Scientific
Research

A State Estimation Method for Sound Environment System with Unknown Observation Mechanism by Introducing Fuzzy Inference

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ABSTRACT

The observed phenomena in real sound environment system often contain uncertainty such as the additional external noise with unknown statistics. Furthermore, there is complex nonlinear relationship between the specific signal and the observations, and it cannot be exactly expressed in any definite functional form. In these situations, it is one of reasonable analysis methods to treat the objective sound environment system as a fuzzy system. In this study, a state estimation method for a specific signal under the existence of an unknown observation mechanism and external noise of unknown statistics is proposed by introducing fuzzy inference. The effectiveness of the proposed theoretical method is experimentally confirmed by applying it to the actually observed data in the sound environment.

Keywords: State Estimation; Sound Environment System; Unknown Observation Mechanism; Fuzzy Inference

1. Introduction

The observation data in actual sound environment system exhibit various types of fluctuation characteristics, and these often contain uncertainty. For example, the observed signal is inevitably contaminated by the concurrent external noise (*i.e.*, background noise) of arbitrary distribution type of unknown statistics. In this situation, in order to evaluate the specific signal based on the observed noisy data, it is indispensable to introduce some unified state estimation methods adaptable to various uncertainty caused by complexity, diversity and unknown property existing in the actual sound environment systems.

Though several state estimation methods have been proposed up to now, these state estimation algorithms have been realized by introducing the additive model of the specific signal and the external noise under an assumption of known statistics of the external noise [1-8]. On the other hand, it is necessary to pay our attention on the fact that the observation data in the sound environment system are often contain uncertainty due to several causes. For example, the observation in sound environment system can be generally expressed in an additive model of the specific signal and the background noise on energy scale by using the additive property of acoustic energy. However, for an assessment and a countermeasure of the sound environment, several evaluation quanti-

ties such as L_{eq} (averaged energy on a decibel scale), L_x ((100 − x) percentile level, x = 5, 10, 50, 90, 95) and L_{max} (maximum level), etc. for the specific signal, have to be considered. There are complex nonlinear and uncertain relations between the noise evaluation quantities of the specific signal and the background noise, and it is difficult in general to find the functional relationship between them. Furthermore, it is actually difficult to know in advance statistical properties of the background noise. Therefore, in order to remove effects of the background noise from the observed evaluation quantities under existence of the background noise, standard state estimation method based on an additive model of the specific signal and the background noise of known statistics cannot be applied. In this situation, the relationship between the observed evaluation quantities and the background noise has to be generally considered as a system model with unknown observation mechanism. In our previous study, a method for estimating the specific signal for the sound environment system with uncertainty has been proposed by adopting the conditional probability distribution reflecting the correlation information between the specific signal and observation as the system characteristics [9]. Though the method proposed previously can be applied effectively to actual complex sound environmental system with uncertainty, it needs a lot of priori information on the statistics of the specific signal and observations.

In this study, a practical state estimation method for sound environment systems with unknown observation mechanism of contamination by the background noise of unknown statistics is proposed by introducing the fuzzy inference in a different form the previous methods. More specifically, a membership function is first introduced for the unknown mechanism of a sound environment system. Next, by applying a fuzzy inference for the relationship between the observed data (*i.e.*, evaluation quantities observed under the existence of a background noise) and the state variable (*i.e.*, evaluation quantities of the specific signal), a state estimation algorithm is derived, based on Bayes' theorem as the fundamental principle of the estimation in the similar manner to the previously reported estimation method [4,5]. The effectiveness of the proposed method is experimentally confirmed by applying it to the estimation of evaluation quantities for road traffic noise under existence of a background noise.

2. State Estimation for Sound Environment System with Unknown Observation Mechanism

2.1. Formulation of Sound Environment System by Introducing Fuzzy Inference

Let x_k and y_k be state variable and observation at a discrete time k for sound environment systems. It is assumed that the mutual relationship between x_k and y_k is unknown. For example, the observations in sound environment are inevitably contaminated by the external noise (*i.e.*, background noise) of arbitrary distribution type. In general, by using the additive property of acoustic energy, the observation energy under the existence of external noise can be expressed in an additive model of the specific signal and the background noises in energy scale. However, for the stochastic evaluation quantities: L_{eq} and L_x which are used in the evaluation of actual sound environment, there are complex nonlinear relations between the evaluation quantities of the specific signal and the observed evaluation quantities. For an assessment of sound environment, it is often necessary to investigate the fluctuation of evaluation quantities for the specific signal. Therefore, the evaluation quantities of the specific signal at the *k*-th short time interval and the observed evaluation quantities at the *k*-th time interval have to be regarded as the unknown state variable x_k and the observation y_k respectively. Since it is difficult in general to find functional relationship between the state variable x_k and the observation y_k, the relationship between x_k and y_k has to be considered as a sound environment system with unknown observation mechanism. In this study, the following IF-THEN rule is introduced for the unknown mechanism of the sound envi-

ronment system:

Rule 1: IF x_k is A_1 THEN y_k is B_1

Rule 2: IF x_k is A_2 THEN y_k is B_2

…

Rule N : IF x_k is A_N THEN y_k is B_N , (1)

where A_i and B_i ($i = 1, 2, \cdots, N$) denote fuzzy sets corresponding to the several local state spaces of x_k and y_k, and have membership functions $\mu_{A_i}(x_k)$ and $\mu_{B_i}(y_k)$ ($i = 1, 2, \cdots, N$). For example, $A_1, A_2, \cdots, A_N$ are fuzzy sets describing the local N regions with vague borders such as "very low level", "low level", …, "very high level", for the fluctuation range of the sound level x_k. Applying fuzzy inference to (1), y_k is given by [10]

$$y_k = \frac{\sum_{i=1}^{N} \overline{y}_i \mu_{A_i}(x_k)}{\sum_{i=1}^{N} \mu_{A_i}(x_k)} \qquad (2)$$

where $\overline{y}_i$ is the value at which the membership function $\mu_{B_i}(y_k)$ achieved its maximum value. Furthermore, as the membership function $\mu_{A_i}(x_k)$, the Gaussian type, defined by [10]

$$\mu_{A_i}(x_k) = \exp\left\{-\frac{1}{2}\left(\frac{x_k - \overline{x}_i}{\sigma_i}\right)^2\right\} \qquad (3)$$

where $\overline{x}_i$ and σ_i are parameters, is adopted.

In the next section, an estimation method for the state variable x_k of the specific signal based on the recursive observation y_k is derived. Though the parameter σ_i in (3) can be generally given based on the prior information (or, through trial and error), it can be regarded as unknown constant parameter α ($= \sigma_i$ for every *i*) and estimated simultaneously with the state variable x_k by introducing the following simple dynamic model:

$$\alpha_{k+1} = \alpha_k. \qquad (4)$$

2.2. Estimation Algorithm by Introducing Bayes' Theorem

In order to derive an estimation algorithm for a state variable x_k, with an arbitrary distribution, we focus our attention on Bayes' theorem for the conditional probability density function (abbr. pdf). Since the parameter α_k is also unknown, the conditional pdf of x_k and α_k must be considered.

$$P(x_k, \alpha_k | Y_k) = \frac{P(x_k, \alpha_k, y_k | Y_{k-1})}{P(y_k | Y_{k-1})} \qquad (5)$$

where $Y_k (= \{y_1, y_2, \cdots y_k\})$ is a set of observation data up to *k*-th time. The conditional joint pdf of the state variable x_k, the parameter α_k and the observation y_k

in (5) can be generally expanded in a statistical orthogonal expansion series [4,5]:

$$P(x_k, \alpha_k, y_k | Y_{k-1})$$
$$= P_0(x_k | Y_{k-1}) P_0(\alpha_k | Y_{k-1}) P_0(y_k | Y_{k-1})$$
$$\sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{lmn} \varphi_l^{(1)}(x_k) \varphi_m^{(2)}(\alpha_k) \varphi_n^{(3)}(y_k) \quad (6)$$

After substituting (6) into (5), taking the conditional expectation of the function $f_{N_1,N_2}(x_k, \alpha_k)$ and using the orthonormal condition for the function $\varphi_l^{(1)}(x_k)$ and $\varphi_m^{(2)}(\alpha_k)$, through a similar calculation process to that used in a previously reported estimation method [4,5], the estimate of an arbitrary polynomial function $f_{N_1,N_2}(x_k, \alpha_k)$ of x_k and α_k of $N(\equiv(N_1, N_2))$ -th order can be derived in an infinite series expression, as follows:

$$\hat{f}_{N_1,N_2}(x_k, \alpha_k) \equiv \left\langle f_{N_1,N_2}(x_k, \alpha_k) | Y_k \right\rangle$$
$$= \frac{\sum_{l=0}^{N_1} \sum_{m=0}^{N_2} \sum_{n=0}^{\infty} C_{lm}^{N_1 N_2} A_{lmn} \varphi_n^{(3)}(y_k)}{\sum_{n=0}^{\infty} A_{00n} \varphi_n^{(3)}(y_k)} \quad (7)$$

with

$$A_{lmn} \equiv \left\langle \varphi_l^{(1)}(x_k) \varphi_m^{(2)}(\alpha_k) \varphi_n^{(3)}(y_k) | Y_{k-1} \right\rangle \quad (8)$$

The three functions $\varphi_l^{(1)}(x_k)$, $\varphi_m^{(2)}(\alpha_k)$ and $\varphi_n^{(3)}(y_k)$ are the orthonormal polynomials of degrees l, m and n, with weighting functions $P_0(x_k | Y_{k-1})$, $P_0(\alpha_k | Y_{k-1})$ and $P_0(y_k | Y_{k-1})$, which can be artificially chosen as the pdfs describing the above dominant parts of the actual fluctuation, or as well-known standard pdfs such as Gaussian or Gamma distribution functions. All the coefficients $C_{lm}^{N_1 N_2}$ are appropriate constants in the case when the function $f_{N_1,N_2}(x_k, \alpha_k)$ is expressed in a series expansion form using $\varphi_l^{(1)}(x_k)$ and $\varphi_m^{(2)}(\alpha_k)$:

$$f_{N_1,N_2}(x_k, \alpha_k) = \sum_{l=0}^{N_1} \sum_{m=0}^{N_2} C_{lm}^{N_1 N_2} \varphi_l^{(1)}(x_k) \varphi_m^{(2)}(\alpha_k) \quad (9)$$

As a concrete example of a standard pdf, the well-known Gaussian distribution is adopted:

$$P_0(x_k | Y_{k-1}) = N(x_k; x_k^*, \Gamma_{x_k}) \quad (10)$$

$$P_0(\alpha_k | Y_{k-1}) = N(\alpha_k; \alpha_k^*, \Gamma_{\alpha_k}) \quad (11)$$

$$P_0(y_k | Y_{k-1}) = N(y_k; y_k^*, \Gamma_{y_k}) \quad (12)$$

with

$$N(x; \mu, \sigma^2) \equiv \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\},$$
$$x_k^* \equiv \left\langle x_k | Y_{k-1} \right\rangle, \Gamma_{x_k} \equiv \left\langle (x_k - x_k^*)^2 | Y_{k-1} \right\rangle$$
$$a_k^* \equiv \left\langle a_k | Y_{k-1} \right\rangle, \Gamma_{a_k} \equiv \left\langle (a_k - a_k^*)^2 | Y_{k-1} \right\rangle$$
$$y_k^* \equiv \left\langle y_k | Y_{k-1} \right\rangle, \Gamma_{y_k} \equiv \left\langle (y_k - y_k^*)^2 | Y_{k-1} \right\rangle. \quad (13)$$

Then, the orthonormal polynomials with three weighting pdfs in (10)-(12) can be given in terms of the well-known Hermite polynomials [11].

$$\varphi_l^{(1)}(x_k) = \frac{1}{\sqrt{l!}} H_l\left(\frac{x_k - x_k^*}{\sqrt{\Gamma_{x_k}}}\right) \quad (14)$$

$$\varphi_m^{(2)}(\alpha_k) = \frac{1}{\sqrt{m!}} H_m\left(\frac{\alpha_k - \alpha_k^*}{\sqrt{\Gamma_{\alpha_k}}}\right) \quad (15)$$

$$\varphi_n^{(3)}(y_k) = \frac{1}{\sqrt{n!}} H_n\left(\frac{y_k - y_k^*}{\sqrt{\Gamma_{y_k}}}\right) \quad (16)$$

Using (2) and (3), the two variables y_k^* and Γ_{y_k} in (13) can be expressed as:

$$y_k^* = \left\langle \sum_{i=1}^{N} \overline{y}_i \exp\left\{-\frac{1}{2}\left(\frac{x_k - \overline{x}_i}{\alpha_k}\right)^2\right\} \Bigg/ \sum_{i=1}^{N} \exp\left\{-\frac{1}{2}\left(\frac{x_k - \overline{x}_i}{\alpha_k}\right)^2\right\} \Bigg| Y_{k-1} \right\rangle \quad (17)$$

$$\Gamma_{y_k} = \left\langle \left[\sum_{i=1}^{N} \overline{y}_i \exp\left\{-\frac{1}{2}\left(\frac{x_k - \overline{x}_i}{\alpha_k}\right)^2\right\} \Bigg/ \sum_{i=1}^{N} \exp\left\{-\frac{1}{2}\left(\frac{x_k - \overline{x}_i}{\alpha_k}\right)^2\right\} - y_k^*\right]^2 \Bigg| Y_{k-1} \right\rangle \quad (18)$$

Furthermore, each expansion coefficient A_{lmn} defined by (8) can be expressed as

$$A_{lmn} = \left\langle \frac{1}{\sqrt{l!}} H_l\left(\frac{x_k - x_k^*}{\sqrt{\Gamma_{x_k}}}\right) \frac{1}{\sqrt{m!}} H_m\left(\frac{\alpha_k - \alpha_k^*}{\sqrt{\Gamma_{\alpha_k}}}\right) \frac{1}{\sqrt{n!}} H_n\left(\frac{\sum_{i=1}^{N} \overline{y}_i \exp\left\{-\frac{1}{2}\left(\frac{x_k - \overline{x}_i}{\alpha_k}\right)^2\right\} \Big/ \sum_{i=1}^{N} \exp\left\{-\frac{1}{2}\left(\frac{x_k - \overline{x}_i}{\alpha_k}\right)^2\right\} - y_k^*}{\sqrt{\Gamma_{y_k}}}\right) \Bigg| Y_{k-1} \right\rangle$$
$$(19)$$

The variables y_k^*, Γ_{y_k} and the expansion coefficient A_{lmn} in (17), (18) and (19) are given by the predictions of x_k and α_k at a discrete time $k-1$ (*i.e.*, the expectation value of arbitrary functions of x_k and α_k conditioned by Y_{k-1}). Consequently, the unified algorithm for estimating the state variable can be obtained. In two special cases when $f_{1,0}(x_k, \alpha_k) = x_k$ and $f_{2,0}(x_k, \alpha_k) = (x_k - \hat{x})^2$, estimates related to mean and variance of the state variable are expressed as follows:

$$\hat{x}_k \equiv \langle x_k | Y_k \rangle$$

$$= \frac{\sum_{n=0}^{\infty} \left\{ C_{00}^{10} A_{00n} + C_{10}^{10} A_{10n} \right\} \frac{1}{\sqrt{n!}} H_n \left(\frac{y_k - y_k^*}{\sqrt{\Gamma_{y_k}}} \right)}{\sum_{n=0}^{\infty} A_{00n} \frac{1}{\sqrt{n!}} H_n \left(\frac{y_k - y_k^*}{\sqrt{\Gamma_{y_k}}} \right)} \quad (20)$$

$$P_{x_k} \equiv \langle (x_k - \hat{x}_k)^2 | Y_k \rangle$$

$$= \frac{\sum_{n=0}^{\infty} \left\{ C_{00}^{20} A_{00n} + C_{10}^{20} A_{10n} + C_{20}^{20} A_{20n} \right\} \frac{1}{\sqrt{n!}} H_n \left(\frac{y_k - y_k^*}{\sqrt{\Gamma_{y_k}}} \right)}{\sum_{n=0}^{\infty} A_{00n} \frac{1}{\sqrt{n!}} H_n \left(\frac{y_k - y_k^*}{\sqrt{\Gamma_{y_k}}} \right)} \quad (21)$$

with

$$C_{00}^{10} \equiv x_k^*, C_{10}^{10} \equiv \Gamma_{x_k}, C_{00}^{20} \equiv \Gamma_{x_k} + (x_k^* - \hat{x}_k)^2$$
$$C_{10}^{20} \equiv 2\sqrt{\Gamma_{x_k}} (x_k^* - \hat{x}_k), C_{20}^{20} \equiv \sqrt{2}\Gamma_{x_k} \quad (22)$$

Furthermore, by focusing on a polynomial function:

$$f_{N_1,0}(x_k - \alpha_k) = \frac{1}{\sqrt{N!}} H_{N_1} \left(\frac{x_k - \hat{x}_k}{\sqrt{P_{x_k}}} \right) \quad (23)$$

the estimate for the expansion coefficient reflecting the non-Gaussian property of the state variable is given by Equation (24), where $C_{l0}^{N_1 0}$ $(l = 0, 1, \cdots, N_1)$ are coefficients satisfying the following relationship:

$$\frac{1}{\sqrt{N_1!}} H_{N_1} \left(\frac{x_k - \hat{x}_k}{\sqrt{P_{x_k}}} \right) = \sum_{l=0}^{N_1} C_{l0}^{N_1 0} \frac{1}{\sqrt{l!}} H_l \left(\frac{x_k - x_k^*}{\sqrt{\Gamma_{x_k}}} \right) \quad (25)$$

Finally, in order to derive the prediction step necessary to perform the recurrence estimation, the fuzzy inference is introduced again. More specifically, after dividing the fluctuation range of x_k into M fuzzy regions S_i ($i =$

1, 2,$\cdots$, M) with membership functions $\mu_i(x_k)$ ($i = 1, 2, \cdots, M$), by creating fuzzy rules from given data sets, the following IF-THEN rules are obtained.

Rule 1: IF x_k is S_1 THEN x_{k+1} is S_1'
Rule 2: IF x_k is S_2 THEN x_{k+1} is S_2'
....

Rule M: IF x_k is S_M THEN x_{k+1} is S_M', (26)

where S_1' is one of M fuzzy sets S_i ($i = 1, 2, \cdots, M$), and is decided adequately based on the prior information or measured data. For example, after evaluating two degrees for conformity of data pair x_k and x_{k+1} to S_i and S_i', by selecting S_i' so as to maximize the synthesized degree of these two degrees, each rule of (26) can be assigned. By applying the center average defuzzification formula [12] to (26), a state transition model can be determined as

$$x_{k+1} = \frac{\sum_{i=1}^{M} \tilde{x}_i \mu_i(x_k)}{\sum_{i=1}^{M} \mu_i(x_k)} \quad (27)$$

$$\mu_i(x_k) = \exp \left\{ -\frac{1}{2} \left(\frac{x_k - \tilde{x}_i}{\sigma} \right)^2 \right\}, \quad (28)$$

where $\tilde{x}_i$ is the value at which the membership function $\mu_i(x_k)$ achieved its maximum value, and σ is a constant parameter. By considering (4) and (27), the prediction algorithm can be given for an arbitrary polynomial function $g_r(x_{k+1}, \alpha_{k+1})$ of x_{k+1} and α_{k+1} of $r (\equiv (r_1, r_2))$-th order, as follows:

$$g_r^*(x_{k+1}, \alpha_{k+1})$$
$$\equiv \langle g_r(x_{k+1}, \alpha_{k+1}) | Y_k \rangle \quad (29)$$
$$= \left\langle g_r \left(\sum_{i=1}^{M} \tilde{x}_i \mu_i(x_k) \right) \middle/ \sum_{i=1}^{M} (\mu_i(x_k), \alpha_k) \middle| Y_k \right\rangle$$

The above prediction can be evaluated by the estimates at a discrete time k. Therefore, by combining (7) with (29), the recurrence estimation of x_k and α_k can be achieved.

3. Application to Evaluation Quantities of Road Traffic Noise

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$$\hat{a}_{N_1} \equiv \frac{1}{\sqrt{N_1!}} \left\langle H_{N_1} \left(\frac{x_k - \hat{x}_k}{\sqrt{P_{x_k}}} \right) \middle| Y_k \right\rangle = \frac{\sum_{n=0}^{\infty} \left\{ C_{00}^{N_1 0} A_{00n} + C_{10}^{N_1 0} A_{10n} + \cdots + C_{N_1 0}^{N_1 0} A_{N_1 0 n} \right\} \frac{1}{\sqrt{n!}} H_n \left(\frac{y_k - y_k^*}{\sqrt{\Gamma_{y_k}}} \right)}{\sum_{n=0}^{\infty} A_{00n} \frac{1}{\sqrt{n!}} H_n \left(\frac{y_k - y_k^*}{\sqrt{\Gamma_{y_k}}} \right)} \quad (24)$$

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In order to confirm experimentally the validity of the proposed method, it is applied to the actually observed data in the sound environment. As the specific signal, the road traffic noise becoming one of the aggravating environmental noise pollution is adopted. By regarding the observations on several evaluation quantities: $L_{Aeq,T}$ and $L_{A50,T}$, measured by use of sound level meter with the frequency characteristic of A weight and the time interval $T = 1$ min. under existence of a background noise, as observations y_k, and regarding the evaluation quantities: $L_{Aeq,T}$ and $L_{A50,T}$ ($T = 1$ min) for only the specific signal as the state variables x_k, the fluctuation of x_k is estimated on the basis of y_k.

By dividing the fluctuation ranges of x_k and y_k into 3 fuzzy regions, A_1, B_1: "low level", A_2, B_2: "middle level", and: A_3, B_3 "high level", (1) is expressed as follows:

Rule 1: IF x_k is A_1 THEN y_k is B_1

Rule 2: IF x_k is A_2 THEN y_k is B_2

Rule 3: IF x_k is A_3 THEN y_k is B_3. (30)

Furthermore, (2) is expressed as follows:

$$y_k = \frac{\overline{y}_1 \mu_{A_1}(x_k) + \overline{y}_2 \mu_{A_2}(x_k) + \overline{y}_3 \mu_{A_3}(x_k)}{\mu_{A_1}(x_k) + \mu_{A_2}(x_k) + \mu_{A_3}(x_k)}. \quad (31)$$

In the above formula, the three parameters are decided as:

$$\overline{y}_1 = \mu_y - 3\sigma_y, \overline{y}_2 = \mu_y, \overline{y}_3 = \mu_y + 3\sigma_y \quad (32)$$

where μ_y and σ_y are mean and standard deviation of y_k. Furthermore, by determining the parameters of (28) so as to express the whole distribution of actual measured data of x_k as precisely as possible, the membership functions in **Figures 1** and **2** are obtained for $L_{Aeq,T}$ and $L_{A50,T}$.

For comparison, the previously reported method [4,5] without considering fuzzy theory is also applied to the observed data after introducing a linear system model:

$$y_k = a_k x_k + b_k v_k, \quad (33)$$

$$x_{k+1} = F x_k + G u_k, a_{k+1} = a_k, b_{k+1} = b_k \quad (34)$$

In (33) and (34), a_k and b_k are unknown parameters to be estimated simultaneously with the state variable x_k. Furthermore, v_k and u_k are random noises with mean 0 and variance 1. Tow parameters F and G are estimated by use of auto-correlation technique [4]. The estimation results for $L_{Aeq,1min}$ and $L_{A50,1min}$ of the road traffic noise by observing $L_{Aeq,1min}$ and $L_{A50,1min}$ under existence of a background noise are shown in **Figures 3** and **4**. The proposed method estimates precisely the fluctuation of the state variables. On the other hand, there are great
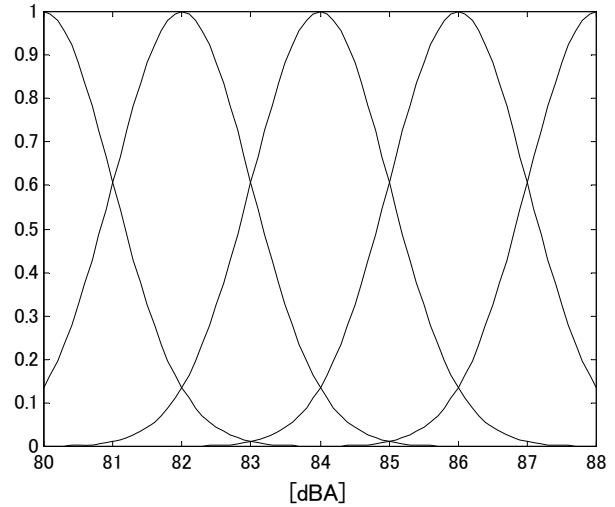

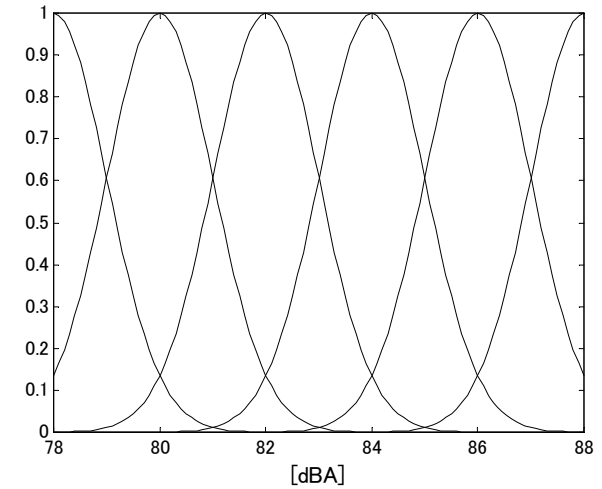**Figure 1. Membership function for $L_{Aeq,1min}$.**

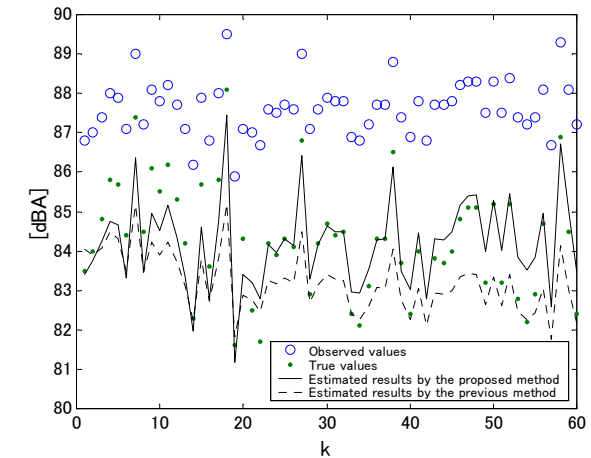**Figure 2. Membership function for $L_{A50,1min}$.**

Figure 3. A comparison between the proposed method and the previous method for state estimation results of $L_{Aeq,1min}$ for the road traffic noise based on the observation of $L_{Aeq,1min}$ under existence of a background noise.
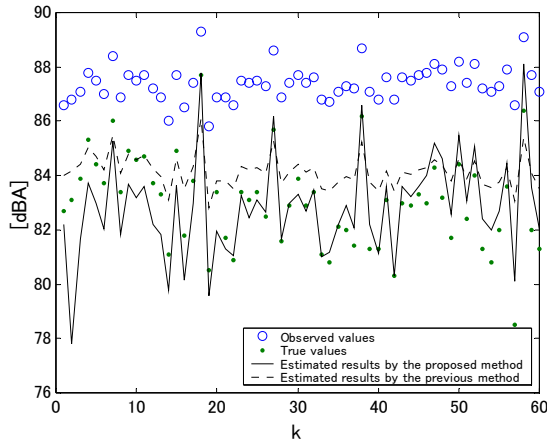
Figure 4. A comparison between the proposed method and the previous method for of state estimation results of $L_{A50,1min}$ for the road traffic noise based on the observation of $L_{A50,1min}$ under existence of a background noise.

discrepancies between the true values and the estimates based on the linear system of (33) and (34). The squared sum of the estimation error is shown in **Table 1**. It is obvious that the proposed method based on the introduction of fuzzy theory shows more accurate estimation than the results based on the usual linear system model like (33) and (34).

Next, as one of reasonable methods for determining the three parameters in (31), by introducing an unknown parameter β in (32) as:

$$\bar{y}_1 = \mu_y - \beta\sigma_y, \bar{y}_2 = \mu_y, \bar{y}_3 = \mu_y + \beta\sigma_y \qquad (35)$$

and focusing our attention on the Bayes' theorem for the conditional pdf of x_k, α_k and β_k:

$$P(x_k, \alpha_k, \beta_k | Y_k) = \frac{P(x_k, \alpha_k, \beta_k, y_k | Y_{k-1})}{P(y_k | Y_{k-1})}, \qquad (36)$$

after introducing a dynamical model for β:

$$\beta_{k+1} = \beta_k \qquad (37)$$

an estimation algorithm for the state variable can be derived through the same calculation process as (7). The estimation results by the proposed method with introduction of the unknown parameter β are shown in **Figures 5** and **6**. In general, the results by the proposed method using (35) estimate precisely the peak values of fluctuation than the results by the method using (32). A comparison between the squared sum of the estimation error in two cases is also shown in **Table 1**. The estimated results by the algorithm with unknown parameter β show more accurate estimation than the results based on the method with the fixed value of β.

Furthermore, the proposed method with flexibility is applied to estimate $L_{Aeq,1min}$ of the road traffic noise based on the observation of $L_{A50,1min}$ under existence of
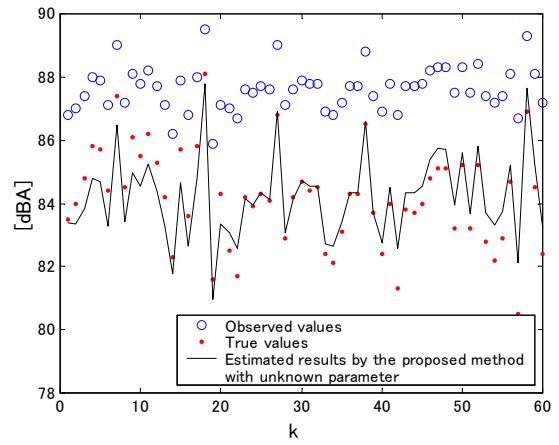

Figure 5. A state estimation result of $L_{Aeq,1min}$ for the road traffic noise based on the observation of $L_{Aeq,1min}$ by the proposed method with unknown parameter.
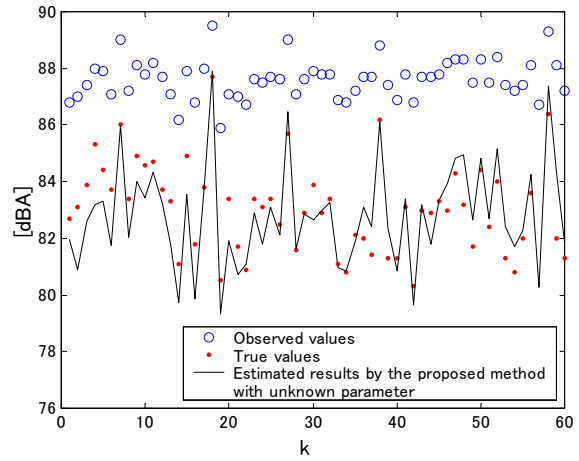

Figure 6. A state estimation result of $L_{A50,1min}$ for the road traffic noise based on the observation of $L_{A50,1min}$ by the proposed method with unknown parameter.

Table 1. Root-mean squared error of the estimation (in dBA).

State Variable	Observation	Proposed Method with Fixed Parameter	Proposed Method with Unknown Parameter	Previous Method
$L_{Aeq,1min}$	$L_{Aeq,1min}$	0.749	0.718	1.31
$L_{A50,1min}$	$L_{A50,1min}$	1.20	1.07	1.61

the background noise as a trial. It must be noticed that the observation and the specific signal to be estimated are different evaluation quantities in this case. **Figure 7** shows a comparison between the proposed method and the previous method, and the estimated result by use of the proposed method with unknown parameter is shown in **Figure 8**. The root mean squared error of the estimation is shown in **Table 2**. From these results, the effectiveness of the proposed method has been confirmed even
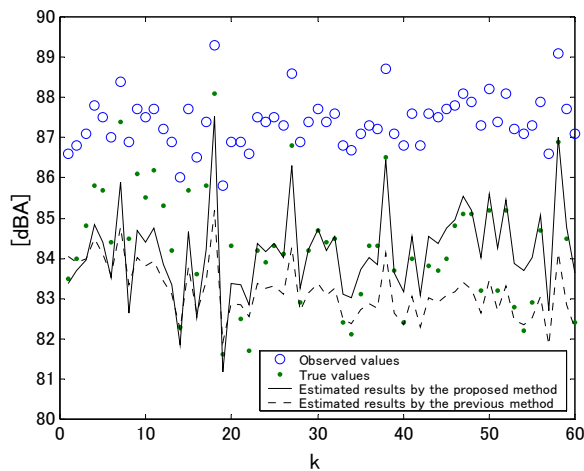
Figure 7. A comparison between the proposed method and the previous method for state estimation results of $L_{Aeq,1min}$ for the road traffic noise based on the observation of $L_{A50,1min}$ under existence of a background noise.
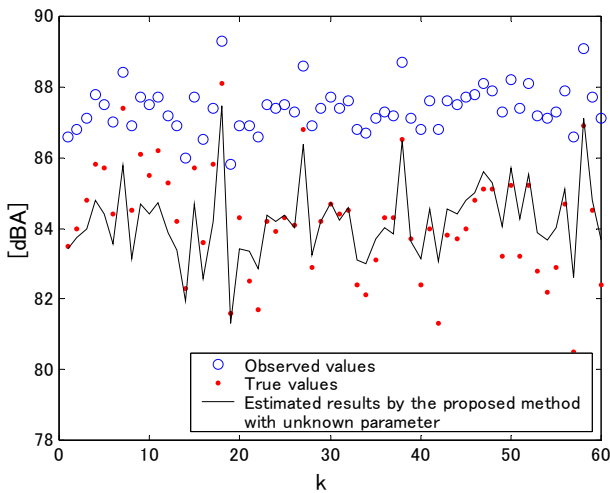

Figure 8. A state estimation result of $L_{Aeq,1min}$ for the road traffic noise based on the observation of $L_{A50,1min}$ by the proposed method with unknown parameter.

Table 2. Root-mean squared error of the estimation in Figures 7 and 8 (in dBA).

State Variable	Observation	Proposed Method with Fixed Parameter	Proposed Method with Unknown Parameter	Previous Method
$L_{Aeq,1min}$	$L_{Aeq,1min}$	0.896	0.875	1.38

in the case of observation of different evaluation quantity.

4. Conclusions

In this study, a practical state estimation method for the sound environment system with uncertainty has been theoretically proposed by introducing the fuzzy inference.

More specifically, after regarding the relationship between the state variable and observed evaluation quantities under existence of a background noise as a sound environment system with unknown observation mechanism, a recursive estimation algorithm for evaluation quantities of the specific signal has been derived. Furthermore, by applying the proposed method to the actually observed data of road traffic noise in the sound environment, the effectiveness of the theory has been confirmed experimentally too.

There are two points to be considered for obtaining improved estimations, as compared with the previous method. The first point is that our proposed method introduces fuzzy theory on the basis of fundamental principles of probability theory. More specifically, based on the Bayes' theorem which is a normative axiom of probability theory, by introducing fuzzy inference for the uncertainty of the objective system, the generalization ability can be enhanced. The second point is that we can express the fuzziness of phenomena in a form reflecting the vague fluctuation around a true value. That is, by estimating simultaneously the parameter of the membership function with the state variable based on the observation data, the new state estimation algorithm has been derived.

The proposed approach is quite different from the traditional standard approach. Thus, it is still at an early stage of study, and there are left a number of practical problems to be continued in the future, starting from the result of the basic study in this paper. Some of the problems are the following.

1) The proposed method should be applied to the other actual data of sound environment, and its practical usefulness should be verified in each actual situation. For example, the proposed estimation method can be applied to the actual sound insulation systems with complex structure. By applying the proposed method to the input and output data of sound insulation systems, the system parameters reflecting the sound insulation properties can be estimated.

2) The proposed theory should be further extended into more complicated situation with multi-signal sources. In the acoustical engineering field, it is very important to separate only the objective specific signal from many other signals based on the compound observation in the actual situation in the presence of external noise. A future challenge is the development of an estimation method under the multi-signal sources, on the basis of the proposed method.

3) The theory should be developed to find more precise estimation methods for the state variables by introducing a hybrid method of fuzzy theory and statistical methods. More specifically, by taking the higher order correlation information between the state variables and

the observation into consideration in the fuzzy theory, a new type of state estimation method can be proposed.

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