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A Closed Model of the Universe

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ABSTRACT

A closed model of the universe was constructed according to the assumption that very minor fraction of the dark energy transfers so slowly to matter and radiation. The cosmological parameter Λ is no longer fixed but represents so slowly decreasing function with time. In this model the universe expands to maximum limit at t_{me} = 26.81253 Gyr, then it will contract to a big crunch at t_{bc} = 53.6251 Gyr. Observational tests to the closed cosmic model were illustrated. Distributions of the universe expansion and contraction speed were established in this model which indicated that the expansion speed in the early universe is appreciably high, then it will decrease rapidly until it vanishes at t_{me}. However, the contraction speed of the universe increases continuously until the time just before t_{bc}. Distributions of the universe expansion and contraction acceleration were performed empirically which confirmed the previous result. In the closed cosmic model the universe history can be categorized into six main stages, these are the first radiation epoch, the first matter epoch, the first dark energy epoch, the last dark energy epoch, the last matter epoch and the last radiation epoch. Distributions of the density parameters of the radiation, matter, dark energy and the total density as well as the distributions of temperature of the radiation and non-relativistic matter were all investigated in this model at all epochs of the universe.

Keywords: Cosmological Parameter; Cosmology; Cosmic Dynamics

1. Introduction

In pervious two articles [1,2] the cosmological parameter Λ was assumed constant in five general cosmic models. However, in some cosmological studies Λ is not actually perfectly constant but exhibits slow variation, so Λ is often described as quintessence [3-6]. In other words, the dark energy density $\left(\rho_\Lambda = \dfrac{\Lambda c^4}{8\pi G}\right)$ does not remain constant with time.

This point of view is in a good agreement with the Heisenberg's Uncertainty Principle that there is an uncertainty in the amount of energy which can exist. This small uncertainty allows non-zero energy ΔE to exist for short intervals of time $\left(\Delta t = \dfrac{h/2\pi}{\Delta E}\right)$, where h is Planck's constant $\left(6.626\times10^{-14}\ \mathrm{m}^2\cdot\mathrm{Kg}\cdot\mathrm{s}^{-1}\right)$.

As a result of the equivalence between matter and energy, these small energy fluctuations can produce virtual pairs of matter particles (particles and their anti-particles must be produced simultaneously) which come into existence for a short time and then disappear to produce photons.

In the present study ρ_Λ is assumed to be very slowly decreasing function of the cosmic time t such that any decrease in $\dfrac{\rho_\Lambda}{c^2}$ say $\Delta\left(\dfrac{\rho_{\Lambda,t}}{c^2}\right)$ should be compensated by increasing each of the matter density $\rho_{m,t}$ and radiation density $\dfrac{\rho_{r,t}}{c^2}$ by $\dfrac{1}{2}\Delta\left(\dfrac{\rho_{\Lambda,t}}{c^2}\right)$.

The importance of this study is to know under what cosmological conditions the universe can be contracting to big crunch rather than expanding for ever as shown in the five general cosmic models investigated in [1].

In Section 2, a detailed description is given for the methodology. Determination of t_{me} is explained in Section 3. Observational tests of the closed cosmic model are illustrated in Section 4. Results and discussion are presented in Section 5. Finally the conclusion is displaced in Section 6.

2. Methodology

From [1] we have seen that the densities of matter $\rho_{m,t}$, radiation $\rho_{r,t}$ and dark energy $\rho_{\Lambda,t}$ at a cosmic time

t are given by

$$\rho_{m,t} = \rho_{c,t}\Omega_{m,t}. \tag{1}$$

$$\rho_{r,t} = \grave{o}_{c,t}\Omega_{r,t}. \tag{2}$$

$$\rho_{\Lambda,t} = \grave{o}_{c,t}\Omega_{\Lambda,t}. \tag{3}$$

where

$$\rho_{c,t} = \frac{3H^2}{8\pi G}. \tag{4}$$

$$\grave{o}_{c,t} = \frac{3H^2c^2}{8\pi G} = \rho_{c,t}c^2. \tag{5}$$

$$\Omega_{m,t} = \left(\frac{H_0}{H}\right)^2 \frac{\Omega_{m,0}}{a^3}. \tag{6}$$

$$\Omega_{r,t} = \left(\frac{H_0}{H}\right)^2 \frac{\Omega_{r,0}}{a^4}. \tag{7}$$

$$\Omega_{\Lambda,t} = \left(\frac{H_0}{H}\right)^2 \Omega_{\Lambda,0}. \tag{8}$$

$$H(t) = \frac{H_0}{a}\left[1-\Omega_{\Lambda,0}\left(1-a^2\right)+\Omega_{m,0}\left(\frac{1}{a}-1\right) + \Omega_{r,0}\left(\frac{1}{a^2}-1\right)\right]^{1/2}. \tag{9}$$

Substituting by (4), (6) in (1) we get

$$\rho_{m,t} = \frac{3H_0^2}{8\pi G}\frac{\Omega_{m,0}}{a^3}$$

Or,

$$\rho_{m,t} = \rho_{c,0}\frac{\Omega_{m,0}}{a^3}. \tag{10}$$

Similarly we can find

$$\frac{\rho_{r,t}}{c^2} = \rho_{c,0}\frac{\Omega_{r,0}}{a^4}. \tag{11}$$

$$\frac{\rho_{\Lambda,t}}{c^2} = \rho_{c,0}\Omega_{\Lambda,0} = \text{constant}. \tag{12}$$

Now assume a very small decrease in $\frac{\rho_{\Lambda,t}}{c^2}$ about 1% per Gyr, so the decrease in $\left(\frac{\rho_{\Lambda,t}}{c^2}\right)$ in cosmic time t is expressed as

$$\Delta\left(\frac{\rho_{\Lambda,t}}{c^2}\right) = 0.01\frac{\rho_{\Lambda,t}}{c^2}t. \tag{13}$$

According to the conservation law of mass and energy the decrease $\Delta\left(\frac{\rho_{\Lambda,t}}{c^2}\right)$ in the energy density $\frac{\rho_{\Lambda,t}}{c^2}$ is compensated by increase of $\frac{1}{2}\left(\frac{\rho_{\Lambda,t}}{c^2}\right)$ in each of $\rho_{m,t}$, and $\frac{\rho_{r,t}}{c^2}$.

Therefore at the cosmic time t the new values of $\frac{\rho_{\Lambda,t}}{c^2}, \rho_{m,t}$ and $\frac{\rho_{r,t}}{c^2}$ are given by

$$\frac{\rho'_{\Lambda,t}}{c^2} = \rho_{c,0}\Omega_{\Lambda,0} - \Delta\left(\frac{\rho_{\Lambda,t}}{c^2}\right). \tag{14}$$

$$\rho'_{m,t} = \rho_{c,0}\frac{\Omega_{m,0}}{a^3} + \frac{1}{2}\Delta\left(\frac{\rho_{\Lambda,t}}{c^2}\right). \tag{15}$$

$$\frac{\rho'_{r,t}}{c^2} = \rho_{c,0}\frac{\Omega_{r,0}}{a^4} + \frac{1}{2}\Delta\left(\frac{\rho_{\Lambda,t}}{c^2}\right). \tag{16}$$

The slowly varying cosmological parameter is

$$\Lambda(t) = \frac{8\pi G\rho'_{\Lambda,t}}{c^4}. \tag{17}$$

Using Equations (1)-(5) and (14)-(16) the new values of density parameters in the expanding cosmic model at time t are

$$\Omega'_{\Lambda,t} = \frac{\rho'_{\Lambda,t}}{c^2\rho_{c,t}}. \tag{18}$$

$$\Omega'_{m,t} = \frac{\rho'_{m,t}}{\rho_{c,t}}. \tag{19}$$

$$\Omega'_{r,t} = \frac{\rho'_{r,t}}{c^2\rho_{c,t}}. \tag{20}$$

Let $s = \frac{H(t)}{H_0}$ then Equations (6)-(8) can be written as

$$\Omega_{m,0} = s^2a^3\Omega_{m,t}. \tag{21}$$

$$\Omega_{r,0} = s^2a^4\Omega_{r,t}. \tag{22}$$

$$\Omega_{\Lambda,0} = s^2\Omega_{\Lambda,t}. \tag{23}$$

Substituting by (21)-(23) in (9) and using (18)-(20) we get the Hubble parameter in the closed cosmic model at time t

$$H'(t) = \frac{H_0}{a}\left[1-s^2\Omega'_{\Lambda,t}\left(1-a^2\right)+s^2a^3\Omega'_{m,t}\left(\frac{1}{a}-1\right) + s^2a^4\Omega'_{r,t}\left(\frac{1}{a^2}-1\right)\right]^{1/2}$$

or,

$$H'(t) = \frac{H_0}{a}\left[1 - s^2\Omega'_{\Lambda,t}\left(1-a^2\right) + s^2\Omega'_{m,t}\left(a^2 - a^3\right) + s^2\Omega'_{r,t}\left(a^2 - a^4\right)\right]^{1/2}. \tag{24}$$

The critical mass density in the closed cosmic model at time t becomes

$$\rho''_{c,t} = \frac{3H'^2(t)}{8\pi G}. \tag{25}$$

The new density parameters in the closed cosmic model at time t are

$$\Omega''_{\Lambda,t} = \frac{\rho'_{\Lambda,t}}{c^2\rho''_{c,t}}. \tag{26}$$

$$\Omega''_{m,t} = \frac{\rho'_{m,t}}{\rho''_{c,t}}. \tag{27}$$

$$\Omega''_{r,t} = \frac{\rho'_{r,t}}{c^2\rho''_{c,t}}. \tag{28}$$

And the total density parameter in the closed cosmic model at time t is

$$\Omega''(t) = \Omega''_{r,t} + \Omega''_{m,t} + \Omega''_{\Lambda,t}. \tag{29}$$

The speed of the universe dynamics in the closed cosmic model is obtained from Equation (24) such that

$$\dot{a}(t) = H'(t)a$$

or,

$$\dot{a}(t) = H_0\left[1 - s^2\Omega'_{\Lambda,t}\left(1-a^2\right) + s^2\Omega'_{m,t}\left(a^2 - a^3\right) + s^2\Omega'_{r,t}\left(a^2 - a^4\right)\right]^{1/2}. \tag{30}$$

The acceleration of the universe dynamics in the closed cosmic model is found empirically as

$$\ddot{a}(t) = \lim_{\Delta t \to 0}\frac{\Delta\dot{a}(t)}{\Delta(t)}. \tag{31}$$

The time interval between two instants with scale factors a_1, a_2 during the universe expansion is given by Equation (16) in [1] as

$$t_2 - t_1 = \frac{1}{H_0}\int_{a_1}^{a_2}\left[1 - \Omega_{\Lambda,0}\left(1-a^2\right) + \Omega_{m,0}\left(\frac{1}{a}-1\right) + \Omega_{r,0}\left(\frac{1}{a^2}-1\right)\right]^{-\frac{1}{2}}\mathrm{d}a. \tag{32}$$

However, during the universe contraction if a2 < a1 then modulus of the right hand side of Equation (32) should be taken. The redshift lookback time relation in the closed cosmic model is given by Equation (18) in [1]. In addition, the distributions of temperature at different epochs of the universe depend on relations similar to Equations (33), (34) and (37) in [1].

3. Determination of t_{me}

The time of the maximum expansion of the universe in the closed model is evaluated by iterative procedure as follows:

1) Start with $a_1 = 0$ at $t_1 = 0$, and let $a_2 = a_1 + \frac{I}{D}$, $I = 1,2,3,4,5,\cdots,1000$, $D = 10^2$.

2) Calculate 1000 values of t_2, $\dot{a}(t_2)$ and $\frac{\dot{a}^2(t_2)}{H_0^2}$ using Equations (32), (30). The value of t_2 corresponding to the minimum positive value of $\dot{a}(t_2)$ is assumed to be t_{me_1} and

$$\dot{a}(t_2) = \dot{a}\left(t_{me_1}\right), \quad \frac{\dot{a}^2(t_2)}{H_0^2} = \frac{\dot{a}^2\left(t_{me_1}\right)}{H_0^2}.$$

3) Select $a_1 = a\left(t_{me_1}\right)$ at $t_1 = t_{me_1}$, and repeat the previous two steps where $D = 10^4$. Now the value of t_2 corresponding to the minimum positive value of $\dot{a}(t_2)$ is supposed to be t_{me_2} and

$$\dot{a}(t_2) = \dot{a}\left(t_{me_2}\right), \quad \frac{\dot{a}^2(t_2)}{H_0^2} = \frac{\dot{a}^2\left(t_{me_2}\right)}{H_0^2}.$$

4) Repeat this method several times using the values $D = 10^6, 10^8, 10^{10}$ and 10^{11}, then estimate the values $t_{me_3}, t_{me_4}, t_{me_5}$ and t_{me_6} and obtain the corresponding values of $\dot{a}(t_{me})$ and $\frac{\dot{a}^2(t_{me})}{H_0^2}$.

5) Denote these results as presented in **Table 1**, where it is noticeable that the values of $\dot{a}(t_{me})$ and $\frac{\dot{a}^2(t_{me})}{H_0^2}$. converge and become very close to zero. In other words the universe stops expending at $t = t_{me_6}$.

6) From **Table 1** one can easily find that the time of maximum expension of the universe in the closed model is $t_{me} = 26.8125327\ \text{Gyr}$. By similarity the time of big craunch is $t_{bc} = 53.6250654\ \text{Gyr}$.

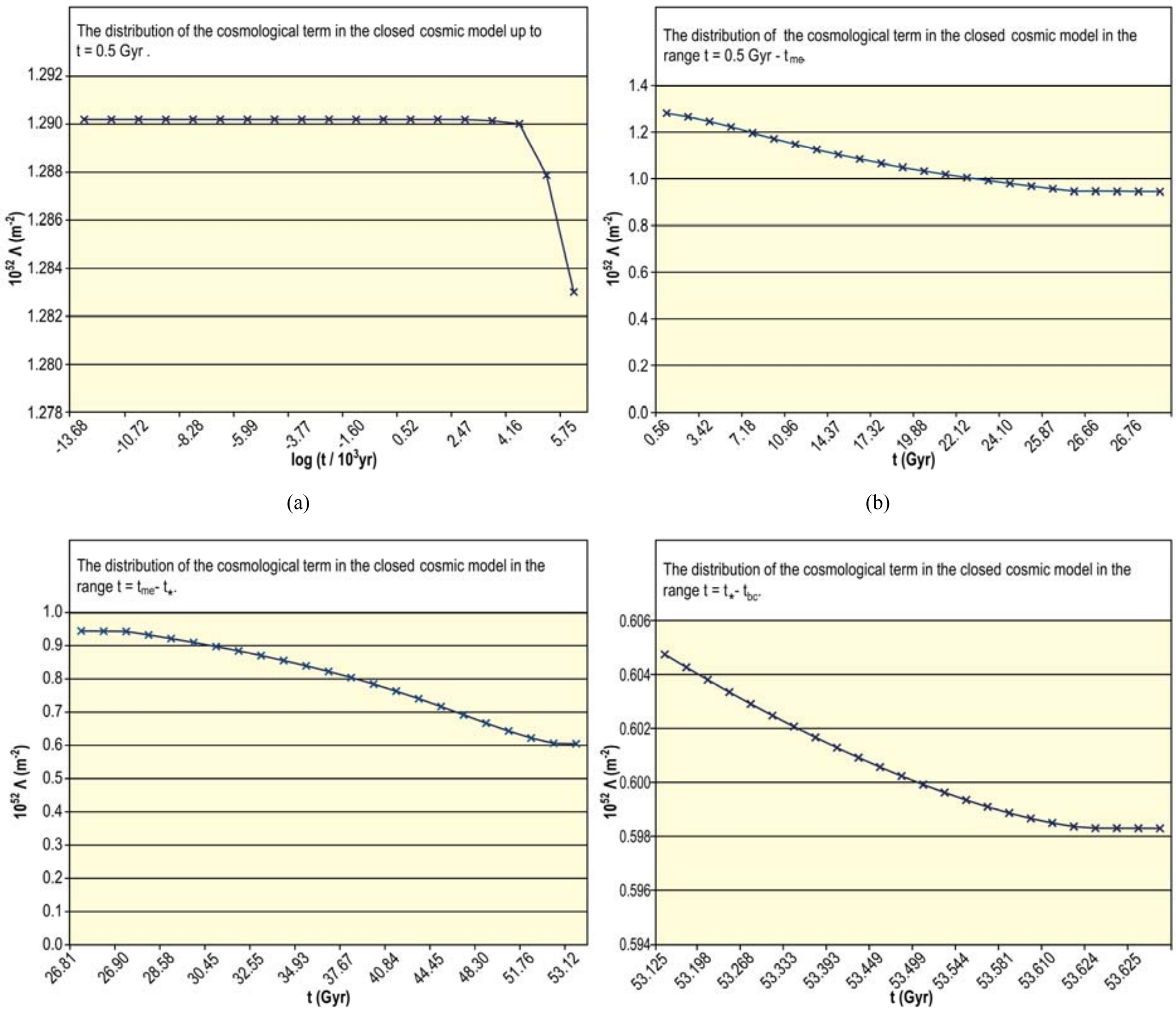
Table 1. Iterative determination of the maximum expansion time of the universe in the closed cosmic model.

t_{me}	$a(t_{me})$	$\dot{a}(t_{me})$	$\dot{a}^2(t_{me})/\mathrm{H}_0^2$
26.775089	2.37	11.278297	0.025233
26.811945	2.3755	1.415359	0.000397
26.81252920	2.37558696	0.09020085	0.00000161
26.81253246	2.37558728	0.0066986483	0.0000000089
26.8125327107	2.3755872677	0.000480250618	0.000000000046
26.812532710799	2.37558726767	0.0000404116057	0.0000000000003

4. Observational Tests to the Closed Cosmic Model

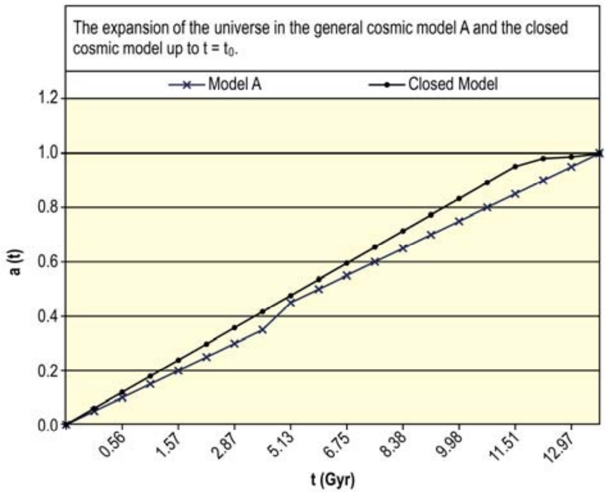
It is convenient to start by investigating the distributions of the cosmological parameter $\Lambda(t)$ in the closed cosmic model at various epochs according to Equation (17). **Figure 1(a)** shows no evident change of $\Lambda(t)$ with cosmic time until $t = 14.36$ Gyr, then $\Lambda(t)$ decreases in relatively higher rate towards $t = 0.5$ Gyr. On the other hand $\Lambda(t)$ exhibits a gradual change with time in the time range $t = 0.5\ \text{Gyr} - t_{me}$ as seen in **Figure 1(b)**, where $t_{me} = 26.81$ Gyr is the time of maximum expansion of the universe in the closed cosmic model. The slow variation of $\Lambda(t)$ with t is also noticeable in the time ranges $t = t_{me} - t_*$, $t = t_* - t_{bc}$, as displaced in **Figures 1(c)** and **(d)** respectively where $t_{bc} = 53.63$ Gyr is the time of big craunch of the universe in the closed cosmic model and $t_* = t_{bc} - 0.5$ Gyr.

Figure 2(a) shows that the expansion distribution of the universe in the closed cosmic model up to $t = t_0$ is found using Equation (32). This distribution is in good agreement with that of the observed general cosmic model A obtained by Equation (16) in [1]. Moreover, at $t = 12.97$ Gyr, these two distributions become identical. The redshift look-back time distributions in these two models up to $t = t_0$ were established and presented in **Figure 2(b)**. Both distributions are in perfect agreement. The obvious agreement between the observed general cosmic model A and the closed cosmic model as seen from **Figures 2(a)** and **(b)** strongly argues in favour of reliability of the closed cosmic model.

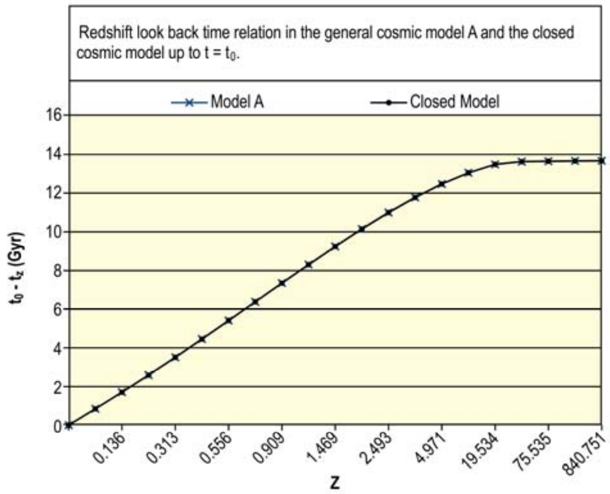


(a) (b) (c) (d)

Figure 1. (a) The distribution of the cosmological term in the closed cosmic model up to t = 0.5 Gyr; (b) The distribution of the cosmological term in the closed cosmic model in the range t = 0.5 Gyr − t_{me}; (c) The distribution of the cosmological term in the closed cosmic model in the range $t = t_{me} - t_*$; (d) The distribution of the cosmological term in the closed cosmic model in the range $t = t_* - t_{bc}$.



(a)



(b)

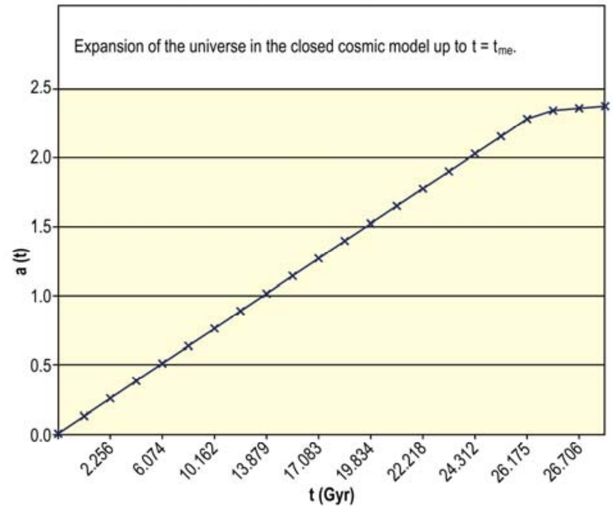
Figure 2. (a) The expansion of the universe in the general cosmic model A and the closed model up to $t = t_0$; (b) Redshift look back time relation in the general cosmic model A and the closed cosmic model up to $t = t_0$.

5. Results and Discussion

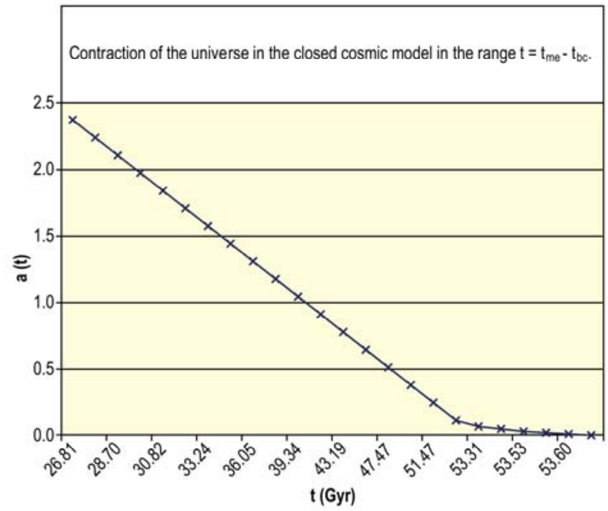
The expansion of the universe in the closed cosmic model up to $t = t_{me}$ is obtained by Equation (32) and presented in **Figure 3(a)**. It is noticeable that the increase of $a(t)$ with t is continuous as a linear relation until about $t = 26.3$ Gyr, then $a(t)$ increases relatively slow with t. Nevertheless, the contraction of the universe in the closed model in the time range $t = t_{me} - t_{bc}$ is illustrated in **Figure 3(b)**. It is obvious that $a(t)$ almost linearly decreases with t. However, $a(t)$ reduces relatively slow with t just before $t = t_{bc}$.

The distribution of the universe expansion speed $\dot{a}(t)$ in the closed model in the range $t = 0.5\ \text{Gyr} - t_{me}$ is performed using Equation (30) and displaced in **Figure 4(a)**. The value of $\dot{a}(t)$ is high in the early universe, then it decreases abruptly up to about $t = 0.26$ Gyr. Afterwards $\dot{a}(t)$ fluctuates gradually with t until $\dot{a}(t) \approx 0$ at $t = t_{me}$. On the other hand, **Figure 4(b)** exhibits the distribution of the universe contraction speed $\dot{a}(t)$ in the closed model in the range $t = t_{me} - t_*$. It is clear that the increase of $\dot{a}(t)$ with t is gradual up to $t \approx 52.4$ Gyr then $\dot{a}(t)$ rapidly increases with t until $t = t_*$.

The distribution of the universe expansion acceleration $\ddot{a}(t)$ in the closed model in the range $t = 0.5\ \text{Gyr} - t_{me}$ is deduced from Equation (31) and exhibited in **Figure 5(a)**. Abrupt increase in $\ddot{a}(t)$ with t is obvious up to $t = 2.0$ Gyr. Then $\ddot{a}(t)$ changes very slightly with t until $t \approx 25$ Gyr, where $\ddot{a}(t)$ starts decreasing gradually up to $t = 26.73$ Gyr. Afterwards, $\ddot{a}(t)$ decreases

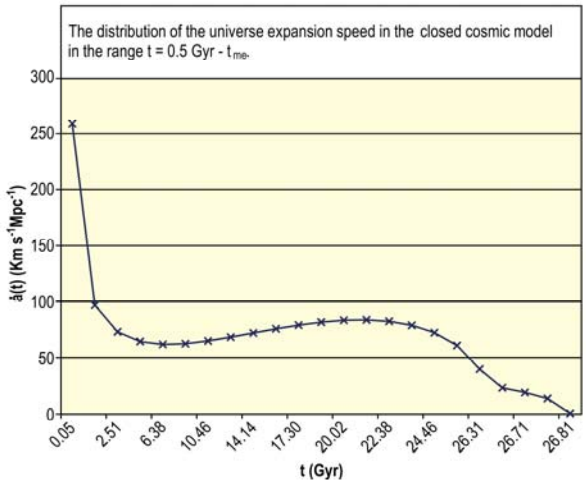


(a)

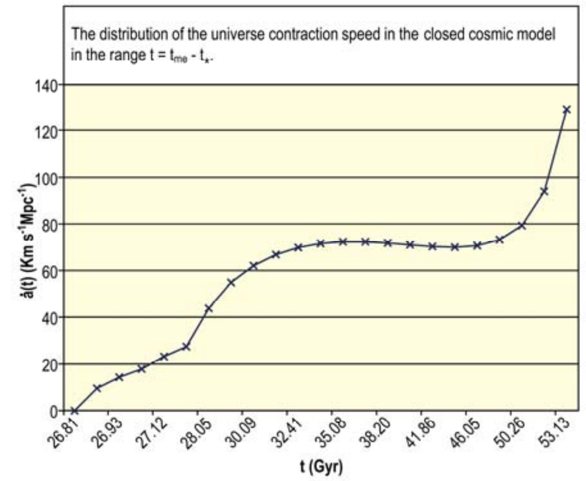


(b)

Figure 3. (a) Expansion of the universe in the closed cosmic model up to $t = t_{me}$; (b) Contraction of the universe in the closed cosmic model in the range $t = t_{me} - t_{bc}$.

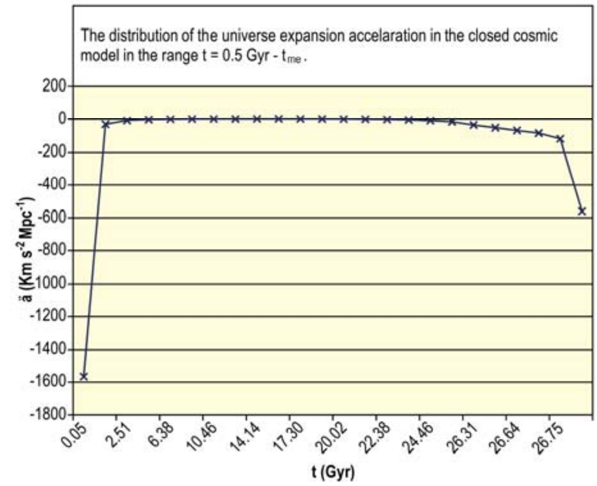


(a)

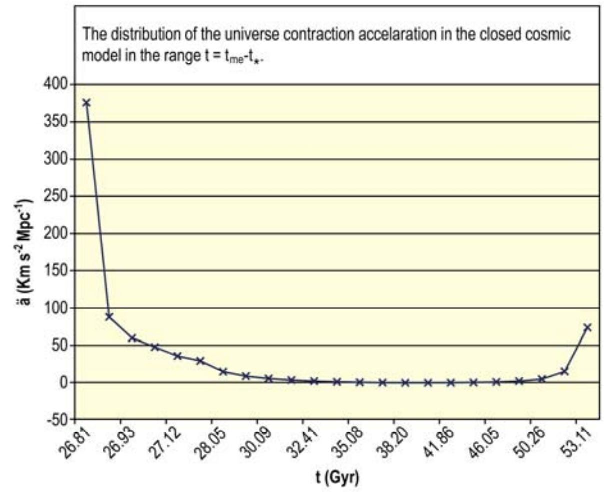


(b)

Figure 4. (a) The distribution of the universe expansion speed in the closed cosmic model in the range *t* = 0.5 Gyr − *t*$_{me}$; (b) The distribution of the universe contraction speed in the closed cosmic model in the range *t* = *t*$_{me}$ − *t*$_*$.



(a)



(b)

Figure 5. (a) The distribution of the universe expansion acceleration in the closed cosmic model in the range *t* = 0.5 Gyr − *t*$_{me}$; (b) The distribution of the universe contraction acceleration in the closed cosmic model in the range *t* = *t*$_{me}$ − *t*$_*$.

rapidly towards the maximum expansion time t_{me}. It is clear that in the range $t = 20.92$ - 20.99 Gyr $\ddot{a}(t) \approx 0$ $\text{Km} \cdot \text{s}^{-2} \cdot \text{M}_{\text{pc}}^{-1}$. Furthermore, **Figure 5(b)** shows the distribution of the universe contraction acceleration in the closed model in the range $t = t_{me} - t_*$. It is noticeable that $\ddot{a}(t)$ suddenly reduces up to $t \approx 26.90$ Gyr, then $\ddot{a}(t)$ reduces gradually until $t = 39.34$ Gyr where $\ddot{a}(t) = -0.446$. Afterwards, $\ddot{a}(t)$ raises gradually up to $t = 52.93$ Gyr where $\ddot{a}(t)$ starts increasing quite rapidly towards t_*. $\ddot{a}(t) = 0\ \text{Km} \cdot \text{s}^{-2} \cdot \text{M}_{\text{pc}}^{-1}$ in the interval $t = 43.49$ - 43.88 Gyr.

It is remarkable to note that the distributions of $\dot{a}(t)$ and $\ddot{a}(t)$ in the closed cosmic model in the ranges $t \le 0.5$ Gyr, $t = t_* - t_{bc}$ will be investigated in details in a separate study, since in these two time ranges the pressure of the cosmic fluid is significant and can not be neglected.

The distribution of the density parameters in the closed cosmic model up to $t = 0.5$ Gyr is disclosed in **Figure 6(a)**. It is prominent that the distribution of the radiation density parameter $\Omega''_{r,t}$ coincides on the distribution of the total density parameter Ω''_t up to $t = 240.2663$ yr. However, the distribution of the matter density parameter $\Omega''_{m,t}$ coincides on the distribution of Ω''_t at $t = 6.2656$ Myr. It is also obvious that the distributions of the dark energy density parameter $\Omega''_{\Lambda,t}$ and the distribution of $\Omega''_{m,t}$ are increasing while the distribution of $\Omega''_{r,t}$ remains almost fixed at the value $\Omega''_{r,t} = 1$ up to $t = 240.2663$ yr, then it starts decreasing. Nevertheless, the distribution of Ω''_t stays almost constant at the value $\Omega''_t = 1$ in this epoch of the universe. Thus

$\Omega''_{r,t} = \Omega''_{m,t} = 0.5$ at $t_{rm1} = 55915$ yr, whereas

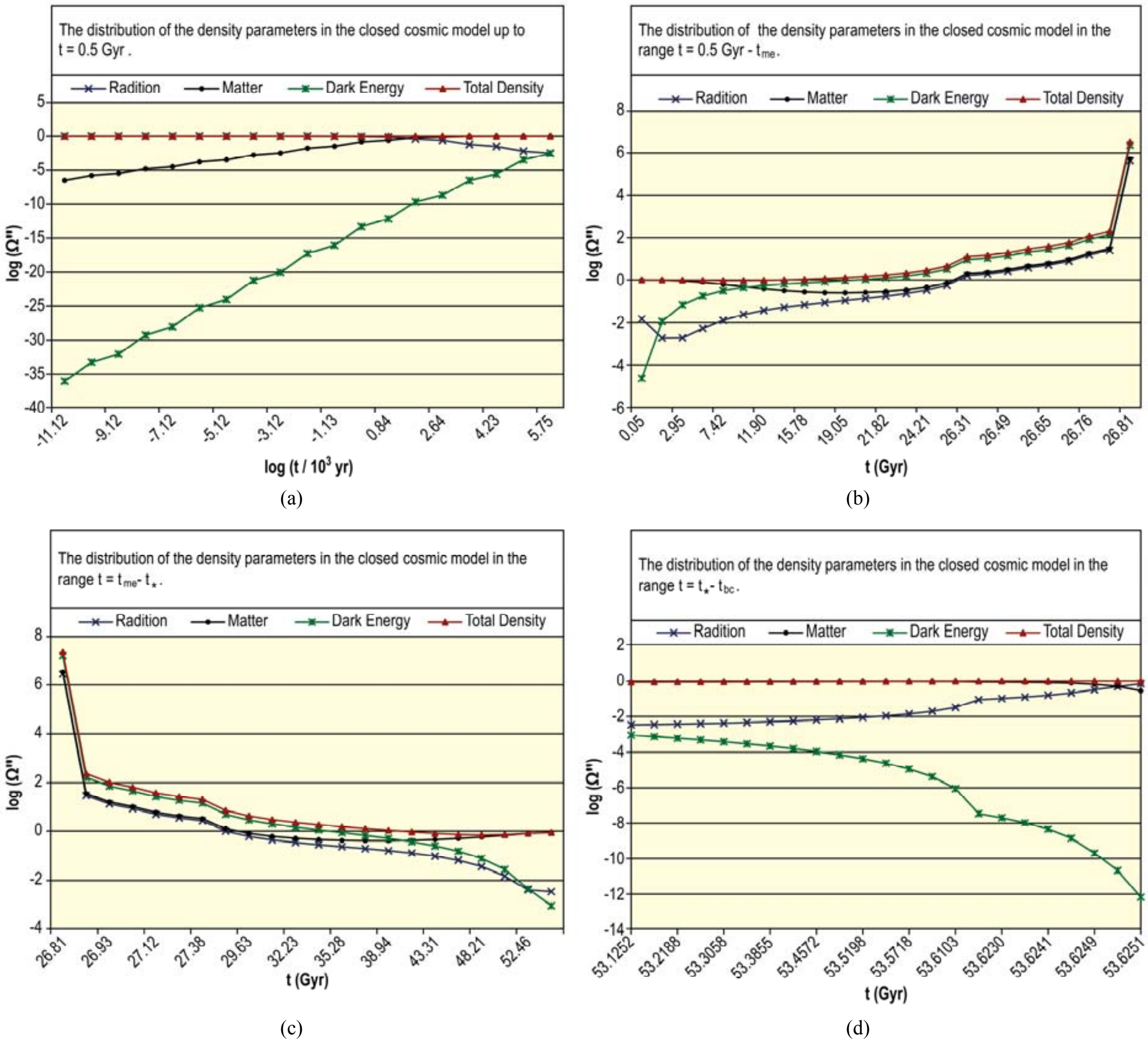


Figure 6. (a) The distribution of the density parameters in the closed cosmic model up to t = 0.5 Gyr; (b). The distribution of the density parameters in the closed cosmic model in the range t = 0.5 Gyr − t_{me}; (c) The distribution of the density parameters in the closed cosmic model in the range $t = t_{me} - t_*$; (d) The distribution of the density parameters in the closed cosmic model in the range $t = t_* - t_{bc}$.

$\Omega''_{r,t} = \Omega''_{\Lambda,t} = 0.00296$ at $t = 585.8445$ Myr . **Figure 6(b)** shows the distribution of the density parameters in the closed cosmic model in the range $t = 0.5\ \text{Gyr} - t_{me}$. It is evident that the distribution of $\Omega''_{\Lambda,t}$ displays rapid increase until the time $t_{m\Lambda 1} = 10.1022$ Gyr where $\Omega''_{m,t} = \Omega''_{\Lambda,t} = 0.4593$, then it raises gradually up to $t \approx 26.7701$ Gyr where it exhibits abrupt increase again. The distributions of $\Omega''_{\Lambda,t}, \Omega''_{t}$ become close together from $t = 16.4243$ Gyr to $t = t_{me}$. The value of Ω''_{t} is almost 1.0 in the time intervals $t = 0.5$ - 6.5121 Gyr , $t = 10.0882$ - 15.1187 Gyr. The distributions of $\Omega''_{r,t}$ and $\Omega''_{m,t}$ change quite slowly up to $t = 26.7701$ Gyr where they also raise up suddenly. They get close together from $t = 23.4536$ Gyr to $t = t_{me}$. The distribution of the density parameters in the closed cosmic model in the range $t = t_{me} - t_*$ is presented in **Figure 6(c)**. All distributions, reveal steep decrease up to $t = 26.8635$ Gyr . Distributions of $\Omega''_{r,t}, \Omega''_{m,t}$ are adjacent to each other until $t = 31.7474$ Gyr , then they diverge apart and decrease slowly. In addition, the distributions of $\Omega''_{\Lambda,t}$ and Ω''_{t} are also near each other up to $t = 28.2604$ Gyr . Afterwards these two distributions reduce gradually and get away from each other. Nevertheless, after the time $t \approx 43.3096$ Gyr the distributions of $\Omega''_{r,t}$ and $\Omega''_{\Lambda,t}$ reduce quite rapidly and intersect with each other at $t = 52.579$ Gyr where $\Omega''_{r,t} = \Omega''_{\Lambda,t} = 0.0037$. However, the distributions of $\Omega''_{m,t}$ and $\Omega''_{\Lambda,t}$ intersect at $t_{m\Lambda 2} = 40.2712$ Gyr where $\Omega''_{m,t} = \Omega''_{\Lambda,t} = 0.4181$. The distributions of $\Omega''_{m,t}$ and Ω''_{t} get close to each other from $t = 45.6450\,\text{Gyr}$ until $t = t_*$. **Figure 6(d)** illustrates the distribution of density parameters in the closed cosmic model in the range $t = t_* - t_{bc}$. It is clear that the distributions of $\Omega''_{m,t}$ and Ω''_t almost coincide on each other up to about $t = 53.5818\,\text{Gyr}$, then the distribution of $\Omega''_{m,t}$ starts decreasing slightly but still close to that of Ω''_t until $t = 53.6248\,\text{Gyr}$, while Ω''_t takes the values between 0.9,1.0 throughout the interval $t = t_* - t_{bc}$. However, the distribution of $\Omega''_{r,t}$ raises gradually and intersects with the distribution of $\Omega''_{m,t}$ at $t_{rm2} = 53.6250\,\text{Gyr}$. In addition the distribution of $\Omega''_{r,t}$ gets closer to the distribution of Ω''_t at $t = t_{bc}$. Finally, the distribution of $\Omega''_{\Lambda,t}$ indicates slow decrease until about $t = 53.5752\,\text{Gyr}$ then it exposes quite rapid decrease towards the time of big Crunch.

It is essential to realize that the universe history has six main stages in the closed model, these are

1) The first radiation epoch in the range $t \le t_{rm1}$.
2) The first matter epoch in the range $t_{rm1} < t \le t_{m\Lambda 1}$.
3) The first dark energy epoch in the range $t_{m\Lambda_1} < t \le t_{me}$.
4) The last dark energy epoch in the range $t_{me} < t \le t_{m\Lambda_2}$.
5) The last matter epoch in the range $t_{m\Lambda_2} < t \le t_{rm2}$.
6) The last radiation epoch in the range $t_{rm2} < t < t_{bc}$.

These epochs of the universe with their relevant density parameters are all summarized in **Table 2**. Forthermore, the geometry of space throughout the universe history in the closed cosmic model is presented in details in **Table 3**.

One can see in **Table 3** that the space of the universe is flat just after the big bang up to $t = 6.5321\,\text{Gyr}$ where the total density parameter lies in the range $0.95 \le \Omega''_t \le 1.044$. Afterwards, the space of the universe becomes open until $t = 10.0751\,\text{Gyr}$ since $\Omega''_t < 0.95$. Then the universe space returns to flat up to $t = 15.1261\,\text{Gyr}$ as $0.95 \le \Omega''_t \le 1.044$. Afterward, the universe space gets curved then closed until $t = t_{me}$ because $1.5 \le \Omega''_t \le 3.259 \times 10^6$. Hence, the universe space remains being closed then curved up to $t = 39.3822\,\text{Gyr}$ since $1.05 \le \Omega''_t \le 1.555 \times 10^7$. Afterward the universe space evolves into flat until $t = 40.7521\,\text{Gyr}$ as $0.95 \le \Omega''_t \le 1.044$. Then the universe space develops into open up to $t = 53.48\,\text{Gyr}$ owing to $\Omega''_t < 0.95$. Eventuall the space of the universe comes back to flat until the time just before the big cranch by the reason of $0.95 \le \Omega_t \le 1.044$.

The distribution of the universe temperature $T_u(t)$ in the closed cosmic model in the first radiation epoch is obtained using Equation(34) in [1] and displayed in **Figure 7(a)**. It is evident that $T_u(t)$ reduces continuously in linear manner during this era. The temperatures of the radiation $T_r(t)$ and non relativistic matter $T_m(t)$ are determined from Equations (33), (37) in [1] respectively. The distributions of $T_r(t)$ and $T_m(t)$ in the first matter and dark energy eras are presented in **Figure 7(b)**. It is prominent that $T_r = T_m = 5609.1493\,\text{K}$ at $t = t_{rm1}$, then the distributions of $T_r(t), T_m(t)$ decrease sharply up to t = 0.0702 Gyr. However, both distributions reduce gradually afterwards. The distribution of $T_r(t)$ is above that of $T_m(t)$ throughout these two epochs. At

Table 2. Epochs of the universe history in the closed cosmic model.

Epoch	Time interval of epoch	Relevant density parameter
First radiation era	$t \le t_{rm1}(55915)$ yr	$\Omega''_{rm1} = 0.5$
First matter era	$t_{rm1} < t \le t_{m\Lambda 1}(10.1022)$ Gyr	$\Omega''_{m\Lambda 1} = 0.4593$
First dark energy rea	$t_{m\Lambda 1} < t \le t_{me}(26.8125)$ Gyr	$\Omega''_{\Lambda me} = 2.321 \times 10^6$
Last dark energy era	$t_{me} < t \le t_{m\Lambda 2}(40.2712)$ Gyr	$\Omega''_{m\Lambda 2} = 0.4181$
Last matter era	$t_{m\Lambda 2} < t \le t_{rm2}(53.6250)$ Gyr	$\Omega''_{rm2} = 0.4998$
Last radiation era	$t_{rm2} < t \le t_{bc}(53.6251)$ Gyr	$\Omega''_{rbc} = 0.7143$

Table 3. Geometry of space throughout the universe history in the closed cosmic model.

Time interval/Gyr	Total density parameter	Geometry of space in the universe	Relevant epochs
$t \le 6.5321$	$0.95 \le \Omega''_t \le 1.044$	Flat space	First radiation era and first matter era
$6.5321 < t \le 10.0751$	$\Omega''_t \le 0.95$	Open space	First matter era
$10.0751 < t \le 15.1261$	$0.95 \le \Omega''_t \le 1.044$	Flat space	First matter era and first dark energy era
$15.1261 < t \le t_{me}$	$1.5 \le \Omega''_t \le 3.259 \times 10^6$	Curved then closed space	First dark energy era
$t_{me} < t \le 39.3822$	$1.05 \le \Omega''_t \le 1.555 \times 10^7$	Closed then curved space	Last dark energy era
$39.3822 < t \le 40.7521$	$0.95 \le \Omega''_t \le 1.044$	Flat space	Last dark energy era and last matter era
$40.7521 < t \le 53.48$	$\Omega''_t \le 0.95$	Open space	Last matter era
$53.48 < t < t_{bc}$	$0.95 \le \Omega''_t \le 1.044$	Flat space	Last matter era and last radiation era

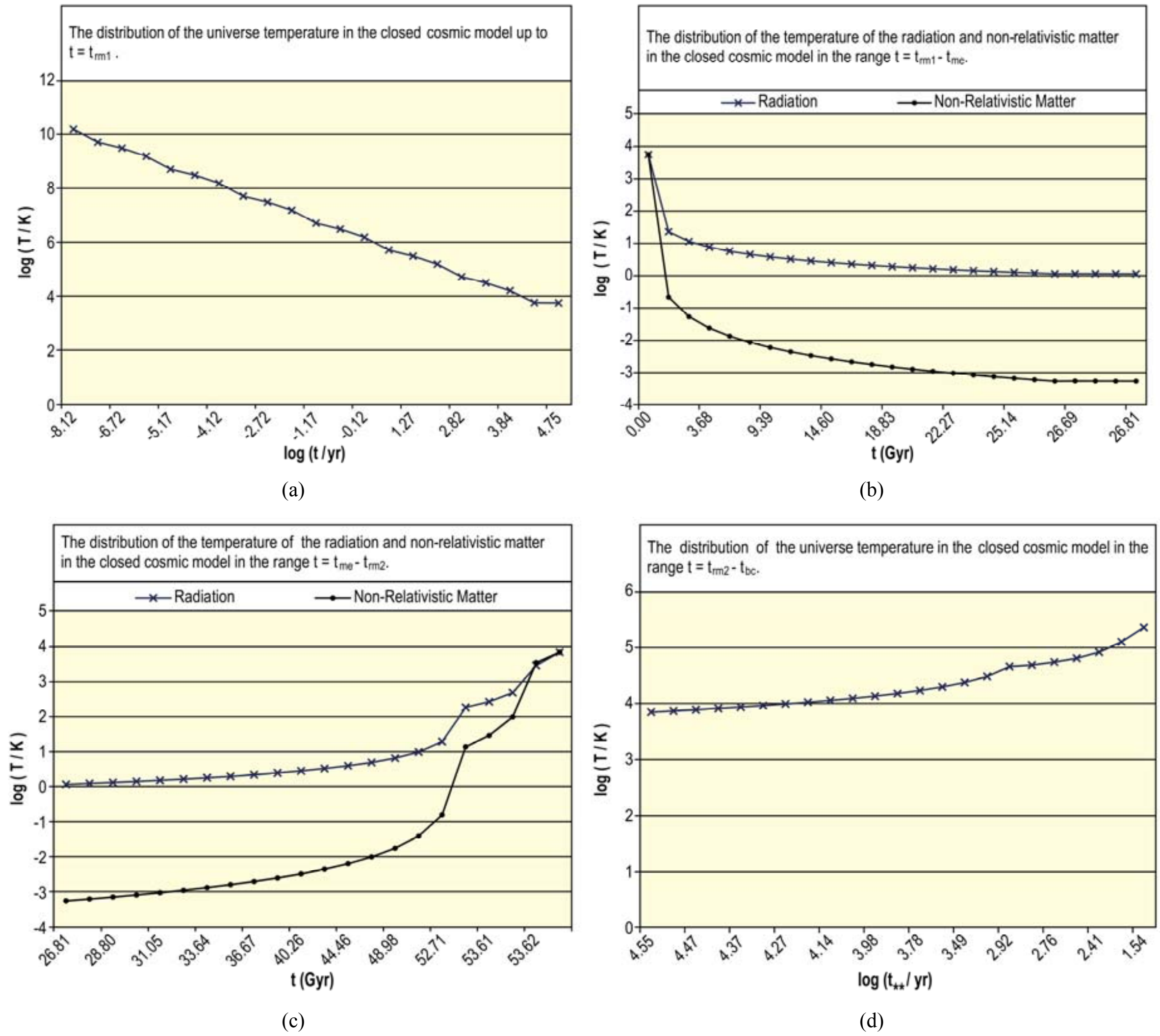


Figure 7. (a) The distribution of the universe temperature in the closed cosmic model up to $t = t_{rm1}$; (b) The distribution of temperature of the radiation and non-relativistic matter in the closed cosmic model in the range $t = t_{rm1} - t_{me}$. (c) The distribution of temperature of the radiation and non-relativistic matter in the closed cosmic model in the range $t = t_{me} - t_{rm2}$; (d) The distribution of the universe temperature in the closed cosmic model in the range $t = t_{rm2} - t_{bc}$.

$t = t_{me}$ T_r = 1.1471 K, T_m = 0.0005 K. The distribution of $T_r(t)$ and $T_m(t)$ in the last dark energy and last matter epochs are exposed in **Figure 7(c)**. Both distributions increase slowly up to t = 53.2567 Gyr, then they start raising rapidly until they join together at t_{rm2} where $T_r = T_m$ = 7032.5366 K. Eventually, **Figure 7(d)** indicates the distribution of the universe temperature in the last radiation epoch. This distribution raises slowly up to $t \approx 6007.3647$ yr before t_{bc}, then it increases rapidly to the value $T_u = 2.2593 \times 10^5$ K at t = 34.4654 yr before t_{bc}.

Further interesting physical properties of the universe in the closed cosmic model would be investigated in separate studies in comparison with the corresponding properties of the universe in the five general cosmic models.

6. Conclusion

In this study a closed model of the universe was developed depending on the assumption that very slow transfer of the dark energy to mater and radiation is allowed. Thus the cosmological parameter is no longer constant but so slowly decreasing function of time. In the light of this model the universe expands to maximum limit at $t_{me} = 26.81253$ Gyr, then it will recollape to a big crunch at $t_{bc} = 53.6251\,\text{Gyr}$. Observational tests to this model were presented. The distributions of the universe expension and contraction speed were investigated in the closed model which disclosed that the expansion speed in the early universe is very high, then it will reduce rapidly until it vanishes at t_{me}. Nevertheless, the contraction speed of the universe raises continuously until the time just before t_{bc}. The distribution of the universe expansion and contraction acceleration were carried out empirically which supported the previous result. In this model the universe history is classified in to six main eras, these are the first radiation epoch, the first matter epoch, the first dark energy epoch, the last dark energy epoch, the last matter epoch and the last radiation epoch. The distributions of the density parameters of the radiation, matter, dark energy and total density in addition to the distributions of temperatures of the radiation and nonrelativistic matter were all determined and discussed in this model in the various eras of the universe.

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