in cosmology which is considered with biggest symmetry of space-time.

Standard cosmology uses the Friedmann equation as basic equation. The Friedmann equation is based on the Einstein's equation of gravity field. But two simplified conditions are used. One is the R-W metric and another is static energy momentum tensors. However, British physicist E. A. Milne proved in 1943 that the Friedmann equation could be deduced simply based on the Newtonian theory of gravity when cosmic constant is not considered [6]. This result indicates that the Friedmann equation is actually equivalent to the Newtonian theory of gravity which is only suitable to describe the universal processes with low expansion speed, unsuitable for the universal processes with high expansion speed. So we have to consider the rationality of these two simplified condition.

In fact, as discussed before, the R-W metric violates the invariability principle of light's speed in vacuum and is not the metric of relativity. Static energy momentum tensor means that the velocity and momentum of celestial body which exist practically in the expansion process of the universe are neglected. So the Friedmann equation is one which is improperly simplified. In the early period of cosmology, what observed were the expansion processes with low speed nearing the Milky Way galaxy, the Friedmann equation can handle it. While cosmology develops to the current period, what observed are the expansion processes with high speed such as the red shift of Ia supernova, the Friedmann equation becomes unsuitable and need relativity revision [7].

If we use metric (33) which satisfies the constrain of special relativity in the Einstein's equation of gravity, because metric tensors $g_{00} \neq 1$ and $g_{01} \neq 0$ contain speed factor $\dot{R}(t)\bar{r}$, the obtained cosmic equation is much more complicated than the Friedmann equation. If dynamic energy momentum tensor means are used simultaneously, speed factor $\dot{R}(t)\bar{r}$ is also introduced. In this case, even though we consider the principle of cosmology to take material density as $\rho(t)$, the equation of cosmology will becomes very complex so that it is impossible to solve [7]. This may be the real reason why the pioneers of cosmology had to use the R-W metric and static energy momentum tensor, for they had no another choice.

By considering this difficulty, we have to look for other more proper method to study cosmology. In fact, we have proved that by transforming the geodesic equation of the Schwarzschild solution of the Einstein's equation of gravity field to flat space-time description, the following revised Newtonian formula of gravity can be obtained [7]

$$m_0 \frac{d^2 \mathbf{r}}{d\tau^2} = -GM_0 m_0 \left(1 + \frac{3L^2}{c^2 r^2} \right) \frac{\mathbf{r}}{r^3} \quad (55)$$

In (55) all quantities are defined in flat space. We have

$$d\tau = \sqrt{1 - \frac{V^2}{c^2}} dt \quad (56)$$

This is just the time delay formula of special relativity, so (55) can be considered as the relativity revision formula of the Newtonian gravity. The space-time singularity in the Einstein's theory of gravity becomes the original point $r=0$ in the Newtonian formula of gravity. The singularity problem of general relativity in curved space-time is eliminated thoroughly. The theory of gravity returns to the traditional form of dynamic description.

When the formula is used to describe the universe expansion, the revised Friedmann equation can be obtained. Based on it, the high red-shift of Ia supernova can be explained well. We do not need the hypotheses of the accelerating universe and dark energy. It is also unnecessary for us to assume that non-baryon dark material is 5 - 6 times more than normal baryon material in the universe if they really exist. The problem of the universal age can also be solved well.

Further more, we prove below that by the method of numerical calculation and the Doppler's formula proposed in [7], even based on the Newtonian theory of gravity, we can also explain well the relation of red shift and distance of Ia supernova.

3.2. Using the Doppler's Formula to Calculate the Red Shift of Ia Supernova

As we know that the solution of differential equation is determined by initial condition. However, according to the Big Bang theory, the universe blew up from a singularity with infinite density. That is to say, all material in the universe has a same initial position. However, infinite density is unimaginable and singularity can not exist in the real world. The practical situation may be that some unknown interactions can avoid material to be compressed into infinite density by gravity.

According to [7], we assume that there exist a certain mechanism so that a uniform material sphere with mass M_0 can only be compress into a finite radius r'_0 . The motion equation of universe expansion can be written as

$$m_0 \ddot{r} = F(r) + F_n(r) \quad (57)$$

Here $F(r)$ is the Newtonian gravity and $F_n(r)$ is the sum of all non-gravities. For convenience of calculation, we assume

$$F_n(r) = \frac{m_0}{2} A(r) \delta(r - r'_0) \quad (58)$$

Here $A(r)$ is an unknown function. $F_n(r)$ corre-

sponds to an infinite barrier at position r'_0 . When a material sphere with radius r is compressed into a sphere with radius r'_0 , it can not be compressed further. For the spheres with different radii r , their r'_0 are different.

Suppose that the material distribution of the universe is uniform with $\rho = \rho(t)$. The static mass contained in the spherical surface with radius r is M_0 . According to (55), the revised Newtonian gravity is related to velocity. Using it to calculate the universe expansion, under the condition $V/c \ll 1$, the speed of a particle located on the spherical surface is

$$\frac{V}{c} = \sqrt{Q_1(r) + K(r'_0)} \\ = \sqrt{\frac{2GM_0}{c^2 r} \left(1 - \frac{3}{20} \frac{\alpha}{r} + \frac{3}{56} \left(\frac{\alpha}{r} \right)^2 + \dots \right) + K(r'_0)} \quad (59)$$

Here $\alpha = 2GM_0/c^2$. $K(r'_0)$ is a constant which describes initial conditions. Let $\alpha/r = 0$ in (59), we get the result of the Newtonian theory of gravity

$$\frac{V}{c} = \sqrt{\frac{2GM}{c^2 r} + K(r'_0)} = \sqrt{\frac{8G\rho r^2}{3c^2} + K(r'_0)} \quad (60)$$

We take an expansive sphere as the expansive universe and use the Doppler's formula to describe red shift with

$$z = \sqrt{\frac{1+V(t)/c}{1-V(t)/c}} \quad (61)$$

Suppose that luminous body moves along the direction of radius and observer is located at the origin point of flat reference frame. The distance between observer and celestial body is $r(t)$ at moment t . The distance between observer and celestial body is r_0 at present moment t_0 . In the expanding process of the universe, celestial body moves from r_1 to r_0 with $r_0 > r_1$, while the light travels from r_1 to observer along opposite direction. Suppose that light's speed is invariable in the process (having nothing to do with the speed of light's source or the expansive speed of space, as shown in (30)), we have following relation

$$\Delta t = \frac{r_1}{c} = \int_{r_1}^{r_0} dt = \int_{r_1}^{r_0} \frac{dr}{V} \quad (62)$$

By astronomical observations, we know the universe material density ρ_0 at present time t_0 , but we do not know $\rho(t)$ at past time t . By the relation $\rho_0 r_0^3 = \rho r^3$, we have

$$\sqrt{\frac{8\pi G \rho r^2}{3c^2}} = \sqrt{\frac{8\pi G \rho_0 r_0^3}{3c^2 r}} \quad (63)$$

Substituting (60) in (62) and using (63), we obtain

$$r_1 = \int_{r_1}^{r_0} \frac{dr}{V/c} = \int_{r_1}^{r_0} \frac{dr}{\sqrt{\frac{8G\rho_0 r_0^3}{3c^2 r} + K(r'_0)}} \quad (64)$$

In principle, we can write (64) as

$$r_1 = f(r_0, K(r'_0)) - f(r_1, K(r'_0)) \quad (65)$$

From (65), we can get $r_0 = g(r_1, K)$ in principle. What observer see at present time is the light emitted by the celestial body located at position $r_1 < r_0$ and time $t_1 < t_0$. At present time t_0 , the celestial body has moved to position r_0 .

In the formulas above, ρ_0 , r_1 and z are known through observations, but r_0 and $K(r'_0)$ are unknown. By connecting (60), (61) and (65), we can determinate r_0 and $K(r'_0)$. The integral of (64) is difficult but can be calculated by the numerical method of computer. Take $\rho_0 = b \times 10^{-27} \text{ Kg/m}^3$, $r_0 = y_0 \times 10^{26} \text{ m}$, $r_1 = y_1 \times 10^{26} \text{ m}$, we have

$$x = \sqrt{\frac{\alpha}{r}} = 0.25 \sqrt{\frac{by_0^3}{y}} \quad (66)$$

We use x as basic variable to calculate y_0 and $K(r'_0)$. In the calculation, we take b , z and y_1 as input parameters. With this method, we actually traced back to the initial situations of the universe expansion based on the present observations of red shift and distances. In other words, as long as the initial conditions of the universe expansion are known, we can know its current situations.

3.3. The Red Shift of Ia Supernova

According to photometry measurement, the density of luminous material in the university is about $\rho_0 \approx 2 \times 10^{-28} \text{ kg/m}^3$ at present time. Because there exists a great mount of non-luminous material, we assume that practical material is 10 times more than luminous material and let $\rho_0 \approx 2 \times 10^{-27} \text{ kg/m}^3$. In **Figure 1**, we take $m_B = 5.5 + 5 \log d_L$ in which d_L is luminosity distance with unit length $10^6 \text{ pc} = 3.09 \times 10^{22} \text{ m}$. Because our discussion is based on flat space-time and the Doppler's effect without considering the gravity red shift in this paper, light's frequency does not change in the process of propagation. It is unnecessary for us to use the concept of luminosity distance (See appendix for detail). So we need to transform d_L back to practical distance r .

The curved line in **Figure 2** shows the relations between the red-shifts, distances and initial condition parameters of Ia supernova based on the unrevised Newtonian formula of gravity. The vertical coordinate is the values of $K(r'_0)$. The bottom horizontal coordinate is the value of red-shift. On the upside, under the line of horizontal coordinate, the numbers are the values of distance r . Above the line of horizontal coordinate, the numbers are the values of r_0 . For $z=1$ and $m_B = 25$, we get $r_1 = 1.23 \times 10^{26} \text{ m}$. By numerical calculation, we obtain $r_0 = 1.83 \times 10^{26} \text{ m}$ and $K(r'_0) = -2.60 \times 10^{-2}$. For

$z = 0.5$ and $m_B = 23.1$ corresponding to $r_1 = 0.67 \times 10^{26}$ m, we obtain $r_0 = 0.91 \times 10^{26}$ and $K(r'_0) = -2.51 \times 10^{-2}$. For $z = 0.1$ and $m_B = 19.1$ which corresponds to $r_1 = 0.15 \times 10^{26}$ m, we obtain $r_0 = 0.16 \times 10^{26}$ and $K(r'_0) = -5.30 \times 10^{-3}$.

We see that by directly using the Doppler's formula and the Newtonian formula of gravity, we can explain the high red shift of Ia supernova well. The hypotheses of dark energy and the accelerating expansion of the universe become unnecessary. The universe began its expansion from a finite volume, rather than a singularity. The difficulty of singularity in cosmology is eliminated. If the revised Newtonian formula (55) is used, according to reference [7], the result is shown in **Figure 3**. Comparing **Figures 2** and **3**, the difference is that for the

Newtonian formula of gravity, we have $K(r'_0) < 0$. But for the revised Newtonian formula of gravity, we have $K(r'_0) < 0$ when $z > 0.7$, and $K(r'_0) > 0$ when $z < 0.7$. When z is very small, we have $K(r'_0) \rightarrow 0$ for both situations.

3.4. The Age of the Universe

We consider the universe as a material sphere with radius $r'_0 = 1.5 \times 10^{11}$ m at initial moment, which is about the distance between the sun and the earth. Long enough later, at time t , an observer located at the original point of reference frame receives the light emitted from a celestial body on the spherical surface with radius $r = 1.23 \times 10^{26}$ m and red shift $z = 1$ at time t_0 . Suppose that

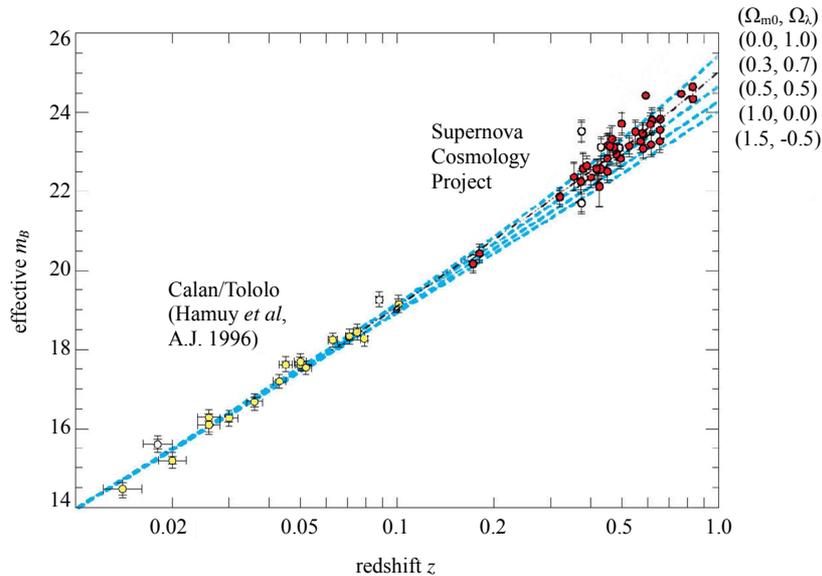


Figure 1. The Hubble diagram for red shift and distance of Ia supernova.

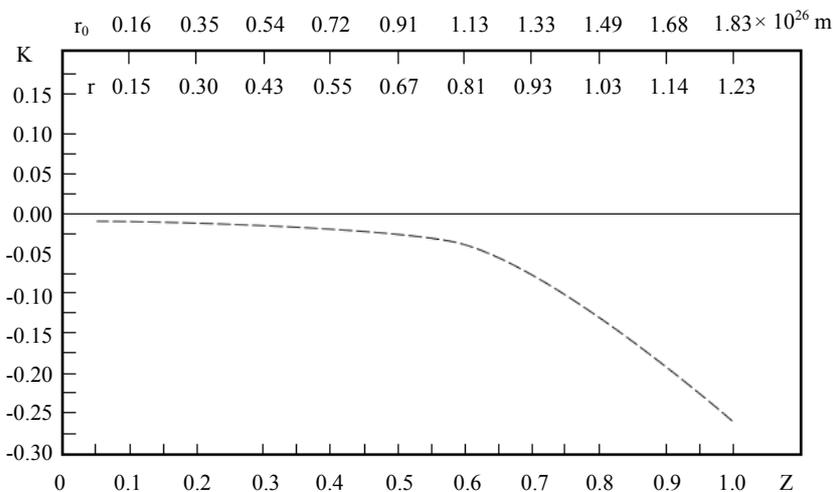


Figure 2. The relations between red-shifts, distances and initial parameters of Ia supernova by using the Doppler's formula and the Newtonian gravity.

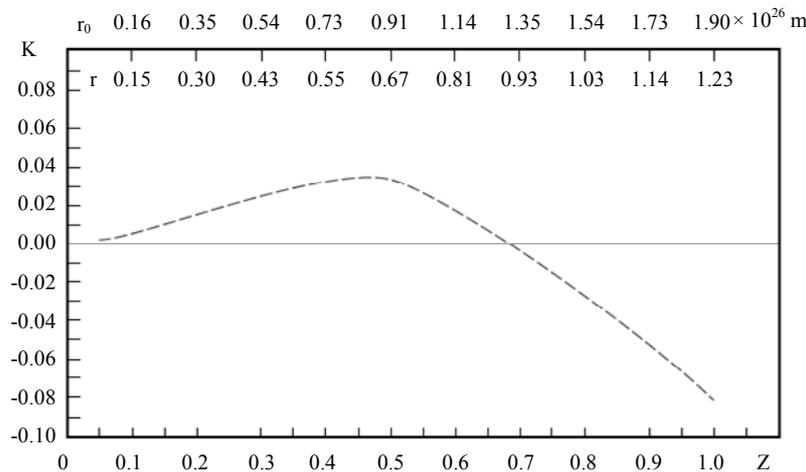


Figure 3. The relations between red-shifts, distances and initial parameters of Ia supernova by using the Doppler’s formula and the revised Newtonian gravity.

the material density of the universe is $\rho_0 = 2 \times 10^{-27}$ kg/m³ at present time, the initial density is $\rho_0 = 5.9 \times 10^{17}$ kg/m³, to be equal to the density of neutron star. According to the calculation before, the real distance of celestial body is $r_0 = 1.83 \times 10^{26}$ m at present time. We consider it as the radius of observable universe and take $K(r'_0) = -0.026$ for the following formula to calculate the time during which the universe expanded from radius $r'_0 = 1.5 \times 10^{11}$ m to $r_0 = 1.83 \times 10^{26}$ m

$$\Delta t = \int_{t_1}^{t_0} dt = \int_{r_1}^{r_0} \frac{dr}{V} = \int_{r_1}^{r_0} \frac{dr}{c \sqrt{8G\rho_0 r_0^3 / (3c^2 r) - 0.026}} \quad (67)$$

The result is $\Delta t = 35$ billion years. But this value is not sensitive to r_1 when it is not very large. Taking $r_1 = 10^{20}$ m, which is about the radius of the Milky Way galaxy, the result is almost the same with $r'_0 = 1.5 \times 10^{11}$ m. It means that the age of the universe mainly depends on the later expansive process. Using (67) to calculates the time during which the radius of sphere expands from 1.23×10^{26} m to 1.83×10^{26} m, the result is 15.4 billion years, so the time during which the radius of sphere expands from 1.23×10^{26} m to 1.83×10^{26} m is 19.5 billion years.

By using the revised Newtonian formula of gravity, for the same red shift $z = 1$, the result in reference [8] is that the time is 30.8 billion years for a sphere’s radius expands from $r_1 = 1.5 \times 10^{11}$ m to $r_0 = 1.95 \times 10^{26}$ m and 13 billion years for radius expands from 1.23×10^{26} m to 1.95×10^{26} m. So the sphere’s radius expands from 1.5×10^{11} m to 1.23×10^{26} m is 17.8 billion years.

Therefore, for the same red shift, by using the revised Newtonian gravity, the age of the universe is smaller than using the unrevised Newtonian gravity. According to the current cosmology, the universe age is estimated to be about 10 - 15 billion years, too short to the formation

of galaxies. The problem does not exist by using the Doppler’s formula to calculate the red shift of cosmology, no matter we use the revised Newtonian formula or the unrevised Newtonian formula of gravity.

4. Conclusions

There are many difficulties in the current cosmology. They are essential and can not be solved by some small revising. We need to suspect whether or not there are problems in the foundation of cosmology. In the process of deducing the Friedmann equation of cosmology based on the Einstein’s equation of gravity, two simplification conditions are used. One is the R-W metric, in which metric tensors are only related to time, not related to space coordinates. Another is static energy momentum tensors, in which celestial body’s velocities are neglected. We need to discuss whether or not these two conditions are proper, for they are really over simplified.

Cosmic red shift is considered to be caused by the receding motions of celestial bodies which are actually the Doppler’s effect. However, in the current cosmology, what used to calculate the relation of red shift and distance of Ia supernova is the formula of metric red shift based on the R-W metric, rather than the formula of the Doppler’s red shift. They are completely different and the inconsistency can not be allowed in physics. We should consider why the metric red shift is used rather than the Doppler’s red shift in cosmology.

The reason is that the Doppler’s effect is related to speed factor $\dot{R}(t)\bar{r}$. If the Doppler’s effect is used directly, we should use dynamic energy momentum tensor in the Einstein’s equation of gravity too. In this case, because speed factor $\dot{R}(t)\bar{r}$ is contained in energy momentum tensor, the obtained equation of cosmology would become too complex to be solved. If the metric

red shift formula is used, it is equivalent to use static energy momentum tensor, the trouble can be avoided. However, the problem is that there exist relative motion speeds practically between observers at rest on the earth and celestial bodies. So it is very irrational to use static energy momentum tensors in cosmology.

In order to avoid using dynamic metric, co-moving coordinate system is used by expansion cosmology. Because observer moves with celestial body in co-moving coordinates system, the speed of celestial body can be eliminated. However, there are three problems here. The first is that astronomers at rest on the earth observe red shift without moving with celestial body. The second is that even though the observer moves with a celestial body, because the speed of expansion is related to the distance between celestial bodies, the relative speeds between the observer and other celestial bodies still exist. The third is that if observer moves with celestial body, it means that observer falls freely in gravitational field. According to the principle of general relativity, gravity is canceled locally. In this case, the metric of space-time is flat and all calculations about cosmic red shift become ineffective.

On the other hand, the formula of metric red shift based on the R-W metric is impossible. The reason is the R-W metric is only a mathematical structure of space. It is not absolutely necessary to use the R-W metric in cosmology. In fact, we can also use common coordinate system to establish cosmic equation based on the Einstein's equation of gravity. So the origin of metric red shift is suspicious. At least, it is not an independent effect caused by independent physical mechanism. But gravity red shift and the Doppler's red shift are caused by independent physical laws. If we can obtain red shift from the R-W metric, it should not be contradicted with gravity red shift or the Doppler's red shift. Otherwise, it is unacceptable.

It is proved strictly that Formula (1) is only the result of first order approximation. When higher order approximations are taken into account, the constriction condition $\dot{R}(t) = \text{constant}$ can be obtained. Therefore, Formula (1) is only suitable to describe the process of spatial uniform expansion; it's unsuitable for the practical process of the universal expansion with acceleration. More strictly, if we do not consider approximation, both relations the formula (6) and (15) do not exist. That is to say, we have no metric red shift actually.

Besides, the R-W metric has many other problems. It is proved that the R-W metric violates the principle of light's speed invariable. When $R(t)$ is related to time, the R-W metric has no constant curvature. The time delay caused by relativity velocity between light's source and observer is neglected according to the R-W metric. So the R-W metric is not a relativity one and unsuitable to be used as the basic space-time frame in cosmology. The red shift based on it can not be correct.

Therefore, the current formula used to calculate the relation of red shift and distance of Ia supernova in cosmology is wrong. We should directly use the Doppler's formula to calculate the red shift of cosmology. By the method of numerical calculation, based on the (revised) Newtonian theory of gravity and the Doppler's formula, it is proved that the red shift of Ia supernova can be explained well. Therefore the hypotheses of dark energy and the accelerating universe are completely unnecessary. The problems of the universe age and the singularity of the Big Bang in the early universe can also be solved simultaneously. In fact, the so-called accelerating expansion of the universe is not the result of direct observation. It depends on the fitting between theoretical model and the observation of Ia supernova. Other proofs of the accelerating universe such as the cosmic background radiation anisotropy and the X ray properties of galaxy clusters and so on are not direct ones, or can be explained by other methods shown in this paper. They remain to be discussed further. The hypothesis of accelerating universe is completely unnecessary.

The procedure we developed and used in this paper and its source code are open to researchers. Demanders, please send us e-mail to ask for it.

REFERENCES

- [1] S. Perlmutter, *et al.*, "Measurements of Ω and Λ from 42 High-Redshift Supernovae," *APJ*, Vol. 517, No. 2, 1999, p. 565. [doi:10.1086/307221](https://doi.org/10.1086/307221)
- [2] B. Leibundgut, *et al.*, "Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant," *The Astronomical Journal*, Vol. 116, No. 3, 1998, p. 1009. [doi:10.1086/300499](https://doi.org/10.1086/300499)
- [3] E. W. Kolb and M. S. Turner, "The Early Universe," Addison-Wesley Publishing Company, Boston, 1990.
- [4] X. Mei, "The R-W Metric Has No Constant Curvature When Scalar Factor $R(t)$ Changes with Time," *International Journal of Astronomy and Astrophysics*, Vol. 1, No. 4, 2011, pp. 177-182.
- [5] S. M. Carroll and W. H. Press, "The Cosmology Constant," *Annual Review of Astronomy and Astrophysics*, Vol. 30, 1992, pp. 499-542. [doi:10.1146/annurev.aa.30.090192.002435](https://doi.org/10.1146/annurev.aa.30.090192.002435)
- [6] E. A. Milne, "A Newtonian Expanding Universe, General Relativity and Gravitation," Springer Netherlands, Berlin, 2000.
- [7] X. Mei and P. Yu, "Revised Newtonian Formula of Gravity and Equation of Cosmology in Flat Space-time Transformed from Schwarzschild Solution," *International Journal of Astronomy and Astrophysics*, Vol. 2, No. 1, 2012, pp. 6-18.
- [8] L. Liao and Z. Zheng, "General Relativity," 2nd Edition, Higher Education Publishing Company, Beijing, 2004.
- [9] S. Weinberg, "Gravitation and Cosmology," John Wiley, New Jersey, 1972.

Appendix: Inconsistence between the R-W Metric and the Doppler's Red Shifts

1. Proper Distance and Expansive Speed Based on the R-W Metric

The standard cosmology uses the R-W metric (2) to describe the universal space-time. According to the R-W metric, at a certain moment t , the proper distance between a celestial body and the original point of coordinate system is [9]

$$r(t) = R(t) \int_0^{\bar{r}} \frac{d\bar{r}}{\sqrt{1 - \kappa \bar{r}^2}} = R(t) l(\bar{r}) \quad (68)$$

$$l(\bar{r}) = \int_0^{\bar{r}} \frac{d\bar{r}}{\sqrt{1 - \kappa \bar{r}^2}}$$

If space is flat with $\kappa = 0$, we get $l(\bar{r}) = \bar{r}$ and

$$r(t) = R(t) \bar{r} \quad (69)$$

Suppose that observer is at the original point of coordinates system and space is flat, due to spatial expansion, the speed of celestial body relative to observer is

$$V(r) = \dot{r}(t) = \dot{R}(t) \bar{r} \quad (70)$$

It is a wrong understanding in cosmology that celestial bodies have no velocities when co-moving coordinates are used. In fact, if $\dot{R}(t) \neq 0$, there is certainly a relative velocity according to (70).

The speed shown in (70) is proportional to \bar{r} . For a certain $\dot{R}(t)$, when \bar{r} is large enough, we could have $V > c$. This result will cause a great trouble for cosmology. Many problems in the current cosmology are related to it. In the early period of cosmology, because the speeds of observed celestial bodies are low with $V \ll c$, (70) is tenable approximately. In fact, the Hubble formula is related to (70). However, if the distance is large enough, the inadaptability of (70) will emerge.

2. Luminosity Distance and Proper Distance

The distances of celestial bodies can not be measured directly in general. The indirect methods are needed and the concept of luminosity distance is used. The energy that celestial body emitted in unit time is called as luminosity L . What measured in astronomy is its brightness B . Let d_L represent luminosity distance, their relation is $B = L / (4\pi d_L^2)$. Suppose that a celestial body emitted N photons at time t_1 with frequency ν_1 , we have $L = Nh\nu_1 / \Delta t_1$. During the time $t_0 \sim t_0 + \Delta t_0$, these photon arrive at the spherical surface with radius $r = R_0 \bar{r}$ and the frequency becomes ν_0 , we have $B = Nh\nu_0 / (4\pi R_0^2 \bar{r}^2 \Delta t_0)$. So we get $d_L = r(1+z) = R_0 \bar{r}(1+z)$.

If space-time is flat with $\Delta t_0 = \Delta t_1$, we have $d_L = R\bar{r}(1+z)^{1/2}$. If light's frequency does not change

with $\nu_1 = \nu_0$ when it propagates in space, luminosity distance is equal to real distance. For example, as shown in this paper, light's source moves in flat space and we only consider the Doppler's shift. In this case, the light's frequency is unchanged and we can take $d_L = r = R_0 \bar{r}$.

3. The Doppler's Red Shift

Suppose that luminous celestial body is at rest in reference frame K , the emitted light's proper frequency and wave length are ν and λ . Reference frame K_0 moves in a uniform speed V relative to K . Observing in K_0 , the received light's frequency and wave length become ν_0 and λ_0 . The definition of frequency shift is

$$z = \frac{\nu}{\nu_0} - 1 = \frac{\lambda_0}{\lambda} - 1 \quad (71)$$

If K moves apart from K_0 along the radial direction with speed $V \ll c$, according to the Doppler's formula, the red shift is

$$z = \sqrt{\frac{1+V(t)/c}{1-V(t)/c}} - 1 \approx \frac{V(t)}{c} \quad (72)$$

The Doppler's frequency shift is only related to speed, having nothing to do with distance. As long as light is emitted, no matter how far the observers are, received frequencies are the same. In this sense, the metric red shift is completely different fundamentally from the Doppler's red shift.

4. The Hubble Red Shift

Suppose that there is no gravitational field in space or space is flat, light's frequency does not change in its propagation process, so luminosity distance is equal to practical distance. By introducing coordinate transformation $r(t) = R(t) \bar{r}$ which is so called space expansion and define the Hubble constant $H(t) = \dot{R}(t)/R(t)$, the Hubble red shift is

$$z = \frac{H(t)}{c} r(t) = \frac{\dot{R}(t)}{cR(t)} R(t) \bar{r} = \frac{\dot{R}(t) \bar{r}}{c} = \frac{V(t)}{c} \quad (73)$$

Comparing (73) with (72), it is obvious that the Hubble formula is the approximation of the Doppler's formula under the condition $V \ll c$. It seems that the Hubble red shift is proportional to distance $r(t)$. However, because the Hubble constant contains speed factor $\dot{R}(t)$ actually, the formula is related to velocity essentially. In fact, the Hubble red shift is considered to be caused by the recession velocity of celestial body. In this way, the universal expansion is deduced. The red shift observed in cosmology is also related to gravity. Strictly speaking, we should consider both the Doppler's and gravitational red shifts simultaneously in cosmology.

5. The Red Shift Caused by the R-W Metric

As shown in **Figure 4**, suppose that a luminous celestial body is located at position $r(t_1) = R(t_1)\bar{r}$ at moment t_1 . The frequency of emitted light is ν_1 . The observer at original point O receives the light at moment t_0 . The frequency of received light is ν_0 . Suppose that light source emits another light in time interval $t_1 + \tau_1$ with period $\tau_1 = c/\nu_1$. If the light is received by observer located at original point O at moment $t_0 + \tau_0$, the frequency of received light is ν_0 .

According to **Figure 4** and based on the R-W metric, we can obtain (1) which is related to scalar $R(t)$ and not related to the speed factor $\dot{R}(t)$ of spatial expansion. But the Doppler's red shift is related to $\dot{R}(t)$, having nothing to do with $R(t)$. Both are completely different.

Let's discuss the real meaning of Formula (1) now. In order to determine red shift, we have to know ν_0 and ν_1 . Because we determine ν_0 by measurement, the key is to determine ν_1 . In practice, ν_1 is considered as light's proper frequency. Suppose that there is no gravity field in space without gravity red shift. Suppose that there is an observer located at point $r(t_1)$ who is at rest with light's source. According to this observer, light's frequency is ν_1 , *i.e.*, proper frequency. When light travels from $r(t_1)$ to original point $r_0(t_0) = 0$, frequency changes from ν_1 to ν_0 . That is to say, the change of frequency is caused by the distance between light's source and observer, rather than the speed of light's source. This is just the essence of metric red shift.

However, this idea will cause serious problem, inconsistent with the Doppler's red shift. According to observer at original point $r_0(t_0) = 0$, the celestial body at point has a moving speed. The frequency of light emitted by the celestial body is not proper frequency with the Doppler's red shift. The change of frequency is related to speed and not related to distance. In fact, if frequency is related to distance, when a static observer measure the frequency of light emitted by a static light's source in distance, red shift would be founded. Obviously, this result violates experiments and basic physical laws and is completely impossible. So the red shift caused by the spatial expansion can only be the Doppler's red shift related to speed, rather than the metric red shift related to distance.

If there is gravity field in space, according to general

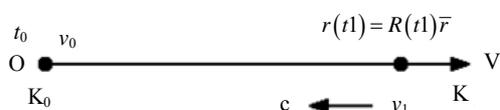


Figure 4. The red shift caused by the R-W metric.

relativity, gravity will cause red shift. In this case, besides the Doppler's red shift, we should consider gravity red shift too. Because the intensities of gravity are different in different places, ν_1 is related to the position where light's source is located. So ν_1 is not proper frequency again.

However, in practical observation, we can not measure the intensity of gravity field in distance place, so we do not know ν_1 actually. We only compare observed frequency ν_0 with proper frequency to calculate red shift. So we can not used (1) to describe gravity red shift. In fact, if the universe is static, we have $R(t_1) = R(t_0)$ and $z = 0$, even though there exist gravity field and gravity red shift.

So metric red shift described by (1) is different from the Doppler's shift and gravity red shift. It contradicts with the Doppler's red shift. We take some concrete examples to prove this point further below.

6. The Red Shift in the Uniform Expansion Process of Space

The R-W metric can be used to describe both the uniform and accelerating expansion of space. The uniform expansion is equal to the uniform motion of light's source. In this case, ν_1 is the proper frequency of light for the observer located at K . For the observers at rest in K_0 , light's proper frequency is also ν_1 , but received frequency is ν_0 . Let $R(t) = a + bt$, a and $b \ll c$ are constants. The speed of spatial expansion is $V(t) = \dot{R}(t)\bar{r} = b\bar{r}$. According to the Doppler's and Hubble's formula, we have the same red shift

$$z = \frac{\sqrt{1+V(t)/c}}{\sqrt{1-V(t)/c}} \approx \frac{V(t)}{c} = \frac{b\bar{r}}{c} \tag{74}$$

The red shift is proportional to coordinate \bar{r} (distance), having nothing to do with the time that light was emitted. The red shift is the same no matter what time observer observes. However, according to the metric red shift formula (1), we have

$$z = \frac{a + bt_0}{a + bt_1} - 1 = \frac{b(t_0 - t_1)}{a + bt_1} \tag{75}$$

z represents the red shift that observer observes at moment t_0 . The Hubble constant at moment t_1 is

$$H(t_1) = \frac{\dot{R}(t_1)}{R(t_1)} = \frac{b}{a + bt_1} \tag{76}$$

The uniform expansion of space corresponds to the uniform motion of light's source, and light's speed is unrelated to the motion of light's source, or light's speed is invariable in vacuum. Let $d_L(t_1) = c(t_0 - t_1)$, in this case, $d_L(t_1)$ is just the distance between observer at original point and luminous celestial body at moment t_1 .

Therefore, we can write (75) in the form of the Hubble's red shift

$$z = H(t_1)(t_0 - t_1) = \frac{H(t_1)}{c} d_L(t_1) \tag{77}$$

It seems that (77) is the same as the Hubble's formula. But it is different from (74) essentially. According to (74), red shift is proportional to \bar{r} and is not a function of time. We observe at present time t_0 , and red shift is $z \neq 0$. According to (77), red shift is related to $d_L(t_1) = c(t_0 - t_1)$. When $t_0 = t_1$, the distance between light's source and observer is zero, red shift is also zero even though the relative speed between them is non-zero.

Speaking simply, according to the Doppler's formula, red shift is related to the velocity of light source, having nothing to do with distance. When the distance is zero and light source's speed is light's speed, red shift infinite. According to the formula based on R-W metric, red shift is related to the distance, and unrelated to the velocity of light source. When the distance is zero and light source's speed is light's speed, red shift is still zero. The difference is so great and essential so that they can not be consistent at all.

7. The Red Shift in the Uniform Accelerating Expansion Process of Space

If light's source is accelerated in vacuum, according to general relativity, non-inertial motion is equal to gravitational field and gravity red shift will be produced. Therefore, in this case, when luminous body moves in gravitational field, gravity red shift should also be considered. Because the red shifts are different in different places with different intensities of gravity, the frequency ν_1 of emitted light is different in different places. So ν_1 shown in (1) is not proper frequency again. The observer located at original point O does not know the initial value ν_1 . He can only compare received frequency ν_0 with proper frequency. So in this case, (1) has nothing to do with practical measurement. That is to say, if gravity or acceleration effects are considered, Formula (1) is unavailable.

If gravity and acceleration effects are omitted, we discuss the differences between the Doppler's and Hubble's red shifts and the metric red shift. Suppose that space is flat, light's source is uniformly accelerated with

$$R(t) = a + bt + \frac{1}{2}gt^2 \tag{78}$$

$$V(t) = \dot{R}(t)\bar{r} = (b + gt)\bar{r} \tag{79}$$

When $V(t) \ll c$, the Doppler's red shift at moment t_1 is

$$z = \sqrt{\frac{1 + (b + gt_1)\bar{r}/c}{1 - (b + gt_1)\bar{r}/c}} - \frac{(b + gt_1)\bar{r}}{c} \tag{80}$$

Because acceleration effect has been omitted, light's frequency is unchanged in the process of light's propagation. The red shift observed by observer at original point at time t_0 is also represented by (80). The Hubble constant at time t_1 is

$$H(t_1) = \frac{\dot{R}(t_1)}{R(t_1)} = \frac{b + gt_1}{a + bt_1 + gt_1^2/2} \tag{81}$$

So the Hubble's red shift is

$$\begin{aligned} z &= \frac{H(t_1)}{c} r(t_1) = \frac{\dot{R}(t_1)}{cR(t_1)} R(t_1)\bar{r} \\ &= \frac{\dot{R}(t_1)\bar{r}}{c} = \frac{(b + gt_1)\bar{r}}{c} \end{aligned} \tag{82}$$

It is the same as the Doppler's red shift (80). If we consider (1), the red shift becomes

$$z = \frac{a + bt_0 + gt_0^2/2}{a + bt_1 + gt_1^2/2} - 1 \tag{83}$$

By using (81) and let $d_L(t_1) = c(t_0 - t_1)$, we can write (82) as

$$z = \frac{H(t_1)}{c} d_L(t_1) - \frac{q(t_1)}{2} \frac{H^2(t_1)}{c^2} d_L^2(t_1) \tag{84}$$

This is considered as the revised Hubble formula in cosmology. In which $d_L(t_1)$ may not be the real distance of light's source, we called it as apparent distance. $q(t_1)$ is the so-called decelerated factor with

$$q(t_1) = -\frac{g(a + bt_1 + gt_1^2/2)}{(b + gt_1)^2} \tag{85}$$

(84) is completely different from (80) and (82), and they are incompatible. According to (80) and (82), red shift is still proportional to coordinate \bar{r} . At present moment, we have $z = (b + gt_0)\bar{r}$. But according to (84), the relation between red shift and $d_L(t_1)$ is not linear again. At present moment, we have $z = 0$, even though the relative speed between them may not be zero. It is obvious that the Doppler's red shift and the Hubble's red shift are consistent, but they are completely different from the metric red shift.

8. The Red Shift of Metric in General Situations

If the acceleration of spatial expansion is arbitrary, according to standard theory, we develop $R(t)$ into the Taylor series in the neighborhood of t_0 with [8]

$$\begin{aligned} R(t) &= R(t_0) \left[1 + \frac{\dot{R}(t_0)}{R(t_0)}(t-t_0) + \frac{\ddot{R}(t_0)}{R(t_0)} \frac{(t-t_0)^2}{2} + \dots \right] \\ &= R(t_0) \left[1 + H(t_0)(t-t_0) - \frac{1}{2}q_0 H^2(t_0)(t-t_0)^2 + \dots \right] \end{aligned} \tag{86}$$

Here the decelerated factor is

$$q_0(t_0) = -\frac{\ddot{R}(t_0)R(t_0)}{\dot{R}^2(t_0)^2} \quad (87)$$

If the items in the bracket of (86) are small enough, let $d_L(t_1) = c(t_0 - t_1)$ and substitute it in (1), we can obtain

$$z = \frac{H(t_0)}{c}d_L(t_1) + \frac{q(t_0)H^2(t_0)}{2c^2}d_L^2(t_1) + \dots \quad (88)$$

(88) is considered as the revised Hubble formula, although $d_L(t_1)$ may not be the real distance of light's source. However, the formula should satisfy the condition that $t_0 - t_1$ is small enough, otherwise the series is divergent. Besides, it should note that $H(t_0)$ and $q_0(t_0)$ in (86) are the quantities at present time. But in (82) and (84), $H(t_1)$ and $q(t_1)$ are the quantities at past time.

However, for the problems of cosmology, $t_0 - t_1$ may be very large. For example, for the supernova of high red shift, $t_0 - t_1$ may be billion years. Because the high order items in (86) are proportional to $(t_0 - t_1)^n$, when $n \rightarrow \infty$, the formula is infinite, so it is unsuitable in general. In fact, as shown in (1), the positions of times t_0 and t_1 are equal. We have two independent variables

t_0 and t_1 actually. When their difference is great, we can only develop $R(t_0)$ into the Taylor series in the neighborhood of t_0 and develop $R(t_1)$ into the Taylor series in the neighborhood of t_1 . The results should be

$$R(t_0) = R(t_0) \left[1 + H(t_0)\Delta t_0 - \frac{1}{2}q_0H^2(t_0)\Delta t_0^2 + \dots \right] \quad (89)$$

$$R(t_1) = R(t_1) \left[1 + H(t_1)\Delta t_1 - \frac{1}{2}q_1H^2(t_1)\Delta t_1^2 + \dots \right] \quad (90)$$

Here $\Delta t_0 \ll 1$ and $\Delta t_1 \ll 1$. So (1) should be

$$\begin{aligned} z &= \frac{1 + H(t_0)\Delta t_0 - \frac{1}{2}q_0H^2(t_0)\Delta t_0^2 + \dots}{1 + H(t_1)\Delta t_1 - \frac{1}{2}q_1H^2(t_1)\Delta t_1^2 + \dots} - 1 \\ &\approx H(t_0)\Delta t_0 - H(t_1)\Delta t_1 - \frac{1}{2}q_0H^2(t_0)\Delta t_0^2 \\ &\quad + \frac{1}{2}q_1H^2(t_1)\Delta t_1^2 \dots \end{aligned} \quad (91)$$

It is completely different from (88). That is to say, we can not obtain the revised Hubble's formula (88) from the metric red shift formula (1) in general situations.