

Estimation of Sensitivity of the DS/AHP Method While Solving Foresight Problems with Incomplete Data

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ABSTRACT

The paper provides mathematical analysis of sensitivity of different combination rules in the DS/AHP method when an alternative is added to the set of decision alternatives while solving foresight problems. Different cases of rank reversals are defined and two sets of conditions for these cases using the method DS/AHP are considered. Rank reversals are illustrated when the DS/AHP method is used to solve practical problem of critical technologies of energy conservation and power efficiency evaluation in Ukraine. It is shown that the DS/AHP method is not sensitive to exclusion (or addition) of an irrelevant decision alternative from (or to) the set of decision alternatives.

Keywords: Analytic Hierarchy Process; Dempster-Shafer Theory; Incomplete Expert Information; Multiple Criteria Analysis; Combination Rules; Rank Reversal; Foresight Problems

1. Introduction

Technological foresight is a decision making relative to complex systems with human factor concerning their potential behavior in future [1]. Foresight problems have innovation character. Mainly information of qualitative character in a form of expert estimates, which is often incomplete, fuzzy and contradictory, serves as input data for these problems. Therefore, technique of decision-making support must include methods for processing information of the mentioned character and also means of estimation of sensitivity and validity for the obtained results.

One of the methods, which are applied in the technique of scenario analysis [1] for solving problems of technological foresight, is the Analytic Hierarchy Process (AHP). Elaborated by T. Saaty AHP and its generalization the Analytic Network Process are popular decision tools used to weight items based on pairwise comparisons in terms of multiple criteria [2-5]. Nowadays AHP and its extensions are used to determine relative weights of items and probabilities of scenarios for solving foresight problems [6,7].

While solving foresight problems, information about decision alternatives may be incomplete due to time limitations, ignorance, intangible nature of some attributes, limited information processing capabilities etc. Besides, a decision maker is not always able to make pairwise comparisons between all decision alternatives. However, this is a prerequisite for the application of AHP. The

DS/AHP method [8,9], which incorporates AHP with the Dempster-Shafer theory (DST) of evidence [10] solves a multiple-criteria decision-making (MCDM) problem directly based on its incomplete expert judgments in terms of criteria. Different combination rules in DST represent different treatment of conflict in the aggregated result.

One of the problems associated with the use of MCDM techniques is the possibility to change the ranking of decision alternatives when an alternative is added or deleted (the rank reversal). A detailed discussion on different types of rank reversals in the AHP method will be found in the literature of the subject [11-16]. Some rank reversal cases also occur when other MCDM methods, in particular the ELECTRE II, III, the TOPSIS methods and others, are used [17,18]. Several numerical examples are given to validate the DS/AHP method with respect to the Pareto optimality and the independence of irrelevant decision alternatives [19].

This paper provides mathematical analysis and simulation study of sensitivity of different combination rules of the DS/AHP method when an alternative is added to the set of decision alternatives while solving foresight problems.

2. The DS/AHP Method and the Combination Rules

The Dempster-Shafer theory (DST) can be regarded as an extension of the Bayesian theory that can deal with incomplete data. To combine aspects of DST and AHP

the DS/AHP method of MCDM is introduced [8]. DS/AHP (as in AHP) is based on a hierarchical structure of a problem.

In DS/AHP measures of probability are assigned to groups of decision alternatives (DAs), each group of identified DAs is compared to the frame of discernment Θ (all possible DAs), and expert expresses some degree of “favorable knowledge” on each of these groups. This differs from AHP, which makes pairwise comparisons between individual DAs. Number of identified groups reflects the amount of knowledge the expert has on a criterion. Let us consider steps of the DS/AHP method (**Figure 1**):

Step 1: to find criteria priority values (CPVs) using the standard eigenvector method of AHP.

Step 2: to identify groups of DAs for each criterion. The number of identified groups is decided by the expert and may reflect the amount of knowledge the expert has on the criterion. Within one group, DAs have equal favorability to the frame of discernment Θ .

Step 3: to construct the knowledge matrix

$D_{r+1} = \{(d_{ij}) | i, j = 1, \dots, r+1\}$ for each criterion where $d_{i,r+1} = d_i w^c$, $i = 1, \dots, r$, values $d_i = d_{i\Theta}$ are the measures of favorability of the groups of DAs $S_i \subset \Theta$ with respect to Θ in terms of the criterion, w^c is the CPV; $d_{r+1,i} = 1/(d_i w^c)$, $i = 1, \dots, r$; $d_{ii} = 1$, $d_{ij} = 0$ for $i \neq j$, $i, j = 1, \dots, r$; r -number of groups of DAs on the criterion.

Step 4: to calculate the associated sets of priority values for the knowledge matrices for each criterion (basic probability assignment (bpa) structures for groups of

DAs and Θ in terms of the criteria) again using the standard eigenvector method of AHP. Analytic functions $m : 2^\Theta \rightarrow [0, 1]$ are constructed to find the bpa [9]:

$$m(S_i) = d_i w^c / \left(\sum_{j=1}^r d_j w^c + \sqrt{r} \right), \quad S_i \subset \Theta, \quad i = 1, 2, \dots, r$$

$$m(\Theta) = \sqrt{r} / \left(\sum_{j=1}^r d_j w^c + \sqrt{r} \right)$$

Step 5: to aggregate the bpa structures founded in step 4 into a single bpa $m_{aggr} = m_1 \oplus m_2 \oplus \dots \oplus m_p$ using a rule of combination, where p is a number of decision criteria. The resulting bpa structure represents the amounts of exact belief in groups of DAs based on the combined evidence from all the criteria. $m_{aggr}(\Theta)$ is the value of combined uncertainty.

Step 6: to calculate values of the belief ($Bel : 2^\Theta \rightarrow [0, 1]$) and plausibility ($Pls : 2^\Theta \rightarrow [0, 1]$) functions and obtain the belief interval $[Bel(\cdot), Pls(\cdot)]$:

$$Bel(A) = \sum_{B \subseteq A} m_{aggr}(B),$$

$$Pls(A) = 1 - \sum_{B \cap A = \emptyset} m_{aggr}(B)$$

for each group of DAs $A \subseteq \Theta$.

There are different ways where evidence can be combined in DST. The original **Dempster’s rule of combination** is considered as AND-operation and ignores the conflict between the evidence. In the DS/AHP method proposed in [8,9] the Dempster’s rule is used instead of distributive and ideal modes of AHP to combine the measures of evidence (bpa values) from different sources (decision criteria). This rule assumes that these sources are independent. According to the rule combined evidence from all criteria is defined by the function $m_1 \oplus m_2 : 2^\Theta \rightarrow [0, 1]$, which is a bpa:

$$(m_1 \oplus m_2)^{Dpstr}(A) = \frac{1}{K} \sum_{X \cap Y = A} m_1(X) m_2(Y)$$

if $A \neq \emptyset$ and $(m_1 \oplus m_2)^{Dpstr}(\emptyset) = 0$, where m_1, m_2 are bpa structures to be aggregated, $X, Y \subseteq \Theta$ are focal elements, the normalization factor

$$K = \sum_{X \cap Y \neq \emptyset} m_1(X) m_2(Y) = 1 - \sum_{X \cap Y = \emptyset} m_1(X) m_2(Y)$$

is interpreted as a measure of conflict between the pieces of evidence.

Other combination rules represent different treatment of conflict in the aggregated result. The **Yager’s rule of combination** is as follows [20]:

$$(m_1 \oplus m_2)^{Ygr}(A) = q(A) = \sum_{X \cap Y = A} m_1(X) m_2(Y)$$

In the rule there is any normalization factor. To repre-

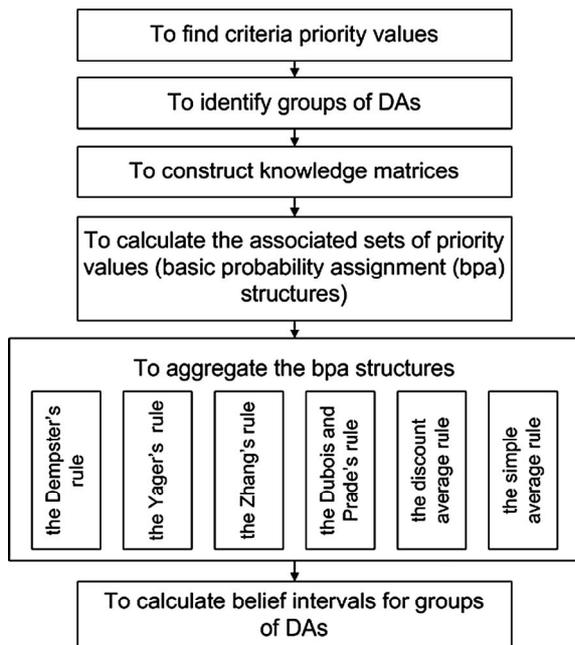


Figure 1. Steps of the DS/AHP method.

sent the conflict the value $q(\emptyset) \geq 0$ is used. This conflict value is added to the aggregated probability assignment of the frame, *i.e.* conflict is attributed to the frame and the evidence is not changed through the normalization.

In the **Zhang's combination rule** the intersection of focal elements is considered as follows [21]:

$$(m_1 \oplus m_2)^{Zhang}(A) = K \sum_{X \cap Y = A} \frac{|A|}{|X||Y|} m_1(X) m_2(Y)$$

where $| \cdot |$ defines a cardinality of a set, K is a normalization factor to provide the sum of the $(m_1 \oplus m_2)^{Zhang}(\cdot)$ to add to 1.

The **Dubois and Prade's combination rule** is OR-based and therefore any normalization is required [22]:

$$(m_1 \oplus m_2)^{DbsPd}(A) = \sum_{X \cup Y = A} m_1(X) m_2(Y),$$

If all sources of evidence are reliable, an AND-based operation is appropriate. If only one source is considered reliable, an OR-based operation has to be used.

The described Yager's, Zhang's, Dubois and Prade's rules represent modified Dempster's rule. Other combination rules are based on a weighted average function. The **discount average combination rule** is as follows [10]:

$$(m_1 \oplus \dots \oplus m_p)^{discnt}(A) = \frac{1}{p} \sum_{i=1}^p (1 - \alpha_i) Bel_i(A),$$

where the $(1 - \alpha_i)$'s are degrees of trust in the belief functions Bel_i 's to be aggregated, $\alpha_i \in [0, 1]$. The rule is used when all belief functions are highly conflicting and also to eliminate the influence of single strongly conflicting belief function.

Also the simple **average combination rule** which may be considered as the distributive synthesis mode is used:

$$(m_1 \oplus \dots \oplus m_p)^{averg}(A) = \frac{1}{p} \sum_{i=1}^p w_i m_i(A),$$

where the w_i 's are degrees of trust in the sources, m_i 's are the bpa's for the belief structures to be aggregated, $w_i \in [0, 1]$.

A combination rule has to aggregate evidence obtained from more than two sources (decision criteria). All of the above rules are commutative. The Dempster's, Dubois and Prade's and average rules are associative. The Yager's rule is quasi-associative.

One of the problems with a MCDM technique is a possibility to change the ranking of DAs when a DA is added or deleted (so called rank reversal). In the next Section the estimation of sensitivity of the above combination rules while adding a DA is provided.

3. Estimation of Sensitivity of the DS/AHP Method

Suppose n DAs A_1, A_2, \dots, A_n are evaluated in terms of two decision criteria C_1 and C_2 . For these criteria d_1 and d_2 groups of DAs $S_{11}, S_{12}, \dots, S_{1d_1}$ and $S_{21}, S_{22}, \dots, S_{2d_2}$, respectively, are identified as being comparable to the frame of discernment Θ , where $S_{1k} \cap S_{1l} = \emptyset$, $S_{2p} \cap S_{2r} = \emptyset$, $k, l = 1, \dots, d_1$, $p, r = 1, \dots, d_2$. Using the DS/AHP method, the belief measures $Bel(\cdot)$ and belief intervals $[Bel(\cdot), Pls(\cdot)]$ are calculated for each group of DAs and the frame Θ .

We are interested in the conditions of changes in the DA ranking orders obtained by the DS/AHP method when a DA is added to the set of DAs. Two sets of such conditions are defined [23]. The first set of conditions deals with the changes in the belief-based ranking orders of DAs. Belief measures $Bel(S_i)$ and $Bel^*(S_i)$ denote the amounts of exact belief in a group S_i based on combined evidence from two criteria before and after a DA is added to the set of DAs, respectively. The second set of conditions of changes in the DA ranking orders concerns the comparison of belief intervals

$[Bel(S_i), Pls(S_i)]$ and $[Bel^*(S_i), Pls^*(S_i)]$, where plausibility measures $Pls(S_i)$ and $Pls^*(S_i)$ denote the maximum probability of possible support to a group S_i before and after a DA is added to the set of DAs, respectively.

Condition #1 of rank reversal: Preference relation defined by belief measures for groups S_i and S_j is changed after a DA is added to the set of DAs:

$$\begin{aligned} & (\Delta Bel_{ij} \Delta Bel_{ij}^* < 0) \vee ((\Delta Bel_{ij} = 0) \wedge (\Delta Bel_{ij}^* \neq 0)) \\ & \vee ((\Delta Bel_{ij} \neq 0) \wedge (\Delta Bel_{ij}^* = 0)) \end{aligned}$$

where $\Delta Bel_{ij} = Bel(S_i) - Bel(S_j)$,

$$\Delta Bel_{ij}^* = Bel^*(S_i) - Bel^*(S_j)$$

Condition #2 of rank reversal: Preference relation defined by belief intervals for groups S_i and S_j is changed after a DA is added to the set of DAs.

To obtain the preference relations among groups of DAs the evidential reasoning ranking method is used [23] to generate the rank of groups of DAs based on their belief intervals.

In this paper, three known criteria are used to test sensitivity of the DS/AHP method while adding a DA. These criteria were applied to test other MCDM methods [17].

Test criterion #1: An effective MCDM method should not change the indication of the best DA when an irrelevant DA (that is dominated by one or more previously existing DAs) is added to the set of DAs given that the relative importance of each decision criterion remains

unchanged. The same should also be true for the relative rankings of the rest of the unchanged DAs.

Test criterion #2: The rankings of DAs by an effective MCDM method should follow the transitivity property.

Test criterion #3: For the same decision problem and while using the same MCDM method, after combining the rankings of the smaller problems that an MCDM problem is decomposed into, the new overall ranking of the DAs should be identical to the original overall ranking of the un-decomposed problem.

Let us consider different cases of rank reversals when the DS/AHP method is used. Suppose that an irrelevant DA is added to the set of DAs, it will be the most interesting result for solving foresight problems.

Case 1: *New DA A_{N+1} is irrelevant and forms a separate group with respect to each one of the decision criteria, i.e. $\{A_{N+1}\} \cap S_{1i} = \emptyset$ and $\{A_{N+1}\} \cap S_{2k} = \emptyset$, $i = 1, \dots, d_1$, $k = 1, \dots, d_2$.*

The intersections' sub-matrix $S_{1i} \cap S_{2k}$ for groups of DAs A_1, \dots, A_N in the Dempster's, Yager's and Zhang's combination rules remains unchanged when such DA A_{N+1} is added. However, the normalization constants K in the Dempster's and Zhang's rules are changed. Therefore the values of aggregated mass function and therefore the corresponding belief measures for groups of DAs A_1, \dots, A_N may be changed disproportionately when these rules are applied. Thus, rank reversals in the rules may occur under the Condition #1. In the Yager's combination rule rank reversals do not occur when such new DA is added.

Case 2: *New DA A_{N+1} is irrelevant and forms a separate group in terms of only one decision criterion. In terms of the other decision criterion this DA A_{N+1} has the same measure of favorability with respect to Θ as one or several previously existing DAs, i.e. A_{N+1} is included into one of the existing groups of DAs.*

Let DA A_{N+1} be included into the group S_{21} (without loss of generality, the choice of S_{21} is not detrimental). Then, after introduction of the DA A_{N+1} the groups $S_{11}, S_{12}, \dots, S_{1D_1}$ and $\{A_{N+1}\}$ are identified in terms of criterion C_1 and the groups $S'_{21}, S_{22}, \dots, S_{2D_2}$ in terms of criterion C_2 , where $S'_{21} = S_{21} \cup \{A_{N+1}\}$. The intersections' sub-matrix in each of the Dempster's, Yager's and Zhang's combination rules for the groups of "previously existing" DAs is changed, since the intersection of group S'_{21} with the frame Θ is changed. The normalization constants in the Dempster's and Zhang's combination rules are also changed. Therefore, the values of aggregated mass function and therefore the corresponding belief measures for groups of DAs A_1, \dots, A_N may be changed disproportionately when these rules are used. Thus, rank reversal in these rules may occur under the Condition #1.

Let us consider the possibility of rank reversal in the DS/AHP method under the test criterion #2. Suppose that the DS/AHP method has ranked a set of DAs of a decision problem. Next, assume that this problem is decomposed into a set of smaller problems each defined on two DAs at a time and the same number of decision criteria as in the original problem. Then, according to the test criterion #2, all the partial rankings deriving from the smaller problems should comply with the transitivity property. Let us denote groups of DAs in terms of two decision criteria C_1 and C_2 as S_{1i} and S_{2j} . Suppose each of the groups consists of a single element, namely $S_{11} = S_{21} = \{A_1\}, \dots, S_{1N} = S_{2N} = \{A_N\}$. Assume that the groups of DAs are considered in pairs and rankings of two arbitrary pairs are $A_i \succ A_j$ and $A_j \succ A_k$. Then the aggregated mass of DA A_i under the Dempster's combination rule is

$$(m_1 \oplus m_2)^{Dpstr}(A) = \frac{1}{K} (m_1(A_i)(m_2(A_i) + m_2(\Theta)) + m_1(\Theta)m_2(A_i))$$

Then rankings $A_i \succ A_j$ and $A_j \succ A_k$ lead to the following inequalities:

$$\begin{aligned} & m_1(A_i)(m_2(A_i) + m_2(\Theta)) + m_1(\Theta)m_2(A_i) \\ & - m_1(A_j)(m_2(A_j) + m_2(\Theta)) + m_1(\Theta)m_2(A_j) > 0 \\ & m_1(A_j)(m_2(A_j) + m_2(\Theta)) + m_1(\Theta)m_2(A_j) \\ & - m_1(A_k)(m_2(A_k) + m_2(\Theta)) + m_1(\Theta)m_2(A_k) > 0 \end{aligned}$$

After combination of these inequalities we have $A_i \succ A_k$. Thus, if each group of DAs consists of a single element, then the transitivity property is satisfied, and, hence, rank reversal does not appear in the Dempster's combination rule under the test criterion #2. The same conclusion is true for the Yager's and Zhang's combination rules.

4. Evaluation of Critical Technologies of Energy Conservation and Power Efficiency in Ukraine

The DS/AHP method was used to calculate relative priority values for critical technologies (CTs) of energy conservation and power efficiency in Ukraine. Quantitative information of passports of CTs and qualitative information in a form of expert estimates serves as input data for this problem.

A list of 14 CTs and their technical passports were presented by leading organizations in energy sector of Ukraine on a first stage of foresight process. Then the CTs were clustered as follows: energy conservation CTs, renewable energy CTs and eco-house CT. Energy conservation CTs include energy conservation while producing energy (cogeneration technologies and power ma-

chine building) and in energy networks (electrical power engineering and technologies of burning). Renewable energy CTs include geothermal, wind, solar and bioenergetics technologies. Problem of power efficiency is included in a notion of eco-house only as a part along with building materials production, construction of eco-house and waste utilization. Therefore the technology of effective eco-house was considered separately. Relative priority values for the 14 CTs A1 - A14 were calculated using the DS/AHP method on basis of information of passports of the CTs and expert judgments about relative importance of CTs in terms of risk factors and relative importance of decision criteria.

Groups of DAs were as follows:

- In terms of criterion C1: $S_1 = \{A1, A2, A5, A6, A12\}$, $S_2 = \{A3, A8, A9, A10, A13\}$, $S_3 = \{A7, A11\}$, $S_4 = \{A4\}$,
- In terms of criterion C2: $S_1 = \{A11, A12, A13\}$, $S_2 = \{A6, A10\}$, $S_3 = \{A2, A3\}$, $S_4 = \{A4, A7, A8, A9\}$, $S_5 = \{A1, A5\}$,
- In terms of criterion C3: $S_1 = \{A12\}$, $S_2 = \{A11, A13\}$, $S_3 = \{A6, A7\}$, $S_4 = \{A3, A4, A5\}$, $S_5 = \{A1, A2, A8, A9, A10\}$,
- In terms of criterion C4: $S_1 = \{A3\}$, $S_2 = \{A1, A2, A4, A8, A9, A10, A11, A12\}$, $S_3 = \{A5, A6, A7, A13\}$.

CPVs of the criteria were equal to 0.3, 0.3, 0.2 and 0.2.

Measures of favorability $d_i = d_{i\ominus}$ of the groups of DAs with respect to \ominus and bpa structures of the groups of DAs in terms of criteria are illustrated in **Table 1**.

Next step is to aggregate the bpa structures into a single bpa $m_{aggr} = m_1 \oplus m_2 \oplus m_3 \oplus m_4$. For example, an aggregated bpa structure $m_{aggr} = m_1 \oplus m_3$ using the Dempster's combination rule is as follows:

$$\begin{aligned} (m_1 \oplus m_3)(A4) &= 0.160, (m_1 \oplus m_3)(\ominus) = 0.093, \\ (m_1 \oplus m_3)(\{A7, A11\}) &= 0.070, \\ (m_1 \oplus m_3)(\{A1, A2, A8, A9, A10\}) &= 0.067, \\ (m_1 \oplus m_3)(\{A3, A4, A5\}) &= 0.059, \\ (m_1 \oplus m_3)(\{A3, A8, A9, A10, A13\}) &= 0.042, \\ (m_1 \oplus m_3)(\{A6, A7\}) &= 0.042, (m_1 \oplus m_3)(\{A7\}) = 0.031 \\ (m_1 \oplus m_3)(\{A8, A9, A10\}) &= 0.030, \\ (m_1 \oplus m_3)(\{A3\}) &= 0.026, \\ (m_1 \oplus m_3)(\{A11, A13\}) &= 0.025, \\ (m_1 \oplus m_3)(\{A11\}) &= 0.019, \\ (m_1 \oplus m_3)(\{A1, A2, A5, A6, A12\}) &= 0.014, \end{aligned}$$

Table 1. Measures of favorability d_i and bpa structures of the groups of DAs in terms of decision criteria C_1 (a), C_2 (b), C_2 (c) and C_4 (d).

(a)						
C_1	S_1	S_2	S_3	S_4	\ominus	
d_i	1	3	5	7		
m_1	0.044	0.132	0.221	0.309	0.294	
(b)						
C_2	S_1	S_2	S_3	S_4	S_5	\ominus
d_i	1	2	3	9	4	
m_2	0.038	0.076	0.113	0.340	0.151	0.282
(c)						
C_3	S_1	S_2	S_3	S_4	S_5	\ominus
d_i	1	3	5	7	8	
m_3	0.028	0.085	0.142	0.199	0.227	0.318
(d)						
C_4	S_1	S_2	S_3	\ominus		
d_i	1	3	5			
m_4	0.057	0.170	0.283	0.490		

$$\begin{aligned} (m_1 \oplus m_3)(\{A13\}) &= 0.011, (m_1 \oplus m_3)(\{A1, A2\}) = 0.010 \\ (m_1 \oplus m_3)(\{A12\}) &= 0.010, (m_1 \oplus m_3)(\{A5\}) = 0.009, \\ (m_1 \oplus m_3)(\{A6\}) &= 0.006. \end{aligned}$$

As a result, based on the aggregated bpa structure $m_{aggr} = ((m_1 \oplus m_3) \oplus m_2) \oplus m_4$, ranking of the CTs was constructed. CTs with the highest priorities are presented in **Table 2**.

Exclusion from consideration priority DA may lead to significant changes of results. Thus, exclusion of DA No3 "Technology of steam compressor thermal pumps" which is in a group of optimal DAs in terms of one of the criteria, results in redistribution of priority rating and DA "Technology of synthetic fuel (gas) production" receives 4th rank. Exclusion of DA of lower rank, which is less priority, in fact has no influence on the solution.

5. Conclusions

In the paper it is shown that the applications of Dempster's, Zhang's and average combination rules of the DS/AHP method may lead to well-known phenomenon of rank reversal. Rank reversal is a changing in overall ranks of decision alternatives when adding or excluding a decision alternative. Of particular interest are changings in these ranks when adding or excluding a decision alternative, which is *irrelevant*, i.e. nonoptimal in terms of

Table 2. Priorities and ranks of the energy conservation and power efficiency CTs.

No	Critical technologies (CTs)	Set of all CTs		Set of CTs without CT No7		Set of CTs without CT No3	
		Priority, *10	Rank	Priority, *10	Rank	Priority, *10	Rank
1	Technology of power efficient eco-house with renewable energy	4.260	1	4.260	1	4.260	1
2	Technology of improvement and structural optimization of energy networks in accordance with a purpose of harmonization with energy system of countries of the European Union	0.815	2	0.829	2	0.815	2
3	Technology of steam compressor thermal pumps	0.591	3	0.632	3	-	-
4, 5	Technology of effective usage of soil and groundwater heat in complex thermal pump systems Technology of diverse renewable energy sources usage in integrated thermal pump systems	0.312	4	0.316	4	0.312	3
6	Technology of energy loss saving in transit power networks	0.191	5	0.210	5	0.191	4
7	Technology of magneto-liquid sealing for considerable increasing energy equipment's service life	0.156	6	-	-	0.156	5
...	...		7		7		6
10	Technology of synthetic fuel (gas) production	0.104	7	0.113	7	0.201	4
...	...		7		7		-
11	Technology of usage of high-temperature conductivity in electrical machines and devices	0.086	8	0.140	6	0.009	7
...	...	0.081	8	0.084	8	0.008	7
12	Technology of production of thermostable and corrosion-proof heat-insulating materials for thermal networks	0.070	9	0.075	-	0.161	5

each decision criterion, and as a result overall less priority. Our investigation reveals that exclusion of such decision alternative while solving foresight problems with incomplete data using the DS/AHP method *does not result* in redistributions in a subset of high priority decision alternatives and, therefore has no influence on the choice of high priority solution. Thus, the DS/AHP method *is not sensitive* to exclusion (or addition) of an irrelevant decision alternative from (or to) the set of decision alternatives.

It is the very first time that the DS/AHP method is used to solve foresight problems, as well as rank reversals are reported to occur while using this method. Therefore, determination of priority critical technologies a priori requires additional investigations concerning possibility of rank reversals when a decision alternative is added or excluded.

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