Intelligent Information Management, **2011, 3, 137-141**
doi: 10.4236/iim.2011.34017 Published Online July 2011 (http://www.SciRP.org/journal/iim)

A Dynamic Cross Contagion Model of Currency Crisis between Two Countries*

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Received March 31, 2011; *revised April* 26, 2011; *accepted May* 10, 2011

Abstract

The contagion aspect of the currency crisis is an important research issue today. In this paper, we set up a dynamic differential model of currency crisis cross contagions between two countries by expanding generalized logistics model, and analyze all kinds of possible equilibrium conditions. It is probably a new idea of studying currency crisis contagion mechanism.

Keywords: Cross Contagion, Currency Crisis, Differential Dynamic Model

1. Introduction

Since 1970s, currency crisis have occurred frequently on a global scale. Especially, in the past twenty years, there have happened several currency crises with significant influence: European exchange rate system crisis in 1992-1993, Mexican crisis in 1994-1995, southeast Asian crisis in 1997-1998, Russian rubles crisis in 1998, Brazilian currency crisis in 1998-1999, Turkish lire crisis in 2000-2001, Argentine peso crisis in 2001-2002 and the global crisis triggered by American subprime crisis in 2007. They may be caused by unreasonable domestic economic structures, heavy debt, unsuccessful monetary policy or external shocks caused by international financial speculators. However the worse thing than crises happening frequently is crisis contagions within a region or global area. This leads to more crises impacts on the economic or financial system over the world. And the crisis contagions tend to be stronger and stronger, wider and wider. From the phenomenon perspective, the simple reason of stronger and wider crises contagions is that the continuous development of international trade and capital flows make closer and closer relations of global economy and finance. But beyond the simple reason, what is complex mechanism of monetary crisis contagion? It will be the focus of this paper.

Since three generations of models were developed to explain financial crises, many scholars have done a lot in the area of currency crisis contagion mechanism and contagion path. Olivier Loisel and Philippe Martin (2001) presented a micro-founded model where governments had an incentive to devalue to increase the national market share in a monopolistically competitive sector. The conclusion showed that the more important trade competition, the more likely self-fulfilling speculative crises and the larger the set of multiple equilibria. They also concluded that coordination decreased the possibility of simultaneous self-fulfilling speculative crises in the region and reduced the set of multiple equilibria. However, regional coordination, even though welfare improving, makes countries more dependent on other countries' fundamentals so that it may induce more contagion [1]. Helmut Stix (2007) studied the effects of France interventions during the 1992-1993 European Monetary System crises. In his paper, a Markov Switching model is estimated where interventions influence the probabilities of transition between one calm and turbulent regime [2]. Eelke de Jong, Willem F. C, Verschoor and Remco C. J. Zwinkels (2009) estimated a dynamic hetero-generous agent model for the British pound during the European monetary system crisis and illustrate the chain of events leading to the suspension of the pound from the exchange rate mechanism in terms of switching beliefs [3]. Li Gang, Pan Hao-min and Jia Wei (2009) empirically studied spatial convergence and contagious pathes of financial crisis using methods of spatial statistical analysis. They concluded that contagious paths of subprime crisis in the United States are the location of geographic regions, G7 political groups, trade

*This work has been supported by the Research Fund of Program Foundation of Education Ministry of China (10YJA790233).

relations and the openness of capital items. Contagious pathes of Southeast Asia's financial crisis are the location of geographic regions, trade relations and the openness of capital items [4]. Juan Gabriel Brida, David Matesanz Gómez and Wiston Adrián Risso (2009) introduced a new method to describe dynamical patterns of the real exchange rate co-movements time series and to analyze contagion in currency crisis. The method combines the tools of symbolic time series analysis with the nearest neighbor single linkage clustering algorithm. By data symbolizing, they obtained a metric distance between two different time series that was used to construct an ultra-metric distance. By analyzing the data of various countries, they derived a hierarchical organization, constructing minimal-spanning and hierarchical trees. From these trees they detected different clusters of countries according to their proximity. They concluded that the methodology permits them to construct a structural and dynamic topology that was useful to study interdependence and contagion effects among financial time series [5].

2. The Dynamic Cross Contagion Model between Two Countries

Logistic models are widely applied to many research areas such as biological, economics, sociology. Ying and Zou (2010) set up a currency crisis infectious differential dynamic model between two countries by expanding generalized logistics model as following [6].

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = \gamma_1 x\left[1 - \dfrac{x}{k_1 + \alpha_1 y}\right] \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = \gamma_2 y\left[1 - \dfrac{y}{k_2 + \alpha_2 x}\right] \end{cases} \tag{1}$$

The system (1) can be easily deduced to a general form.

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = \gamma_1 x \dfrac{\dfrac{k_2}{\alpha_1} - x - \alpha_2 y}{k_1 - \alpha_2 y} \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = \gamma_2 y \dfrac{\dfrac{k_1}{\alpha_2} - \alpha_1 x - y}{k_2 - \alpha_1 x} \end{cases} \tag{2}$$

In this paper, we consider a more general dynamic model of financial crisis cross contagion between two countries.

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = \gamma_1 x \dfrac{\dfrac{k_2}{\alpha_1} - x - \alpha_2 y}{k_1 + \beta_1 x - \alpha_2 y} \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = \gamma_2 y \dfrac{\dfrac{k_1}{\alpha_2} - \alpha_1 x - y}{k_2 - \alpha_1 x + \beta_2 y} \end{cases} \tag{3}$$

where x, y stand for exchange rate of country A and exchange rate of country B respectively, γ_1, γ_2 the intrinsic growth of exchange rate of country A and country B respectively, k_1, k_2 the superior limit of exchange rate changing of country A and country B respectively, α_1 the crisis infect coefficient B to A, α_2 the crisis infect coefficient A to B, β_1 the "addicted-to-absorption" coefficient of country A toward the crisis infect, β_2 "addicted-to-absorption" coefficient of country B toward the crisis infect, $0 < \alpha_i < 1$, $\gamma_i > 0$, $k_i > 0$ and $\beta_i > -1$ ($i = 1, 2$).

In order to be convenient, we transform (3) to (4).

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = xF_1(x, y) \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = yF_2(x, y) \end{cases} \tag{4}$$

where

$$F_1(x, y) = \gamma_1 \frac{\dfrac{k_2}{\alpha_1} - x - \alpha_2 y}{k_1 + \beta_1 x - \alpha_2 y} \tag{5}$$

$$F_2(x, y) = \gamma_2 \frac{\dfrac{k_1}{\alpha_2} - \alpha_1 x - y}{k_2 - \alpha_1 x + \beta_2 y} \tag{6}$$

The parameters α_i, γ_i, k_i ($i = 1, 2$) are all positive constants and $\beta_i > -1$ ($i = 1, 2$).

We denote the region (**see Figure 1**)

$$D = \left\{ (x, y) \,\middle|\, 0 < x < \frac{k_2}{\alpha_1}, 0 < y < \frac{k_1}{\alpha_2} \right\} \tag{7}$$

3. Stability Analysis

Proposition 1: If $\beta_i > -1$ ($i = 1, 2$), $0 < \alpha_i < 1$, $\alpha_1 k_1 > \alpha_2^2 k_2$, $\alpha_2 k_2 > \alpha_1^2 k_1$, then there exists an unique equilibrium point $Q(\bar{x}, \bar{y})$ in the region D, and it is a stable node or focus.

Proof: After a simple calculation, we can obtain an equil-brium point $Q(\bar{x}, \bar{y})$ in addition to another three equilbrium points $(0, 0)$,

$$\left(\frac{k_1}{\alpha_1 \alpha_2}, 0\right), \quad \left(0, \frac{k_2}{\alpha_1 \alpha_2}\right) \quad \text{and} \quad Q(\bar{x}, \bar{y})$$

where

$$\bar{x} = \frac{\alpha_2 k_2 - \alpha_1^2 k_1}{(1 - \alpha_1 \alpha_2)\alpha_1 \alpha_2}, \quad \bar{y} = \frac{\alpha_1 k_1 - \alpha_2^2 k_2}{(1 - \alpha_1 \alpha_2)\alpha_1 \alpha_2} \tag{8}$$

If it satisfies the conditions of proposition 1, then we can easily obtain

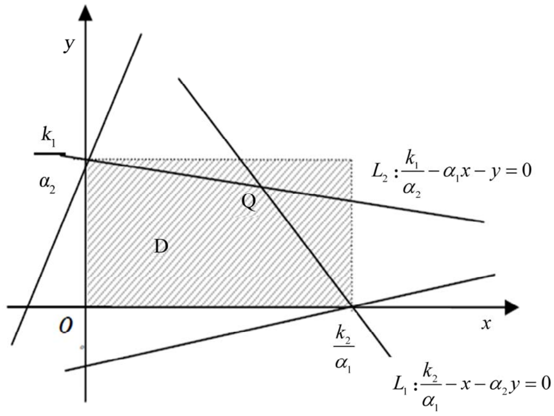

Figure 1. The equilibrium point Q in the region D.

$$0<\overline{x}<\frac{k_1}{\alpha_2},\quad 0<\overline{y}<\frac{k_2}{\alpha_1} \tag{9}$$

We can conclude that there is a unique equilibrium point in region D.

By using Taylor formula, we can transform (4) to a form as following,

$$\begin{cases}\dfrac{\mathrm{d}x}{\mathrm{d}t}=x\left[\dfrac{\partial F_1}{\partial x}\Big|_Q\cdot\left(x-\overline{x}\right)+\dfrac{\partial F_1}{\partial y}\Big|_Q\cdot\left(y-\overline{y}\right)\right]\\ \dfrac{\mathrm{d}y}{\mathrm{d}t}=y\left[\dfrac{\partial F_2}{\partial x}\Big|_Q\cdot\left(x-\overline{x}\right)+\dfrac{\partial F_2}{\partial y}\Big|_Q\cdot\left(y-\overline{y}\right)\right]\end{cases} \tag{10}$$

where Q point of coordinates are

$$x_Q=\overline{x}+\theta_1\left(x-\overline{x}\right),\ y_Q=\overline{y}+\theta_2\left(y-\overline{y}\right) \tag{11}$$

and $0<\theta_i<1\ (i=1,2)$.

The equivalent form for system (10) is

$$\begin{cases}\dfrac{\mathrm{d}x}{\mathrm{d}t}=x\left[a_{11}\left(x_Q,y_Q\right)\cdot\left(x-\overline{x}\right)+a_{12}\left(x_Q,y_Q\right)\cdot\left(y-\overline{y}\right)\right]\\ \dfrac{\mathrm{d}y}{\mathrm{d}t}=y[a_{21}\left(x_Q,y_Q\right)\cdot\left(x-\overline{x}\right)+a_{22}\left(x_Q,y_Q\right)\cdot\left(y-\overline{y}\right)]\end{cases} \tag{12}$$

After simple calculating, we can obtain

$$a_{11}\left(x,y\right)=\gamma_1\frac{\left(\dfrac{k_2}{\alpha_1}-2x-\alpha_2 y\right)\left(k_1-\alpha_2 y\right)-\beta_1 x^2}{\left(k_1+\beta_1 x-\alpha_2 y\right)^2} \tag{13-1}$$

$$a_{12}\left(x,y\right)=\gamma_1\alpha_2\frac{x\left[\dfrac{k_2}{\alpha_1}-k_1-(1+\beta_1)x\right]}{\left(k_1+\beta_1 x-\alpha_2 y\right)^2} \tag{13-2}$$

$$a_{21}\left(x,y\right)=\gamma_2\alpha_1\frac{y\left[\dfrac{k_1}{\alpha_2}-k_2-\left(1+\beta_2\right)y\right]}{\left(k_2-\alpha_1 x+\beta_2 y\right)^2} \tag{13-3}$$

$$a_{22}\left(x,y\right)=\gamma_2\frac{\left(\dfrac{k_1}{\alpha_2}-\alpha_1 x-2y\right)\left(k_2-\alpha_1 x\right)-\beta_2 y^2}{\left(k_2-\alpha_1 x+\beta_2 y\right)^2} \tag{13-4}$$

Embedding the coordinates of point $Q\left(\overline{x},\overline{y}\right)$ into equations from (13-1) to (13-4) one by one, we can obtain expression as following respectively.

$$a_{11}\left(x_Q,y_Q\right)=-\frac{\gamma_1\left(\beta_1+\alpha_1\alpha_2\right)\overline{x}^2}{A^2} \tag{14-1}$$

$$a_{12}\left(x_Q,y_Q\right)=\frac{\gamma_1\alpha_2\overline{x}\left[\dfrac{k_2}{\alpha_1}-k_1-\left(1+\beta_1\right)\overline{x}\right]}{A^2} \tag{14-2}$$

$$a_{21}\left(x_Q,y_Q\right)=\frac{\gamma_2\alpha_1\overline{y}\left[\dfrac{k_1}{\alpha_2}-k_2-\left(1+\beta_2\right)\overline{y}\right]}{B^2} \tag{14-3}$$

$$a_{22}\left(x_Q,y_Q\right)=-\frac{\gamma_2\left(\beta_2+\alpha_1\alpha_2\right)\overline{y}^2}{B^2} \tag{14-4}$$

where

$$A=\frac{\beta_1+\alpha_2{}^2}{\left(1-\alpha_1\alpha_2\right)\alpha_1\alpha_2}\left(\alpha_2 k_2-\alpha_1{}^2 k_1\right) \tag{15}$$

$$B=\frac{\beta_2+\alpha_1{}^2}{\left(1-\alpha_1\alpha_2\right)\alpha_1\alpha_2}\left(\alpha_1 k_1-\alpha_2{}^2 k_2\right) \tag{16}$$

In order to judge the state of the equilibrium point Q, we consider the characteristic equation of equation (4) at the point Q.

$$\lambda^2+p\lambda+q=0 \tag{17}$$

where

$$p=-\left[a_{11}\left(x_Q,y_Q\right)+a_{22}\left(x_Q,y_Q\right)\right] \tag{18}$$

$$q=a_{11}\left(x_Q,y_Q\right)a_{22}\left(x_Q,y_Q\right)-a_{12}\left(x_Q,y_Q\right)a_{21}\left(x_Q,y_Q\right) \tag{19}$$

We can judge the state of the equilibrium point Q according ODE theory when we embed (14-1), (14-2), (14-3) and (14-4) into equation (18) one by one.

Proposition 2: If $\min\{\beta_1,\beta_2\}+\alpha_1\alpha_2>0$, then equilibrium point Q is a stable node or focus.

Proof: Embed（14-1）and 14-4）into inequality as following

$$-\left[a_{11}\left(x_Q,y_Q\right)+a_{22}\left(x_Q,y_Q\right)\right]>0 \tag{20}$$

After calculating, we can easily obtain

$$\frac{\gamma_1\left(\beta_1+\alpha_1\alpha_2\right)}{\left(\beta_1+\alpha_2^{\,2}\right)^2}+\frac{\gamma_2\left(\beta_2+\alpha_1\alpha_2\right)}{\left(\beta_2+\alpha_1^{\,2}\right)^2}>0 \tag{21}$$

So the proposition 2 is obviously true.

Proposition 3: If $\min\{\beta_1,\beta_2\}+\alpha_1\alpha_2<0$, then equilibrium point Q is an unstable node or focus.

Proof: Embed（14-1）and （14-4）into inequality as fol-lowing

$$-\left[a_{11}\left(x_Q,y_Q\right)+a_{22}\left(x_Q,y_Q\right)\right]<0 \tag{22}$$

After calculating, we can easily obtain

$$\frac{\gamma_1\left(\beta_1+\alpha_1\alpha_2\right)}{\left(\beta_1+\alpha_2^{\,2}\right)^2}+\frac{\gamma_2\left(\beta_2+\alpha_1\alpha_2\right)}{\left(\beta_2+\alpha_1^{\,2}\right)^2}<0 \tag{23}$$

So the proposition 3 is obviously true.

Proposition 4: There exists at least a pair of β_1 and β_2 Which satisfies conditions

$$\left(\beta_1+\alpha_1\alpha_2\right)\left(\beta_2+\alpha_1\alpha_2\right)<0 \tag{24}$$

and such that $p=0$. In this situation, problem of judging center or focus appears.

Proof: We can find a pair of β_1 and β_2 as following

$$\begin{cases}\beta_1=\dfrac{\gamma_2}{\gamma_1+\gamma_2}\left(\alpha_1^{\,2}-\alpha_2^{\,2}\right)-\alpha_1\alpha_2\\ \beta_2=\beta_1+\left(\alpha_2^{\,2}-\alpha_1^{\,2}\right)\end{cases} \tag{25}$$

which satisfies condition $\left(\beta_1+\alpha_1\alpha_2\right)\left(\beta_2+\alpha_1\alpha_2\right)<0$ and such that $p=0$.

So the proposition 4 is true.

Proposition 5: If $q<0$, then equilibrium point Q is unstable saddle point.

Proof: Embed (14-1), (14-2), (14-3) and (14-4) into (19), we can obtain

$$\beta_1\beta_2+\alpha_1\alpha_2\left(\beta_1+\beta_2\right)+\alpha_1^{\,2}\alpha_2^{\,2}<0 \tag{25}$$

Here α_1， α_2 are both real numbers. Thus，if (25) is true， β_1， β_2 must satisfy the condition as following

$$\Delta=\left(\beta_1+\beta_2\right)^2-4\beta_1\beta_2<0 \tag{26}$$

However, In fact, β_1 and β_2 can not meet inequality (26) in any case. So the proposition 5 is true.

4. Conclusions

In nature some vegetables tend to absorb pollutants like phenol. This is known as "addicted-to-absorption" phenomenon. Alternatively, some plants have a pronounced tendency for "anti-absorption." Financial markets exhibit similar characteristics. When a country is in financial crisis, financial panic ensues and the investors withdraw their money from neighbor or other countries. This makes the financial crisis spread to other countries, and accelerates the decline of prices or the return on assets. This is called the "herding effect." If the affected countries, however, give investors confidence in a timely manner or show stronger economical and financial support, panic among investors can be alleviated. This makes investors to bring more funds to the affected country making asset prices and yields to rise.

Therefore, based on Ying & Zou's (2010) contagion model, we introduce a factor $\beta_1 x$ or $\beta_2 y$ to reflect such an accelerating phenomenon for better describing the dynamic behavior of financial contagion between the two countries. Inspired by the "addicted-to-absorption" phenomenon of the plant, we defined β_i as the "addicted-to-absorption" coefficient of i country's financial markets reflecting to financial contagion. And we define: if $0>\beta_i>-1$, the changes of exchange rate in country i will accelerate its volatility; $\beta_i>0$ States that the exchange rate changes will decrease its volatility.

Furthermore, after analyzing the stability of the promoting model by Ordinary Differential Equation Qualitative Theory, we find: under the assumption that the upper limit of exchange rate change in a country depends on the ratio between the fluctuation limit of another country's exchange rate and its transmission coefficients, there exists some linear combinations of β_i and α_i to reflect that the financial crisis contagions are becoming uncontrollable or controllable. At some angle, the finding is closer to the actual state of the financial contagions. For further study, we will find the approximate calculation method of β_i and α_i by discretizing the promoting differential dynamic model. It is a simple way to do empirical analysis of financial contagions between two countries. This will be our main task of the future.

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