Intelligent Control and Automation, 2017, 8, 126-138
http://www.scirp.org/journal/ica
ISSN Online: 2153-0661
ISSN Print: 2153-0653

Method of Dynamic VaR and CVaR Risk Measures Forecasting for Long Range Dependent Time Series on the Base of the Heteroscedastic Model

Nataliya D. Pankratova, Nataliia G. Zrazhevska

Institute for Applied Systems Analysis, National Technical University of Ukraine "Kyiv Polytechnic Institute", Kyiv, Ukraine
Email: natasha.zrazhevska@gmail.com

How to cite this paper: Pankratova, N.D. and Zrazhevska, N.G. (2017) Method of Dynamic VaR and CVaR Risk Measures Forecasting for Long Range Dependent Time Series on the Base of the Heteroscedastic Model. *Intelligent Control and Automation*, **8**, 126-138.
https://doi.org/10.4236/ica.2017.82010

Received: March 30, 2017
Accepted: May 23, 2017
Published: May 26, 2017

Abstract

The paper proposes a new method of dynamic VaR and CVaR risk measures forecasting. The method is designed for obtaining the forecast estimates of risk measures for volatile time series with long range dependence. The method is based on the heteroskedastic time series model. The FIGARCH model is used for volatility modeling and forecasting. The model is reduced to the AR model of infinite order. The reduced system of Yule-Walker equations is solved to find the autoregression coefficients. The regression equation for the autocorrelation function based on the definition of a long-range dependence is used to get the autocorrelation estimates. An optimization procedure is proposed to specify the estimates of autocorrelation coefficients. The procedure for obtaining of the forecast values of dynamic risk measures VaR and CVaR is formalized as a multi-step algorithm. The algorithm includes the following steps: autoregression forecasting, innovation highlighting, obtaining of the assessments for static risk measures for residuals of the model, forming of the final forecast using the proposed formulas, quality analysis of the results. The proposed method is applied to the time series of the index of the Tokyo stock exchange. The quality analysis using various tests is conducted and confirmed the high quality of the obtained estimates.

Keywords

Dynamic VaR, CVaR, Forecasting, Long Range Dependence, Hurst Parameter, Heteroscedastic Model

1. Introduction

VaR (Value-at-Risk) and CVaR (Conditional Value-at-Risk) have become the standard measures of market risk management. Their popularity has led to a large number of publications on this topic in recent years. Definition, description of the properties and comparative analysis of these risk measures can be found, for example, in [1] [2] [3]. Various methods for their evaluation and forecasting that represents different approaches are proposed. Most of the methods that provide explicit formulas for CVaR estimation are described in [4]. Optimization approach for CVaR evaluation is given in [5] [6]. Non-parametric methods of estimation can be found, for example, in [7] [8]. A large number of works devoted to the method of VaR and CVaR estimating based on the stochastic time series model. The basic ideas of the approach can be found for example in [2] [9] [10]. A significant number of works show the practical application of the approach for estimating and forecasting of stock indices, see for example [10] [11] [12] [13].

At the same time, during the global financial turmoil, the problem of constructing of new approaches for VaR and CVaR estimating and forecasting remains relevant. In this paper, we propose a new method for VaR and CVaR prediction for financial time series. The method takes into account the most statistically significant extreme values of data and the presence of the long-range dependence that is typical for financial time series [2] [14]. For the convenience of practical application, the method is formulated as an incremental algorithm. At each step, the system of tests is proposed to evaluate the quality of the obtained results.

The proposed algorithm is used for forecasting VaR and CVaR for the time series of daily log return *Nikkey*225 *Stock Index*. The analysis of the obtained forecast estimates confirms their high quality. The formatter will need to create these components, incorporating the applicable criteria that follow.

2. Key Definitions

The continuously distributed random variable $\{X_t, t \in Z\}$ with finite mean defined on the probability space $(\Omega, \Psi_t, \mathrm{P}_t)$ is considered. Here Ψ_t is the information set containing all available at the time t information about the time series. Series $\{X_t^2, t \in Z\}$ is assumed to be stationary. It is accepted that the time series has the property of the long-range dependence [15]: there is $0 < \gamma < 1$ and $c_r > 0$ so that:

$$\lim_{k\to\infty} \rho(k)\big/\left(c_r k^{-\gamma}\right) = 1,\quad \rho(k) = \mathrm{Corr}\left(X_t, X_{t+k}\right),\quad k \in N \cup \{0\} \tag{1}$$

For a fixed confidence level α dynamic risk measures VaR and CVaR are defined as [9]:

$$\mathrm{VaR}_\alpha^t(t+h) = \inf\left\{x \in R \,\middle|\, \mathrm{P}_t\left[X_{t+h} \le x\right] \ge \alpha\right\},$$

$$\mathrm{CVaR}_\alpha^t(t+h) = E_{\Psi_t}\left[X_{t+h} \,\middle|\, X_{t+h} \ge \mathrm{VaR}_\alpha^t(t+h)\right],$$

$E_{\Psi_t}[\cdot]$ denotes expectation with respect to Ψ_t.

The aim of the study is to construct a model for $\mathrm{VaR}_\alpha^{\tilde{t}}(\tilde{t}+h)$ and

$\mathrm{CVaR}_{\alpha}^{\tilde{t}}\left(\tilde{t}+h\right)$ where $\tilde{t}$ is an arbitrary moment of time. The forecasting values are determined by extrapolation of the values of this model (21 cm × 28.5 cm).

3. Forecast Methodology

In the article [16], the most popular methods for dynamic VaR and CVaR estimating are analyzed, their classification is given and the recommendations for their use are proposed. In accordance with the formulated in the article the structural scheme of selection of dynamic risk measures estimation the approach based on a stochastic time series model is chosen.

Suppose that the time series $\{X_t, t \in Z\}$ is a trajectory of stochastic process, that is:

$$X_t = \mu_t + \varepsilon_t = \mu_t + \sigma_t Z_t \tag{2}$$

where conditional mean μ_t and variation σ_t are defined on the information space Ψ_t, $\{Z_t\} \overset{iid}{\sim} F_t(0,1)$ (independent, identically distributed random variables with a conditional distribution function $F_t(0,1)$). Let Z is a random variable with the same distribution as any random variable from $\{Z_t\}$. Then [2] [9] [10]:

$$\begin{aligned} \mathrm{VaR}_{\alpha,k}^{t} &= \mu_{t+k} + F^{-1}(\alpha)\sigma_{t+k} = \mu_{t+k} + \mathrm{VaR}_{\alpha}(Z)\sigma_{t+k}, \\ \mathrm{CVaR}_{\alpha,k}^{t} &= \mu_{t+k} + \mathrm{CVaR}_{\alpha}(Z)\sigma_{t+k}. \end{aligned} \tag{3}$$

It is necessary to construct the forecast model for σ_t to determine its P days forecast and to estimate VaR and CVaR for a random variable Z. Then the forecasting values for dynamic risk measures can be found under the following formulas:

$$\begin{aligned} \mathrm{VaR}_{\alpha}^{t+P} &= \mu_{t+P} + \mathrm{VaR}_{\alpha}(Z)\sigma_{t+P}, \\ \mathrm{CVaR}_{\alpha}^{t+P} &= \mu_{t+P} + \mathrm{CVaR}_{\alpha}(Z)\sigma_{t+P}.. \end{aligned} \tag{4}$$

Hereinafter it is assumed that the trend, that defines μ_t, is absent (or removed from the data) [2]. Please do not revise any of the current designations.

4. An Algorithm for Constructing the Dynamic Risk Measures VaR and CVaR Forecast Taking into Account the Long-Range Dependence Presence

For the convenience of the practical application the proposed method for VaR and CVaR forecasting is formulated as an incremental algorithm.

Step 1. For the time series a time series of variances (TSV) is constructed. General analysis of the studied time series and the TSV is carried out, the dependence of time series members (and their squares) from their previous values; the volatility and normality are analyzed.

Step 2. The TSV is tested on the long-range dependence. The Hurst parameter is estimated using five standard methods: the aggregated variance method, the method of absolute values of the aggregated series, the periodogram method, the method of residuals of regression, the R/S method [17]. Average value $\widehat{H}_{mn}$

is chosen as the Hurst parameter estimation.

Step 3. The model for σ_t forecasting is estimated using the FIGARCH model and taking into account the long-range dependence of the WFD. The actualization of the model by reducing it to the model AR (∞) is performed. The method of smoothing of the autocorrelation function (ACF) proposed by the authors in [18] (the new method) is used. The least square method is used to determine the autoregression coefficients $(a_1,\cdots,a_N,\cdots)'$. So the problem is reduced to the infinite system of Yule-Walker equations [18]:

$$\sum_{j=0}^{\infty}\rho_{|i-j|}a_j=\rho_{i+1},\quad i=0,\cdots,\infty. \tag{5}$$

The regression equation for ACF based on the definition of the long-range dependence (1) is used to get estimates for $\rho_i:\rho(k)=\alpha_1 H(2H-1)k^{2H-2}+\alpha_2+\varepsilon_k$, $\varepsilon_k - iid$, $k_0\le k\le N$. With the help of the optimization procedure [17] the Hurst parameter estimate and the estimates $\widehat{\rho}(k)$ are corrected.

Using $\widehat{\rho}(k)$ instead of ρ_k the reduced system of normal Equations (5) is constructed and using the Holetskogo method the vector of assessments $\hat{\vec{a}}_N=(\hat{a}_1,\cdots,\hat{a}_N)'$ is found. As it is shown in [19] the solution of the reduced system converges to the exact solution.

The lag of the reduced AR model $M\le N$ is determined using the information criterions: AIK (Akaike information criterion), HQC (Hannan-Quinn information criterion), SBIC (Bayesian information criterion) [14]. The lag value is chosen on the basis of minimum deviation.

The quality of the obtained AR model is checked. The variance ratio test [20] is used to test if the residuals of the model are *iid* (independent and identically distributed). The resulting model is used to obtain $\hat{\sigma}_t$.

Step 4. The residuals of the model (2) are analyzed. Using $\hat{\sigma}_t$ (step 3) the implementations of a random variable $Z_t:\hat{Z}_t=X_t/\hat{\sigma}_t$ are built. Z_t are analyzed on *iid* (the variance ratio test) and other properties. In accordance with the results using the classification scheme given in [21], the method to get $\mathrm{VaR}_\alpha(Z)$ and $\mathrm{CVaR}_\alpha(Z)$ estimates is chosen. The estimates $\widehat{\mathrm{VaR}}_\alpha(Z)$, $\widehat{\mathrm{CVaR}}_\alpha(Z)$ are obtained.

Step 5. With the results of steps 3 and 4 the model for dynamic risk measures estimating (3) is ready. After building the dynamic risk measures estimations $\widehat{\mathrm{VaR}}_\alpha^t$ и $\widehat{\mathrm{CVaR}}_\alpha^t$ their quality is analyzed using the Kupiec test, the Kristoffersen test and the V test [10] [16].

Step 6. The built dynamic risk measures model is used to get the forecast. Using the model from step 3 the P-step forecast for σ_t is built by the formulas:

$$\hat{\sigma}_{l+p}^2=\sum_{i=p}^{M+p}\hat{a}_i\hat{\sigma}_{l-i+1}^2,\quad l=N+1,\cdots,p=1,\cdots,P. \tag{6}$$

Using the estimates $\widehat{\mathrm{VaR}}_\alpha(Z)$, $\widehat{\mathrm{CVaR}}_\alpha(Z)$ (step 4) the P-step forecast for dynamic risk measures $\widehat{\mathrm{VaR}}_\alpha^{t+P}$ and $\widehat{\mathrm{CVaR}}_\alpha^{t+P}$ (4) is obtained. Then the index of the time series is increased by P and the procedure is repeated as many times as necessary. Thus in each cycle of the algorithm application the model is updated to take into account new data.

Step 7. Using the back testing procedure, the quality of the predicted values $\widehat{\text{VaR}}_{\alpha}^{t+P}$ and $\widehat{\text{CVaR}}_{\alpha}^{t+P}$ (step 6) is checked, the prediction errors ME, MAE, MSE are calculated. For $\widehat{\text{CVaR}}$ estimates the BPoE-test [22] is used.

Schematic description of the proposed method is shown in Figure 1. Please take note of the following items when proofreading spelling and grammar.

5. Numerical Testing of the Algorithm

To demonstrate the proposed algorithm a forecast for dynamic risk measures $(\alpha = 0.9)$ for the time series of log returns on a daily basis is built. Data are collected from the oldest and the most well-known index of Asian markets Nikkey 225 Stock Index (the time series $N225\_RED$)—a composite index of the 225 largest companies publicly traded in Tokyo Stock Exchange for the period from 2005 to 2015. $N225\_RED$ has a relatively low homogeneous volatility. The aim of this study is to forecast risk measures at a regular market behavior, so data without three time intervals with high volatility of the global financial system (01.07.2008-01.07.2009, 01.01.2011-01.07.2011, 01.02.2013-01.12.2013) are considered. Historical data of Nikkey 225 Stock Index are not available online, but upon request.

Table 1 demonstrates the descriptive statistics for the time-series (X_t) and the squared series (X_t^2).

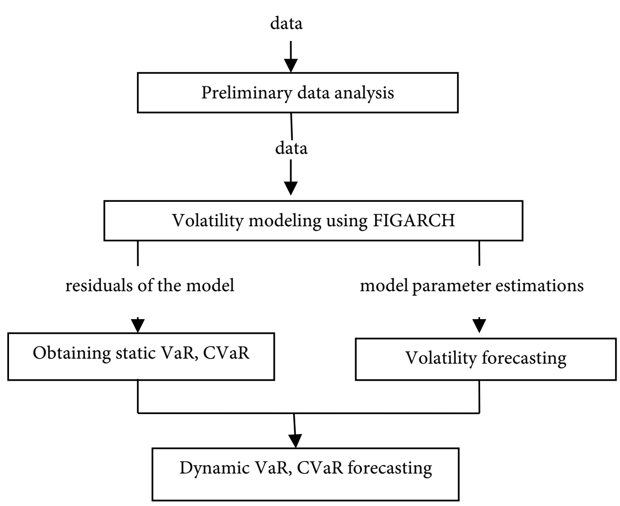

Figure 1. Schematic description of the proposed method of dynamic risk measures VaR and CVaR forecasting.

Table 1. Basic descriptive statistics of the $N225\_RED$.

TS/statistics	Sample size	Mean	Std. deviation	Skewness	Kurtosis	Ljung-box test
X_t	1686	−0.00018	0.013	0.055	3.268	18.493
X_t^2	1686	0.000016	0.0003	1.557	4.372	91.019

Skewness is about 0 and kurtosis is about 3, so the distributions are close to normal. Ljung-Box test [2] results for $m = 7$ confirm the dependence of data (and squared data) on their previous values (the values of *Q*-statistic are larger than critical value 12.017).

Consider the half of the general sample-843 values. The estimates of the Hurst parameter are $\widehat{H}_{mn} = 0.7387$ and $\widehat{H}_{opt} = 0.7281$ (step 2). These values confirm the long-range dependence of the time series.

Simulate σ_t (step 3) using the method SACF (the designation $\_\mathrm{SACF}$) and for comparison the standard methodology (the designation $\_st$). The standard methodology uses the $AR(M)$ model with the coefficients found by the maximum likelihood method (MLH). The lag of the reduced AR model is $M = 55$. The results of the variance ratio test (0.99 < 1.96 for the SACF method and 0.69 < 1.96 for the standard method) confirm that the residuals of the models are *iid*.

For both models Z_t are found (step 4) and their analysis is carried out. The results of the variance ratio test (0.98 < 1.96 for the method SACF and 0.97 < 1.96 for the standard method) show that the residuals of the model (2) are *iid*.

Estimates $\widehat{\mathrm{VaR}}_{0.9}(Z)$, $\widehat{\mathrm{CVaR}}_{0.9}(Z)$ are obtained using the following methods [16]: the historical simulation method (hist), the explicit formulas under the assumption of a normal distribution with the maximum likelihood estimates of the parameters $(\mathrm{paramdistr})$,the explicit formulas using *GEV* and *GPD* functions with the maximum likelihood estimates of the parameters (GEV_quant and GPD_quant respectively), the empirical *POT* method (POT_emp). Table 2 demonstrates the results of the estimating.

Using the results of steps 3, 4 estimates (3) of the dynamic $\widehat{\mathrm{VaR}}_{0.9}^{t}$ and $\widehat{\mathrm{CVaR}}_{0.9}^{t}$ (step 5) are obtained. Figure 2 demonstrates the simulated dynamic $\widehat{\mathrm{VaR}}_{0.9}^{t}\_\mathrm{SACF}$ and $\widehat{\mathrm{CVaR}}_{0.9}^{t}\_\mathrm{SACF}$ (first 836 values) where the explicit formulas under the assumption of a normal distribution $(\mathrm{paramdistr})$ were used for risk measures model residuals estimating.

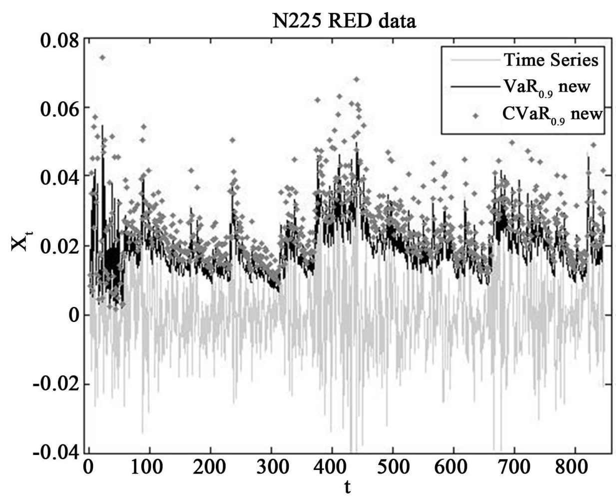

Figure 2. Simulated and predicted values (last 5 values) of dynamic risk measures with new method.

Table 2. The estimates of the statics $\widehat{\mathrm{VaR}}_{0.9}(Z)$, $\widehat{\mathrm{CVaR}}_{0.9}(Z)$.

Risk/method	*hist*	*paramdistr*	*GEV*_quant	*GEV*_quant	*POT*_emp
$\widehat{\mathrm{VaR}}_{0.9}(Z)$_SACF	1.4442	1.5721	1.4942	1.5350	1.2281
$\widehat{\mathrm{CVaR}}_{0.9}(Z)$_SACF	2.4396	2.1375	2.3901	2.4128	2.1501
$\widehat{\mathrm{VaR}}_{0.9}(Z)\_st$	1.3959	1.4993	1.3680	1.3954	1.1940
$\widehat{\mathrm{CVaR}}_{0.9}(Z)\_st$	2.1735	2.0475	2.1719	2.2244	1.9417

Conduct the analysis of quality (step 5) for $\widehat{\mathrm{VaR}}_{0.9}^{t}$ estimates using the Kupiec test ($p-$values of statistics LRpof), the Kristoffersen test ($p-$ values of statistics LRind) and their combination ($p-$ values of statistics LRcc). The obtained estimates are reliable if $p-$ values exceed the given significance level (0.1 in our case). To analyze the $\widehat{\mathrm{CVaR}}_{0.9}^{t}$ estimates the V test with statistics V_1, V_2, V is used. If the estimates are good the statistics, V_1, V_2, V are close to zero. Table 3 presents the results of the analysis of the dynamic risk measures estimates for the SACF method and Table 4 for the standard method.

The analysis of the results shows that the method paramdistr (on the assumption of the normal distribution of residuals) gives the best $\widehat{\mathrm{VaR}}_{0.9}^{t}$ estimates for both methods: $p-$ values of statistics are essentially more than 0.1. This is consistent with the results of the basic analysis (Table 1) and is confirmed by the results of the Jarque-Bera test [2] conducted for Z_t (5.624 < 5.649 and 4.38 < 5.649 for both proposed and standard methods, respectively). At the same time, all estimates obtained with POT_emp method show the poor quality. The popular historical simulated method (hist) gives quality estimates only with the SACF method. In addition all values of statistics for the SACF method are greater than the appropriate values for the standard method. V-test shows good results for both SACF and standard methods.

The built models are used for dynamic risk measures forecasting. Forecasting procedure is performed on the window length equal to the half of the general sample power (843 values). 5-day $(P=5)$ forecast is built (see Figure 2). Thus, it is assumed that the parameters of the model are adequate for a period 5 (or more) days, the estimates of static risk measures at the forecast horizon are unchanged.

The forecasting procedure (steps 2 - 6) is repeated 168 times, and each time 5 new values (the accumulation window) are added. Table 5 presents minimum $H_{\min}$, maximum $H_{\max}$ and average H_{mn} values of the Hurst parameter obtained for the windows.

Table 5 shows, the values of the Hurst parameter confirm the long-range dependence for all windows (all values are essentially greater than 0.5). The minimum range of values (about 7%) shows stability of this characteristic.

Table 3. The results of the analysis of the dynamic risk measures estimates (SACF method).

Method/statistics	*LRpof_SACF*	*LRind_SACF*	*LRcc_SACF*	V_1_SACF	V_2_SACF	V_SACF
hist	0.1289	0.5949	0.6058	−0.0002	−0.0233	0.0118
paramdistr	0.7968	0.6467	0.7536	0.0019	−0.0189	0.0110
GEV_quant	0.3477	0.3359	0.6088	0.0012	−0.0228	0.0120
GPD_quant	0.1986	0.4498	0.6688	0.0013	−0.0230	0.0122
POT_emp	0.0253	0.2610	0.0206	0.0011	−0.0249	0.0130

Table 4. The results of the analysis of the dynamic risk measures estimations (standard method).

Method/statistics	*LRpof_st*	*LRind_st*	*LRcc_st*	V_1_*st*	V_2_*st*	V_*st*
hist	0.0349	0.2980	0.1049	0.0005	−0.0212	0.0109
paramdistr	0.6113	0.5060	0.5434	0.0007	−0.0200	0.0104
GEV_quant	0.4775	0.4273	0.2783	0.0005	−0.0220	0.0113
GPD_quant	0.4775	0.4273	0.2783	0.0001	−0.0227	0.0114
POT_emp	0.0330	0.6259	0.0030	0.0011	−0.0177	0.0094

Table 5. Hurst parameter estimates.

H/method	abs. values	Aggregated variance	Residuals of regression	Periodogram	R/S	Optimization
H_{min}	0.7262	0.6872	0.6241	0.6777	0.7044	0.7199
H_{max}	0.7837	0.7872	0.7246	0.8854	0.7680	0.7438
H_{mn}	0.7546	0.7445	0.6758	0.7926	0.7356	0.7276

Figure 3 demonstrates the results of variance forecasting using (6) with SACF and standard methods.

Visual comparison of the predicted and real values shows that the proposed new method better describes the dynamic behavior of the time series. Extreme values obtained with the new method are much closer to real values. The new method also exhibits less lag in extreme values determination. This can be explained by the fact that the new method uses the ACF prediction and takes into account the property of the long-range dependence. It should also be noted that the optimization procedure in the determination of the Hurst parameter has significantly improved the forecast stability.

Minimum, maximum and average values of the static risk measures for different windows are shown in Table 6 (the SACF method) and Table 7 (the standard method).

Table 6 and Table 7 can be used to compare the quality of static risk measures estimations obtained by SACF and standard methods. The range of val-

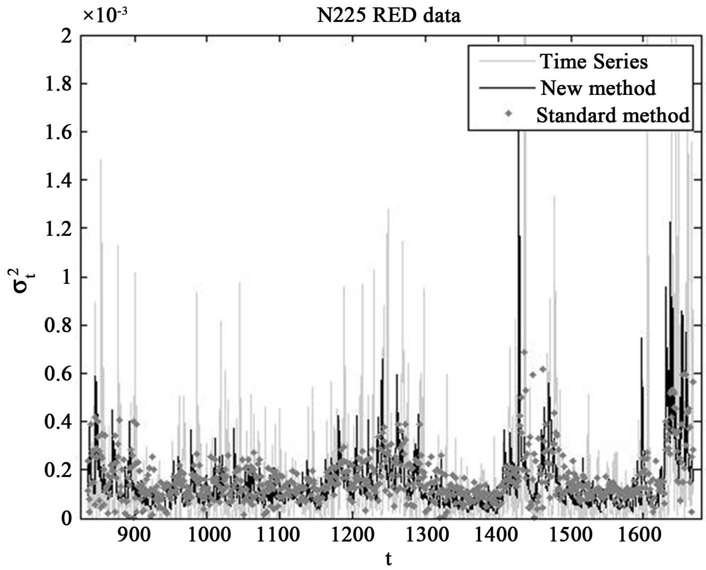

Figure 3. Real values of variance for real data and forecast estimates obtained by SACF and standard methods.

Table 6. $\widehat{\text{VaR}}_{0.9}(Z)$, $\widehat{\text{CVaR}}_{0.9}(Z)$ estimates for different windows (SACF method).

Method	*hist*	*paramdistr*	*GEV_quant*	*GEV_quant*	*POT_emp*
$\widehat{\text{VaR}}_{0.9}(Z)$_SACF min	1.4121	1.5115	1.4465	1.4811	1.2280
$\widehat{\text{VaR}}_{0.9}(Z)$_SACF max	1.4645	1.5849	1.5325	1.5490	1.7623
$\widehat{\text{VaR}}_{0.9}(Z)$_SACF*mn*	1.4379	1.5414	1.4898	1.5135	1.5103
$\widehat{\text{CVaR}}_{0.9}(Z)$_SACF min	2.2466	1.5414	2.2278	2.2607	2.1501
$\widehat{\text{CVaR}}_{0.9}(Z)$_SACF max	2.4645	2.1536	2.4217	2.4364	2.6978
$\widehat{\text{CVaR}}_{0.9}(Z)$_SACF*mn*	2.3254	2.1038	2.3013	2.3298	2.4001

Table 7. $\widehat{\text{VaR}}_{0.9}(Z)$, $\widehat{\text{CVaR}}_{0.9}(Z)$ estimates for different windows(standardmethod).

Method	*hist*	*paramdistr*	*GEV_quant*	*GEV_quant*	*POT_emp*
$\widehat{\text{VaR}}_{0.9}(Z)$_*st* min	1.2462	1.2927	1.2541	1.2717	1.1921
$\widehat{\text{VaR}}_{0.9}(Z)$_*st* max	1.4305	2.3181	1.4411	1.4011	1.5898
$\widehat{\text{VaR}}_{0.9}(Z)$_*stmn*	1.3125	1.4398	1.3241	1.3372	1.3706
$\widehat{\text{CVaR}}_{0.9}(Z)$_*st* min	1.9061	1.7723	1.9038	1.9390	1.8842
$\widehat{\text{CVaR}}_{0.9}(Z)$_*st* max	2.3166	3.1949	2.3825	2.4033	2.4764
$\widehat{\text{CVaR}}_{0.9}(Z)$_*stmn*	2.0032	1.9729	2.0206	2.0479	2.0647

ues (max-min) for the SACF method is less than the range of values for the standard method due to the fact that the proposed method explicitly uses the smoothing procedure of ACF and as a result the distribution function is more stable.

The obtained results are used to get the time series of dynamic risk measures estimates (3). As an example Figure 4 shows the forecast estimates for $\widehat{\mathrm{VaR}}_{0.9}^{t}$ and $\widehat{\mathrm{CVaR}}_{0.9}^{t}$, obtained with the use of the *paramdistr* method.

The prediction errors of $\widehat{\mathrm{VaR}}_{0.9}^{t}$ and $\widehat{\mathrm{CVaR}}_{0.9}^{t}$ for both methods (for different methods of static risk measures estimating) are shown in Table 8 and Table 9.

Table 8 and Table 9 show that the prediction errors obtained for the SACF method is less than the prediction errors obtained for the standard method. This proves the advantage of the proposed method. In addition Table 8 shows that methods hist and paramdistr gives the best estimates. This once again confirms the previously accepted hypothesis of data normal distribution.

The quality of built $\widehat{\mathrm{CVaR}}_{0.9}^{t}$ forecast estimates is analyzed with BPoE test. Table 10 shows the BPoE values obtained for the initial data (real) and for the

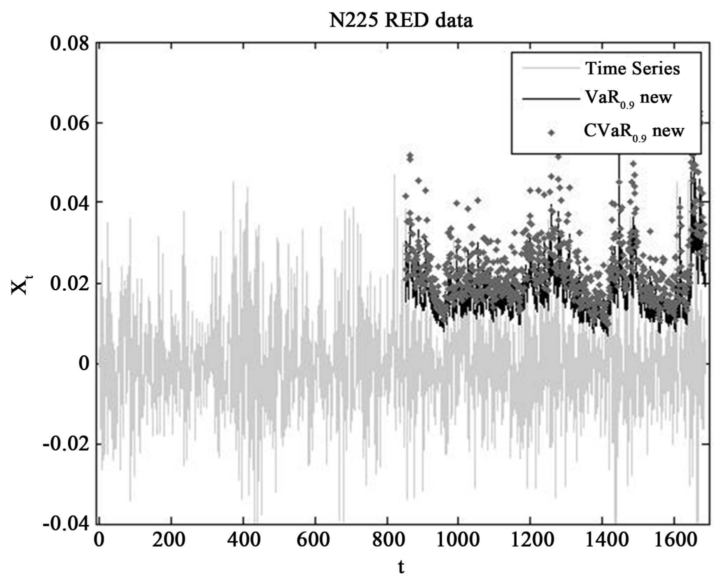

Figure 4. Forecast estimates for $\widehat{\mathrm{VaR}}_{0.9}^{t}$ and $\widehat{\mathrm{CVaR}}_{0.9}^{t}$ for 168 windows.

Table 8. The prediction errors of $\widehat{\mathrm{VaR}}_{0.9}^{t}$ and $\widehat{\mathrm{CVaR}}_{0.9}^{t}$ ($SACF$ method).

Method	$\widehat{\mathrm{VaR}}_{0.9}^{t}$_SACF			$\widehat{\mathrm{CVaR}}_{0.9}^{t}$_SACF		
	ME $(\times 10^{-4})$	MAE $(\times 10^{-3})$	MSE $(\times 10^{-5})$	ME $(\times 10^{-4})$	MAE $(\times 10^{-3})$	MSE $(\times 10^{-5})$
hist	4	4.4	4	1.83	6.96	1.0
paramdistr	5	4.4	4	1.03	6.30	0.8
GEV_quant	8	4.7	4	2.04	6.35	1.0
GPD_quant	6	4.6	4	1.90	6.92	1.0
POT_empt	30	5.1	6	4.24	7.78	1.5

Table 9. The prediction errors of $\widehat{\text{VaR}}_{0.9}^{t}$ and $\widehat{\text{CVaR}}_{0.9}^{t}$ (standard method).

Method	$\widehat{\text{VaR}}_{0.9}^{t}$ _ *st*			$\widehat{\text{CVaR}}_{0.9}^{t}$ _ *st*		
	ME $(\times 10^{-4})$	MAE $(\times 10^{-3})$	MSE $(\times 10^{-5})$	ME $(\times 10^{-4})$	MAE $(\times 10^{-3})$	MSE $(\times 10^{-5})$
hist	−10.0	4.3	3	−9.48	10.25	1.2
paramdistr	−13.0	4.5	4	−6.42	7.93	0.8
GEV_quant	−11.4	4.4	3	−8.87	9.73	1.1
GPD_quant	−13.3	4.5	3	−9.19	9.99	1.2
POT_empt	14.6	4.5	5	−7.27	8.97	1.0

Table 10. Results of BPoE-test for $\widehat{\text{CVaR}}_{0.9}^{t}$.

Method	*hist*	*paramdistr*	*GEV_quant*	*GEV_quant*	*POT_emp*
SACF meth	0.9018	0.9062	0.9102	0.9114	0.9102
st meth	0.8838	0.9154	0.8838	0.8886	0.8958
real	0.9102	0.9034	0.9162	0.9198	0.8898

forecast estimates with the use of the new method (SACF meth) and the standard method (*st* meth). These values are compared with the chosen level of risk measures $\alpha = 0.9$. The results show the high quality of the forecast estimates $\widehat{\text{CVaR}}_{0.9}^{t}$ obtained by the new method.

Table 10 may be used to liken the methods used for forecasting by comparing the value of confidence level α for predicted and real risk measures values. The standard method based on paramdistr demonstrates the best results. At the same time, the proposed method shows the best results with the historical simulation method (hist). Table 10 shows that the deviation of α for the new method (0.84%) is substantially less than the deviation for the standard method (1.8%).

6. Conclusion

In the article, a multi-step procedure for constructing the dynamic risk measures VaR and CVaR forecast is proposed. The procedure is designed for volatile series with the long-range dependence and is based on the heteroscedastic time series model. The optimization procedure for constructing and forecasting of ACF is used to find the model parameters. For the convenience of practical application, the prediction procedure is formulated as an algorithm. To test the proposed algorithm, the risk measures forecast for the time series of daily log return Nikkey 225 Stock Index is built. Different tests carried out at different stages of the algorithm confirm the good quality of the obtained estimates.

References

[1] Artzner, P., Delbaen, F., Eber, J.M. and Heath, D. (1999) Coherent Measures of

Risk. *Mathematical Finance*, **9**, 203-228. https://doi.org/10.1111/1467-9965.00068

[2] Tsay, R.S. (2010) Analysis of Financial Time Series. 3rd Edition, John Wiley & Sons, Hoboken. https://doi.org/10.1002/9780470644560

[3] Yamai, Y. and Yoshiba, T. (2002) Comparative Analysis of Expected Shortfall and Value-at-Risk: Their Estimation Error, Decomposition and Optimization. *Monetary and Economic Studies*, **20**, 57-86.

[4] Nadarajah, S., Zhang, B. and Chan, S. (2014) Estimation Methods for Expected Shortfall. *Quantitative Finance*, **14**, 271-291. https://doi.org/10.1080/14697688.2013.816767

[5] Rockafellar, R.T. and Uryasev, S.P. (2000) Optimization of Conditional Value-at-Risk. *Journal of Risk*, **2**, 21-42. https://doi.org/10.21314/JOR.2000.038

[6] Rockafellar, R.T. and Uryasev, S.P. (2002) Conditional Value-at-Risk for General Loss Distributions. *Journal of Banking & Finance*, **26**, 1443-1471.

[7] Scaillet, O. (2004) Nonparametric Estimation and Sensitivity Analysis of Expected Shortfall. *Mathematical*, **14**, 115-129. https://doi.org/10.1111/j.0960-1627.2004.00184.x

[8] Chen, S.X. (2008) Nonparametric Estimation of Expected Shortfall. *Journal of Financial Econometrics*, **6**, 87-107. https://doi.org/10.1093/jjfinec/nbm019

[9] Embrechts, P., Kaufmann, R. and Patie, P. (2005) Strategic Long-Term Financial Risks: Single Risk Factors. *Computational Optimization and Applications*, **32**, 61-90. https://doi.org/10.1007/s10589-005-2054-7

[10] Kjellson, B. (2013) Forecasting Expected Shortfall. An Extreme Value Approach. Bachelor's Thesis in Mathematical Sciences, K7, 43.

[11] Koksal, B. and Orhan, M. (2013) Market Risk of Developed and Emerging Countries during the Global Financial Crisis. Emerging Markets. *Finance & Trade*, **49**, 20-34. https://doi.org/10.2753/REE1540-496X490302

[12] Lee, T.-H., Bao, Y. and Saltoglu, B. (2006) Evaluating Predictive Performance of Value-at-Risk Models in Emerging Markets: A Reality Check. *Journal of Forecasting*, **25**, 101-128. https://doi.org/10.1002/for.977

[13] Yoon, S.-M. and Kang, S.-H. (2013) VaR Analysis for the Shanghai Stock Market. *The Macrotheme Review*, **2**, 89-95.

[14] Zivot, E. and Wang, J. (2003) Modeling Financial Time Series with S-PLUS. Springer-Verlag, New York. https://doi.org/10.1007/978-0-387-21763-5

[15] Palma, W. (2007) Long-Memory Time Series: Theory and Methods. John Wiley & Sons, Hoboken. https://doi.org/10.1002/9780470131466

[16] Zrazhevska, N. (2016) Construction and Application of the Classification Scheme of Dynamic Risk Measures Estimating. *Eureka*: *Physics and Engineering*, **5**, 67-80. https://doi.org/10.21303/2461-4262.2016.00162

[17] Pankratova, N.D. and Zrazhevska, N.G. (2015) Model of the Autocorrelation Function of the Time Series with Strong Dependence. *Problemi Upravleniya Informatik*, **5**, 102-112.

[18] Zrazhevska, N.G. (2015) Smoothed Autocorrelation Function Method for Predicting of Variation of Heteroscedastictime. *Systemnidoslidzhennya ta informacijni-texnologiyi*, **2**, 97-108.

[19] Zrazhevskij, A.G. (2011) System Approach to Restoring Functional Dependencies of Non-Stationary Time Series with Different Structures. Abstract of Dissertation for Scientific Degree of PhD, 20. (In Ukrainian)

[20] Cochrane, J.H. (1988) How Big Is the Random Walk in GNP? *Journal of Political*

Economy, **96**, 893-920. https://doi.org/10.21303/2461-4262.2016.00162

[21] Zrazhevskaja, N.G. and Zrazhevskij, A.G. (2016) Classification of Methods for Risk Measures VaR and CVaR Calculation and Estimation. Systemnidoslidzhennya ta informacijnitexnologiyi, 3.

[22] Shang, D., Kuzmenko, V. and Uryasev, S. (2015) Cash Flow Matching with Risks Controlled by bPOE and CVaR. Research Report 2015-3, ISE Dept., University of Florida.