

# Improvement of Chen-Zhang-Liu's IRPB Signature Scheme

Dezhi Gao

*College of Information Science and Engineering, Shandong University of Science and Technology,  
Qingdao, Shandong, China*

*E-mail: dezhi\_gao@yahoo.com.cn*

*Received June 8, 2010; revised July 18, 2010; accepted August 21, 2010*

## Abstract

Restrictive partially blind signatures incorporate the advantages of restrictive blind signatures and partially blind signatures, which play an important role in electronic commerce. Recently, Chen-Zhang-Liu first proposed an ID-based restrictive partially blind (IRPB) signature from bilinear pairings. Later, Hu-Huang showed that the Chen-Zhang-Liu's scheme has a security weakness, and pointed out that their scheme does not satisfy the property of restrictiveness as they claimed. In this paper, we improve Chen-Zhang-Liu's scheme and propose a new signature scheme from bilinear pairings. The improved scheme can resist the Hu-Huang's attack.

**Keywords:** Cryptography, Bilinear Pairings, Restrictiveness, Partially Blind Signature, ID-Based Restrictive Partially Blind Signature

## 1. Introduction

Blind signature scheme were first introduced by Chaum [1] to protect the right of an individual privacy. A blind signature allows a user to acquire a signature without giving the signer any information about the actual message or the resulting signature. And blind signature techniques have been widely used in anonymous electronic cash(e-cash) and anonymous voting systems.

Restrictive blind signatures firstly introduced by Brands [2], which allows a user to receive a blind signature on a message not known to the signer but the choice of message is restricted and must conform to certain rules. Furthermore, he proposed a highly efficient electronic cash system, where the bank ensures that the user is restricted to embed his identity in the resulting blind signature.

A partially blind signature scheme allows a signer to produce a blind signature on a message for a user, where the signature explicitly includes common agreed information which remains clearly visible despite the blinding process. The concept was first introduced by Abe and Fujisaki[3].

In an electronic cash system, embedding user identity information in the blind signature enables the bank to

learn the identity of double spenders. Maitland and Boyd [4] first proposed a provably secure restrictive partially blind signature scheme, which satisfies the partial blindness and restrictive blindness.

Recently, Chen-Zhang-Liu[5] proposed the first ID-based restrictive partially blind signature scheme(IRPB) by combining an ID-based partially blind signature scheme proposed by Chow *et al.*[6] with an ID-based restrictive blind signature scheme proposed by Chen-Zhang-Liu[7]. However, X. M. Hu. and S. T. Huang.[8] found that Chen-Zhang-Liu's scheme had an important weakness. Their scheme does not achieve the restrictiveness property as they claimed. They showed, in an electronic cash system constructed by Chen-Zhang-Liu, an account-holder cannot be caught when he performs double-spending. In this paper, we propose an improvement of Chen-Zhang-Liu's scheme, and the new scheme can resist the Hu-Huang *et al.*'s attack.

The rest of the paper is organized as follows: In Section 2, we briefly review Chen-Zhang-Liu's scheme. We propose an improvement of Chen-Zhang-Liu's scheme in Section 3. The completeness and security of improved scheme are discussed in Section 4. Finally, conclusions will be made in Section 5.

## 2. Review of Chen-Zhang-Liu's Scheme

### 2.1. Basic Concepts on Bilinear Pairings

Let  $G_1$  be an additive cyclic group with prime order  $q$ ,  $G_2$  be a multiplicative cyclic group of same order and  $P$  be a generator of  $G_1$ . Let  $e: G_1 \times G_1 \rightarrow G_2$  be a bilinear mapping with the following properties:

1) bilinear:  $e(aP, bQ) = e(P, Q)^{ab}$  for all  $P, Q \in G_1$  and  $a, b \in \mathbb{Z}_q^*$ ;

2) non-degenerate: there exists  $P$  and  $Q \in G_1$  such that  $e(P, Q) \neq 1$ ;

3) computable: there exists an efficient algorithm to compute  $e(P, Q)$  for  $P, Q \in G_1$ .

The bilinear pairings can be derived from the Weil or Tate pairings.

### 2.2. Complexity Assumptions

Let  $G$  be a cyclic multiplicative group generated by  $g$ , whose order is a prime  $q$ , assume that the inversion and multiplication in  $G$  can be computed efficiently.

**Discrete Logarithm Problem (DLP):** Given two elements  $g$  and  $h$ , to find an integer  $n \in \mathbb{Z}_q^*$ , such that  $h = g^n$  whenever such an integer exists.

We assume that the discrete logarithm problem (DLP) in both  $G_1$  and  $G_2$  are hard.

### 2.3. Chen-Zhang-Liu's ID-based Restrictive Partially Blind Signature Scheme

Chen-Zhang-Liu's ID-based restrictive partially blind signature scheme (Chen *et al.*, 2007) consists of four phases: *system parameters generation, key generation, signature generation and signature verification*. For simplicity, we will use the same notation as Chen-Zhang-Liu's scheme.

Define two cryptographic secure hash functions

$$H: \{0,1\}^* \rightarrow G_1 \quad \text{and} \quad H_1: G_1^3 \times G_2^4 \rightarrow \mathbb{Z}_q.$$

-*System parameters generation*: On input security parameter  $k$ , output the master key  $s \in_R \mathbb{Z}_q^*$  and the system parameter

$$\text{params} = \{G_1, G_2, e, q, P, P_{pub}, k, H, H_1\}.$$

-*Key generation*: Given *params* and the signer's identity information ID, output the signer's private key

$$S_{ID} = sQ_{ID} = sH(ID).$$

-*Signature generation*: Let  $\Delta$  be the shared information and a message  $M$  be from the receiver.

Publish  $g = e(P, Q_{ID})$  and  $y = e(P_{pub}, Q_{ID})$ .

- The signer randomly chooses  $Q \in_R G_1, r \in_R \mathbb{Z}_q^*$  and computes  $z = e(M, S_{ID}), a = e(P, Q), b = e(M, Q)$ ,

$U = rP$  and  $Y = rQ_{ID}$ . He then sends  $(z, a, b, U, Y)$  to the receiver.

- The receiver randomly generates  $\alpha, \beta, u, v, \lambda, \mu, \gamma \in_R \mathbb{Z}_q^*$ , and computes  $M' = \alpha M + \beta P$ ,  $z' = z^\alpha y^\beta$ ,  $A = e(M', Q_{ID})$ ,  $a' = a^u g^v$ ,  $b' = a^{u\beta} b^{u\alpha} A^v$ ,  $Y' = \lambda Y + \lambda \mu Q_{ID} - \gamma H(\Delta)$ ,  $U' = \lambda U + \gamma P_{pub}$ ,  $h = \lambda^{-1} H_1(M', Y', U', A, z', a', b') + \mu$  and  $c' = hu$ . He then sends  $h$  to the signer.

- The signer computes  $S_1 = Q + hS_{ID}$ ,  $S_2 = (r + h)S_{ID} + rH(\Delta)$  and sends  $(S_1, S_2)$  to the receiver.

- If the equations  $e(P, S_1) = ay^h$  and  $e(M, S_1) = bz^h$  hold, the receiver computes  $s_1' = uS_1 + vQ_{ID}$  and  $s_2' = \lambda S_2$ . Thus, the tuple  $(Y', U', z', c', s_1', s_2')$  is the signature for  $\Delta$  and  $M'$ .

-*Signature verification*: Given a tuple  $(Y', U', z', c', s_1', s_2')$  for  $\Delta$  and  $M'$ , the verifier computes  $A = e(M', Q_{ID})$ ,  $a' = e(P, s_1')y^{-c'}$  and  $b' = e(M', s_1')z'^{-c'}$ . The verifier accepts the signature if the following equation holds:

$$e(s_2', P) = e(Y' + H_1(M', Y', U', A, z', a', b')Q_{ID}, P_{pub}) \times e(H(\Delta), U')$$

Unfortunately, Hu-Huang [8] pointed out that above scheme is insecure and can not achieve the property of restrictiveness as they claimed. Any adversary receiver can obtain a valid signature for a message.

## 3. Improvement of Chen-Zhang-Liu's Scheme

In [8], X.M. Hu *et al.* found that above scheme had an important weakness. Any adversary receiver can obtain a valid signature for a message  $M'$  with any form. The main reason is that the Chen-Zhang-Liu's scheme had more variable parameters. In this section, we present an improvement of Chen-Zhang-Liu's scheme. The system initialization phase is the same as the one presented in Section 2. In the following, we only describe the *Signature generation* and *Signature verification*.

-*System parameters generation*: The parameters generation is just as before.

-*Key generation*: The key generation is just as before.

-*Signature generation*: Let  $\Delta$  be the shared information and a message  $M$  be from the receiver.

Publish  $g = e(P, Q_{ID})$  and  $y = e(P_{pub}, Q_{ID})$ .

- The signer randomly chooses  $Q \in_R G_1, r \in_R \mathbb{Z}_q^*$  and computes  $z = e(M, S_{ID}), a = e(P, Q), b = e(M, Q)$ ,  $U = rP$  and  $Y = rQ_{ID}$ . He then sends  $(z, a, b, U, Y)$  to the receiver.

- The receiver randomly generates  $\alpha, \beta, u, v \in_R \mathbb{Z}_q^*$ , and computes  $M' = \alpha M + \beta P$ ,  $A = e(M', Q_{ID})$ ,  $z' = z^\alpha y^\beta$ ,  $a' = a^u g^v$ ,  $b' = a^{u\beta} b^{u\alpha} A^v$ ,  $Y' = (\alpha + \beta)Y + (\alpha + \beta)(\beta + u)Q_{ID} - (u + v)H(\Delta)$ ,  $U' = (\alpha + \beta)U + (u + v)P_{pub}$ ,  $h = (\alpha + \beta)^{-1} H_1(M', Y', U', A, z', a', b')$

$+(\beta+u)$  and  $c'=hu$ . He then sends  $h$  to the signer.

- The signer computes  $S_1 = Q + hS_{ID}$ ,  $S_2 = (r+h)S_{ID} + rH(\Delta)$ , and sends  $(S_1, S_2)$  to the receiver.
- If the equations  $e(P, S_1) = ay^h$  and  $e(M, S_1) = bz^h$  hold, the receiver computes  $s_1' = uS_1 + vQ_{ID}$  and  $s_2' = \lambda S_2$ . Thus, the tuple  $(Y', U', z', c', s_1', s_2')$  is the signature for  $\Delta$  and  $M'$ .

-*Signature verification*: Given a tuple  $(Y', U', z', c', s_1', s_2')$  for  $\Delta$  and  $M'$ , the verifier computes  $A = e(M', Q_{ID})$ ,  $a' = e(P, s_1')y^{-c'}$  and  $b' = e(M', s_1')z^{-c'}$ . The verifier accepts the signature if the following equation holds:

$$e(s_2', P) = e(Y' + H_1(M', Y', U', A, z', a', b')Q_{ID}, P_{pub}) \times e(H(\Delta), U')$$

#### 4. Discussion

In this section, we first discuss the completeness of our improved scheme, and then show the new scheme can resist against the proposed attack by X.M. Hu *et al.*

**Theorem 1.** The improved scheme achieves the property of completeness.

**Proof.** Note that

$$e(P, s_1') = e(P, S_1)^u \cdot e(P, Q_{ID})^v = (ay^h)^u g^v = a^u y^{hc}$$

$$e(M', s_1') = e(M', S_1)^u \cdot e(M', Q_{ID})^v = e(\alpha M + \beta P, S_1)^u \cdot A^v = b^u z^{hc}$$

and

$$\begin{aligned} e(s_2', P) &= e((\alpha + \beta)S_2, P) \\ &= e((\alpha + \beta)(r+h)S_{ID} + (\alpha + \beta)H(\Delta), P) \\ &= e((\alpha + \beta)r + H_1(M', Y', U', A, z', a', b') \\ &\quad + (\alpha + \beta)(\beta + u)Q_{ID}, P_{pub}) \cdot e(H(\Delta), (\alpha + \beta)rP) \\ &= e((\alpha + \beta)r + H_1(M', Y', U', A, z', a', b') \\ &\quad + (\alpha + \beta)(\beta + u)Q_{ID}, P_{pub}) \cdot e(H(\Delta), U' - \gamma P_{pub}) \\ &= e((\alpha + \beta)r + H_1(M', Y', U', A, z', a', b') \\ &\quad + (\alpha + \beta)(\beta + u)Q_{ID} - \gamma H(\Delta), P_{pub}) \cdot e(H(\Delta), U') \\ &= e((Y' + H_1(M', Y', U', A, z', a', b')Q_{ID}, P_{pub}) \\ &\quad \cdot e(H(\Delta), U')) \end{aligned}$$

Thus, the improved scheme achieves the property of completeness.

**Theorem 2.** The improved scheme is secure and can resist the attack proposed by Hu-Huang.

**Proof.** Hu-Huang pointed out that the Chen-Zhang-Liu's scheme is insecure, the key reason is that their scheme was constructed by simply assembling Chow *et al.*'s scheme (2005) and Chen *et al.*'s scheme (2005). In their scheme, the receiver randomly generated seven numbers,

and these numbers have redundancy. Any adversary can construct  $(z', a', b')$  without using the values of  $(z, a, b)$  in step in the Chen-Zhang-Liu's scheme. Comparison with the Chen-Zhang-Liu's scheme, our improved scheme only choose  $\alpha + \beta$ ,  $\beta + u$  and  $u + v$ , instead of choosing the parameters  $\lambda$ ,  $\mu$  and  $\gamma$ , respectively, and the verification equation is same as that of Chen-Zhang-Liu. Thus, we reduce the variable parameters and only need four parameters instead of seven parameters in Chen-Zhang-Liu's scheme. Since the parameters  $\alpha + \beta$ ,  $\beta + u$  and  $u + v$  are depend on the parameters  $\alpha$ ,  $\beta$ ,  $u$  and  $v$  in the improved scheme, the recipient obtains a signature on a message that can only be the form  $M' = \alpha M + \beta P$  with  $\alpha$  and  $\beta$  randomly chosen by the recipient. Similar to Chen-Zhang-Liu's analysis, our scheme achieves the property of restrictiveness and can resist the Hu-Huang's attack.

The other properties of the improved scheme are same as the Chen-Zhang-Liu's scheme, we omit them.

#### 5. Conclusions

Chen-Zhang-Liu proposed the first ID-based restrictive partially blind signature scheme (IRPB) by combing the ID-based partially blind signature scheme with the ID-based restrictive blind signature scheme. Recently, X.M Hu *et al.* showed the Chen-Zhang-Liu's scheme did not satisfy the property of restrictiveness as they claimed. In this paper, we give an improved version of Chen-Zhang-Liu's scheme, and the improved scheme can resist the attack proposed by X. M. Hu *et al.*

#### 6. Acknowledgements

The author wishes to thank the anonymous referees for their valuable comments and suggestions.

#### 7. References

- [1] D. Chaum, "Blind Signatures for Untraceable Payments," *Advances in Cryptology Crypto'82*, Springer-Verlag, Germany, 1982, pp. 199-203.
- [2] S. Brands, "Untraceable off-Line Cash in Wallets with Observers," *Advances in Cryptology Crypto'93, LNCS 773*, Springer-Verlag, Germany, 1993, pp. 302-318.
- [3] M. Abe and E. Fujisaki, "How to Date Blind Signatures," *Advances in Cryptology-Asiacrypt 1996, LNCS 1163*, Springer-Verlag, Germany, 1996, pp. 244-251.
- [4] G. Maitland and C. Boyd, "A Provably Secure Restrictive Blind Signature Scheme," *PKC'02, LNCS 2274*, Springer-Verlag, Germany, 2002, pp. 99-114.
- [5] X. F. Chen, F. G. Zhang and S. L. Liu, "ID-Based Restrictive Partially Blind Signatures and Applications," *The Journal of Systems and Software*, Vol. 80, No. 2,

- 2007, pp. 64-71.
- [6] S. M. Chow, C. K. Hui and S. M. Yin, "Two Improved Partially Blind Signature Schemes from Bilinear Pairings," *ACISP'05, LNCS 3574*, Springer-Verlag, Germany, 2005, pp. 316-328.
- [7] X. F. Chen, F. G. Zhang and S. L. Liu, "ID-Based Restrictive Partially Blind Signatures and Applications," 2007. <http://eprint.iacr.org/2005/3/319/>.
- [8] X. M. Hu and S. T. Huang, "Analysis of ID-Based Restrictive Partially Blind Signatures and Applications," *The Journal of Systems and Software*, Vol. 81, No. 11, 2008, pp. 1951-1954.