International Journal of Astronomy and Astrophysics, **2011, 1, 232-236**
doi:10.4236/ijaa.2011.14028 Published Online December 2011 (http://www.SciRP.org/journal/ijaa)

Application of a Probabilistic Neural Network in Radial Velocity Curve Analysis of the Spectroscopic Binary Stars ROXR1 14, RX J1622.7-2325Nw, RR Lyn, 12 Boo and HR 6169

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Received September 20, 2011; *revised October* 26, 2011; *accepted November* 23, 2011

Abstract

Using measured radial velocity data of five double-lined spectroscopic binary systems ROXR1 14, RX J1622.7-2325Nw, RR Lyn, 12 Boo and HR 6169, we find corresponding orbital and spectroscopic elements via a Probabilistic Neural Network (PNN). Our numerical results are in good agreement with those obtained by others using more traditional methods.

Keywords: Stars, Binaries, Eclipsing -- Stars, Binaries, Spectroscopic

1. Introduction

Analysis of both light and radial velocity (hereafter V_R) curves of binary systems helps us to determine the masses and radii of individual stars. One historically well-known method to analyze the V_R curve is that of Lehmann-Filhés [1]. Other methods were also introduced by Sterne [2] and Petrie [3]. The different methods of the V_R curve analysis have been reviewed in ample detail by Karami & Teimoorinia [4]. Karami V_R Teimoorinia [4] also proposed a new non-linear least squares velocity curve analysis technique for spectroscopic binary stars. They showed the validity of their new method to a wide range of different types of binary See Karami & Mohebi [5-7] and Karami *et al.* [8].

Probabilistic Neural Network (PNN) is a new tool to derive the orbital parameters of the spectroscopic binary stars. In this method the time consumed is considerably less than the method of Lehmann-Filhés and even less than the non-linear regression method proposed by Karami & Teimoorinia [4].

In the present paper we use a Probabilistic Neural Network (PNN) to find the optimum match to the four parameters of the V_R curves of the five double-lined spectroscopic binary systems: ROXR1 14, RX J1622.7-2325Nw, RR Lyn, 12 Boo and HR 6169. Our aim is to show the validity of our new method to a wide range of different types of binary.

ROXR1 14 and RX J1622.7-2325Nw are very young, low-mass pre-main sequence (PMS) stars, and double-lined spectroscopic binaries that recently discovered in the Ophiuchus star forming region. The spectral type of both systems is M1 and the orbital period of ROXR1 14 is P = 5.72 days and the orbital period of RX J1622.7-2325Nw is P = 3.23 days [9]. RR Lyn is a double-lined spectroscopic binary with a well-determined orbital inclination and primary and secondary masses of 1.927 ± 0.008 and 1.507 ± 0.004 $M_\odot$, respectively. The components have spectral classes of A3/A8/A6 and the orbital period is P = 9.945080 days [10]. 12 Boo is a double-lined eclipsing binary system with a primary and secondary masses of 1.416 ± 0.003 and 1.375 ± 0.002 $M_\odot$, respectively. The two components have very similar mass and the system's combined spectral type is F8 IV and the orbital period is P = 9.6045529 days [10]. HR 6169 is a double-lined spectroscopic binary and consists of primary and secondary components. The minimum masses of the primary and secondary are 2.20 ± 0.01 and 1.64 ± 0.02 $M_\odot$, respectively. This system have spectral classes of A2 V and the orbital period is P = 10.559435 days [10].

This paper is organized as follows. In Sect. 2, we introduce a Probabilistic Neural Network (PNN) to estimate the four parameters of the V_R curve. In Sect. 3, the numerical results are reported, while the conclusions are given in Sect. 4.

2. V_R Curve Parameters Estimation by the Probabilistic Neural Network (PNN)

Following Smart [11], the V_R of a star in a binary system is defined as follows

$$V_R = \gamma + K\left[\cos(\theta+\omega) + e\cos\omega\right] \quad (1)$$

where γ is the V_R of the center of mass of system with respect to the sun. Also K is the amplitude of the V_R of the star with respect to the center of mass of the binary. Furthermore θ, ω and e are the angular polar coordinate (true anomaly), the longitude of periastron and the eccentricity, respectively.

Here we apply the PNN method to estimate the four orbital parameters, γ, K, e and ω of the V_R curve in Equation (1). In this work, for the identification of the observational V_R curves, the input vector is the fitted V_R curve of a star. The PNN is first trained to classify V_R curves corresponding to all the possible combinations of γ, K, e and ω. For this one can synthetically generate V_R curves given by Equation (1) for each combination of the parameters:

- $-100 \le \gamma \le 100$ in steps of 1;
- $1 \le K \le 300$ in steps of 1;
- $0 \le e \le 1$ in steps of 0.001;
- $0 \le \omega \le 360^\circ$ in steps of 5;

This gives a very big set of k pattern groups, where k denotes the number of different V_R classes, one class for each combination of γ, K, e and ω. Since this very big number of different V_R classes **leads to** some computational limitations, hence one can first start with the big step sizes. Note that from Petrie [3], one can guess γ, K and e from a V_R curve. This enable one to limit the range of parameters around their initial guesses. When the preliminary orbit was derived after several stages, then one can use the above small step sizes to obtain the final orbit. The PNN has **four layers** including input, pattern, summation, and output layers, respectively (see **Figure 5** in Bazarghan *et al.* [12]). When an input vector is presented, the pattern layer computes distances from the input vector to the training input vectors and produces a vector whose elements indicate how close the input is to a training input. The summation layer sums these contributions for each class of inputs to produce as its net output a vector of probabilities. Finally, a competitive transfer function on the output layer picks the maximum of these probabilities, and produces a 1 for that class and a 0 for the other classes [13,14]. Thus, the PNN classifies the input vector into a specific k class labeled by the four parameters γ, K, e and ω because that class has the maximum probability of being correct.

3. Numerical Results

Here, we use the PNN to derive the orbital elements for the five different double-lined spectroscopic systems ROXR1 14, RX J1622.7-2325Nw, RR Lyn, 12 Boo and HR 6169. Using measured V_R data of the two components of these systems obtained by Rosero *et al.* [9] for ROXR1 14 and RX J1622.7-2325Nw and Tomkin & Fekel [10] for RR Lyn, 12 Boo and HR 6169, the fitted velocity curves are plotted in terms of the photometric phase in **Figures 1-5**.

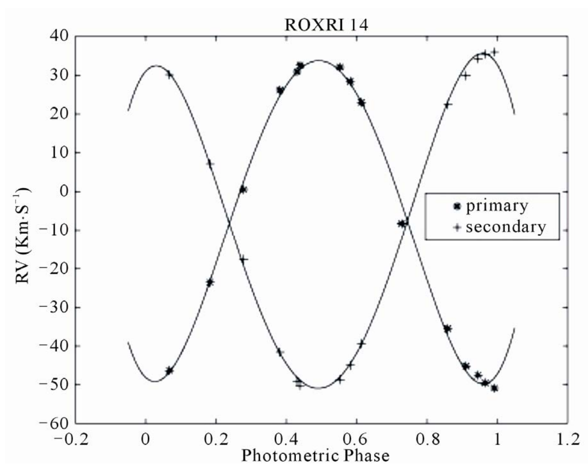

Figure 1. Radial velocities of the primary and secondary components of ROXR1 14 plotted against the photometric phase. The observational data have been measured by Rosero *et al.* [9].

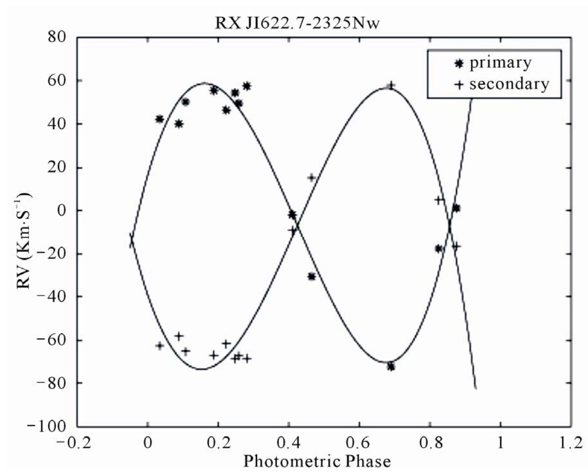

Figure 2. Radial velocities of the primary and secondary components of RX J1622.7-2325Nw plotted against the photometric phase. The observational data have been measured by Rosero *et al.* [9].

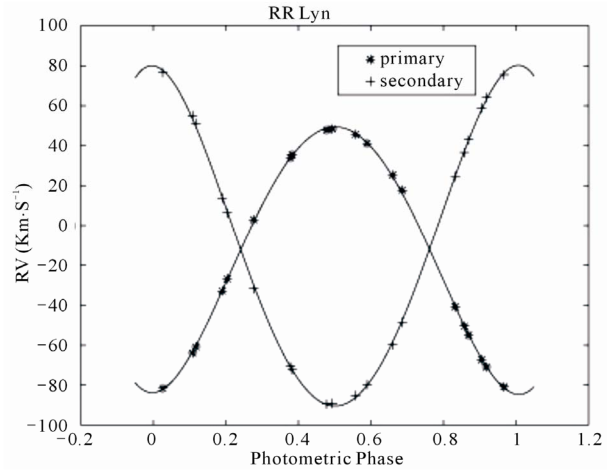

Figure 3. Radial velocities of the primary and secondary components of RR Lyn plotted against the photometric phase. The observational data have been measured by Tomkin & Fekel [10].

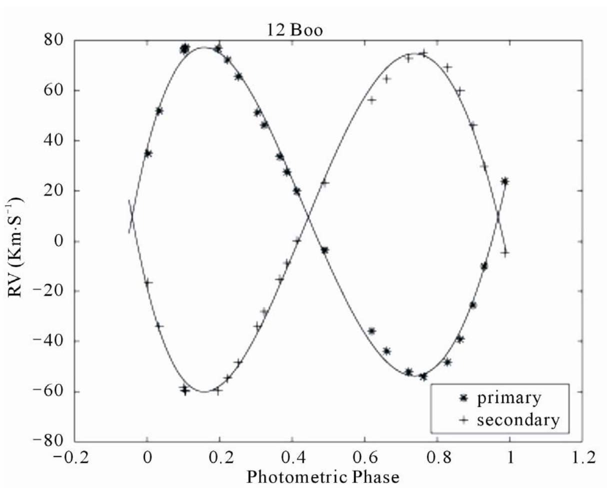

Figure 4. Radial velocities of the primary and secondary components of 12 Boo plotted against the photometric phase. The observational data have been measured by Tomkin & Fekel [10].

The orbital parameters obtaining from the PNN for ROXR1 14, RX J1622.7-2325Nw, RR Lyn, 12 Boo and HR 6169 are tabulated in **Tables 1**, **3**, **5**, **7** and **9**, respectively. Tables show that the results are in good accordance with the those obtained by Rosero *et al.* [9] for ROXR1 14 and RX J1622.7-2325Nw and Tomkin & Fekel [10] for RR Lyn, 12 Boo and HR 6169.

Note that the Gaussian errors of the orbital parameters in **Tables 1**, **3**, **5**, **7** and **9** are the same selected steps for generating V_R curves, *i.e.* $\Delta\gamma = 1, \Delta K = 1, \Delta e = 0.001$ and $\Delta\omega = 5$. These are close to the observational errors reported in the literature. Regarding the estimated errors, following Specht [14], the error of the decision boundaries depends on the accuracy with which the underlying

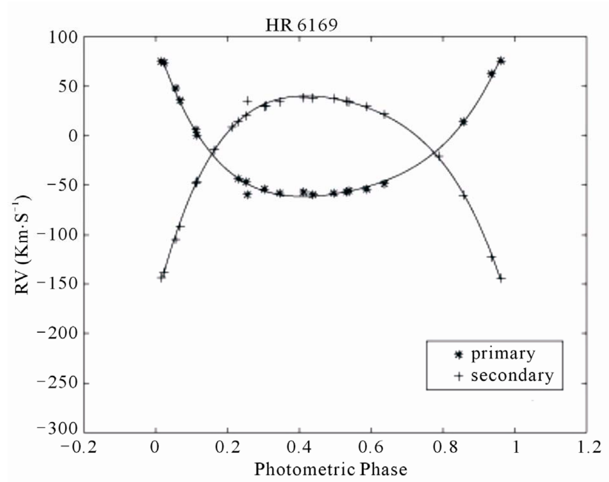

Figure 5. Radial velocities of the primary and secondary components of HR 6169 plotted against the photometric phase. The observational data have been measured by Tomkin & Fekel [10].

Table 1. Orbital parameters of ROXR1 14.

	This Paper	Rosero *et al.* [9]
γ(km/s)	–8 ± 1	–7.98 ± 0.18
K_p(km/s)	43 ± 1	42.66 ± 0.33
K_s(km/s)	44 ± 1	43.94 ± 0.33
e	0.021 ± 0.001	0 .020 ± 0.007
ω(°)	5 ± 5	3 .87 ± 17.04

Probability Density Functions (PDFs) are estimated. Parzen [15] proved that the expected error gets smaller as the estimate is based on a large data set. This definition of consistency is particularly important since it means that the true distribution will be approached in a smooth manner. Specht [14] showed that a very large value of the smoothing parameter would cause the estimated errors to be Gaussian regardless of the true underlying distribution and the misclassification rate is stable and does not change dramatically with small changes in the smoothing parameter.

The combined spectroscopic elements including $m_p \sin^3 i$, $m_s \sin^3 i$, $\left(m_p + m_s\right)\sin^3 i$, $\left(a_p + a_s\right)\sin i$ and $\frac{m_s}{m_p}$ are calculated by substituting the estimated parameters K, e and ω into Equations (3), (15) and (16) in Karami and Teimoorinia [4]. The results obtained for the five systems are tabulated in **Tables 2**, **4**, **6**, **8** and **10** show that our results are in good agreement with the those obtained by Rosero *et al.* [9] for ROXR1 14 and RX J1622.7-2325Nw and Tomkin & Fekel [10] for RR Lyn, 12 Boo and HR 6169, respectively. Here the errors

Table 2. Combined spectroscopic elements of ROXR1 14.

Parameter	This Paper	Rosero *et al.* [9]
$m_p \sin^3 i/M_\odot$	0.1972 ± 0.0003	—
$m_s \sin^3 i/M_\odot$	0.1928 ± 0.0003	—
$(m_p + m_s)\sin^3 i/M_\odot$	0.3900 ± 0.0006	—
$a_p \sin i/10^6\,\text{km}$	3.3832 ± 0.0786	3.36± 0.03
$a_s \sin i/10^6\,\text{km}$	3.4619 ± 0.0786	3.46± 0.03
$(a_p + a_s)\sin i/10^6\,\text{km}$	6.8450 ± 0.1572	—
m_s/m_p	0.9773 ± 0.0005	0.97± 0.01

Table 3. Orbital parameters of RX J1622.7-2325Nw.

	This Paper	Rosero *et al.* [9]
$\gamma(\text{km/s})$	–7 ± 1	–6.75 ± 1.09
$K_p(\text{km/s})$	79 ± 1	78.71 ± 3.52
$K_s(\text{km/s})$	80 ± 1	80.31 ± 3.58
e	0.301 ± 0.001	0.30 ± 0.037
$\omega(°)$	130 ± 5	133.45 ± 4.41

Table 4. Combined spectroscopic elements of RX J1622.7-2325Nw.

Parameter	This Paper	Rosero *et al.* [9]
$m_p \sin^3 i/M_\odot$	0.5869 ± 0.0006	—
$m_s \sin^3 i/M_\odot$	0.5796 ± 0.0005	—
$(m_p + m_s)\sin^3 i/M_\odot$	1.1666 ± 0.0011	—
$a_p \sin i/10^6\,\text{km}$	3.3478 ± 0.0413	3.34± 0.14
$a_s \sin i/10^6\,\text{km}$	3.3902 ± 0.0413	3.40± 0.15
$(a_p + a_s)\sin i/10^6\,\text{km}$	6.7380 ± 0.0825	—
m_s/m_p	0.9875 ± 0.0002	0.98± 0.06

Table 5. Orbital parameters of RR Lyn.

	This Paper	Tomkin & Fekel [10]
$\gamma(\text{km/s})$	–12 ± 1	–12.03 ± 0.04
$K_p(\text{km/s})$	66 ± 1	65.65 ± 0.06
$K_s(\text{km/s})$	84 ± 1	83.92 ± 0.17
e	0.079 ± 0.001	0.0793 ± 0.0009
$\omega(°)$	175 ± 5	179.4 ± 0.6

of the combined spectroscopic elements in **Tables 2**, **4**, **6**, **8** and **10** are obtained by the help of orbital parameters errors. See again Equations (3), (15) and (16) in Karami and Teimoorinia [4].

Table 6. Combined spectroscopic elements of RR Lyn.

Parameter	This Paper	Tomkin & Fekel [10]
$m_p \sin^3 i/M_\odot$	1.9292 ± 0.0006	1.921 ± 0.008
$m_s \sin^3 i/M_\odot$	1.5158 ± 0.0006	1.503 ± 0.004
$(m_p + m_s)\sin^3 i/M_\odot$	3.4450 ± 0.0012	—
$a_p \sin i/10^6\,\text{km}$	9.0022 ± 0.1357	8.950 ± 0.008
$a_s \sin i/10^6\,\text{km}$	11.4574 ± 0.1355	11.441 ± 0.024
$(a_p + a_s)\sin i/10^6\,\text{km}$	20.4596 ± 0.2712	—
m_s/m_p	0.7857 ± 0.0041	—

Table 7. Orbital parameters of 12 Boo.

	This Paper	Tomkin & Fekel [10]
$\gamma(\text{km/s})$	9 ± 1	9.578 ± 0.022
$K_p(\text{km/s})$	67 ± 1	67.286 ± 0.037
$K_s(\text{km/s})$	69 ± 1	69.30 ± 0.05
e	0.192 ± 0.001	0.19268 ± 0.00042
$\omega(°)$	290 ± 5	286.87 ± 0.14

Table 8. Combined spectroscopic elements of 12 Boo.

Parameter	This Paper	Tomkin & Fekel [10]
$m_p \sin^3 i/M_\odot$	1.2004 ± 0.0005	1.218 ± 0.002
$m_s \sin^3 i/M_\odot$	1.1656 ± 0.0005	1.183 ± 0.001
$(m_p + m_s)\sin^3 i/M_\odot$	2.3659 ± 0.0010	—
$a_p \sin i/10^6\,\text{km}$	8.6887 ± 0.1279	8.720 ± 0.005
$a_s \sin i/10^6\,\text{km}$	8.9480 ± 0.1279	8.981 ± 0.007
$(a_p + a_s)\sin i/10^6\,\text{km}$	17.6367 ± 0.2558	—
m_s/m_p	0.9710 ± 0.0004	—

Table 9. Orbital parameters of HR 6169.

	This Paper	Tomkin & Fekel [10]
$\gamma(\text{km/s})$	–18 ± 1	–18.33 ± 0.09
$K_p(\text{km/s})$	71 ± 1	71.35 ± 0.38
$K_s(\text{km/s})$	96 ± 1	95.46 ± 0.13
e	0.413 ± 0.001	0.4140 ± 0.0012
$\omega(°)$	15 ± 5	10.69 ± 0.20

4. Conclusions

A Probabilistic Neural Network to derive the orbital elements of spectroscopic binary stars was applied. PNNs

Table 10. Combined spectroscopic elements of HR 6169.

Parameter	This Paper	Tomkin&Fekel [10]
$m_p \sin^3 i / M_\odot$	2.2126 ± 0.0006	2.197 ± 0.012
$m_s \sin^3 i / M_\odot$	1.6364 ± 0.0005	1.642 ± 0.015
$(m_p + m_s)\sin^3 i / M_\odot$	3.8489 ± 0.0011	—
$a_p \sin i / 10^6\,\mathrm{km}$	9.3939 ± 0.1276	9.431 ± 0.051
$a_s \sin i / 10^6\,\mathrm{km}$	12.7016 ± 0.1260	12.618 ± 0.019
$(a_p + a_s)\sin i / 10^6\,\mathrm{km}$	22.0955 ± 0.2536	—
m_s / m_p	0.7396 ± 0.0050	—

are used in both regression (including parameter estimation) and classification problems. However, one can discretize a continuous regression problem to such a degree that it can be represented as a classification problem [13,14], as we did in this work.

Using the measured V_R data of ROXR1 14, RX J1622.7-2325Nw, RR Lyn, 12 Boo and HR 6169 obtained by Rosero *et al.* [9] and Tomkin & Fekel [10], we find the orbital elements of these systems by the PNN. **Our numerical results show that** the results obtained for the orbital and spectroscopic parameters **agree well with** those obtained by others **using traditional methods.**

This method is applicable to orbits of all eccentricities and inclination angles. In this method the time consumed is considerably less than the method of Lehmann-Filhés. It is also more accurate as the orbital elements are deduced from all points of the velocity curve instead of four in the method of Lehmann-Filhés. The present method enables one to vary all of the unknown parameters γ, K, e and ω simultaneously instead of one or two of them at a time. It is possible to make adjustments in the elements before the final result is obtained. There are some cases, for which the geometrical methods are inapplicable, and in these cases the present one may be found useful. One such case would occur when observations are incomplete because certain phases **could have not been observed**. Another case in which this method is useful is that of a star attended by two dark companions with commensurable periods. In this case the resultant velocity curve may have several unequal maxima and the geometrical methods fail altogether.

5. Acknowledgements

This work has been supported financially by Islamic Azad University, Marivan Branch, Iran.

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