Intelligent Control and Automation, **2012, 3, 59-70**
http://dx.doi.org/10.4236/ica.2012.31008 Published Online February 2012 (http://www.SciRP.org/journal/ica)

$\mathcal{H}_\infty$ Control of Uncertain Fuzzy Networked Control Systems with State Quantization

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Received September 11, 2011; revised October 11, 2011; accepted October 18, 2011

ABSTRACT

The problem of robust $\mathcal{H}_\infty$ control for uncertain discrete-time Takagi and Sugeno (T-S) fuzzy networked control systems (NCSs) is investigated in this paper subject to state quantization. By taking into consideration network induced delays and packet dropouts, an improved model of network-based control is developed. A less conservative delay-dependent stability condition for the closed NCSs is derived by employing a fuzzy Lyapunov-Krasovskii functional. Robust $\mathcal{H}_\infty$ fuzzy controller is constructed that guarantee asymptotic stabilization of the NCSs and expressed in LMI-based conditions. A numerical example illustrates the effectiveness of the developed technique.

Keywords: Networked $\mathcal{H}_\infty$ Control; Fuzzy Systems; Discrete Time-Varying Delay; Linear Matrix Inequality (LMI)

1. Introduction

Fuzzy system models have been widely adopted to represent certain classes of nonlinear dynamic systems following the T-S fuzzy model [1]. Since then there have been several approaches for the study of stability analysis and robust controller synthesis using the so-called parallel distributed compensation (PDC) method for uncertain nonlinear systems [2,3]. Sufficient conditions have been derived based on the feasibility testing of a linear matrix inequality (LMI) in [4-7] and extended for classes of nonlinear discrete-time systems with time delays in [8-10] via different approaches. Recently, much attention has been paid to the stability issue of network based control systems [11]. Several results pertaining to the analysis and design of networked control systems (NCSs) enhanced their wide benefits such as reducing system wiring, ease of system diagnosis and maintenance, and increasing system agility, to name a few. However, communication network in the control loops gave rise to some new issues, especially the intermittent losses or delays of the communicated information due to use of a network, which imposes a challenge to system analysis and design. To address this challenge, many results have been developed in consideration of network-induced delay and packet dropout [12-18], with focus on stability analysis and controller design with random delays.

Further consideration of the communication of the NCSs over the channel emphasized the importance of signal quantization, which has significant impact on the performance of NCSs. In this regard, the problem of guaranteed cost control and quantized controller design were discussed in [17] by using two quantizers in the network both from sensor to controller and from controller to actuator, and the network-induced delay and data dropped were considered as well.

Recent advances converted the quantized feedback design problem into a robust control problem with sector bound uncertainties, [11] and [16-18]. Parallel investigations to the class of switched discrete-time systems with interval time-delays were developed in [19-23].

Despite the potential of these developments, the problem of how to analyze the stability of nonlinear NCSs with data drops still open. On the other hand, most industrial plants have severe nonlinearities, which lead to additional difficulties for the analysis and design of control systems. Though some issues on nonlinear NCSs have been investigated [23,24], limited work has been found on robust $\mathcal{H}_\infty$ state feedback controller design of networks for fuzzy systems with consideration of both network conditions and signal quantization.

The guaranteed cost networked control and robust $\mathcal{H}_\infty$ problem based on the T-S fuzzy model was treated in [25]. The results were derived by using a single Lyapunov function (SLF) method, which in general leas to a conservative result. Designing fuzzy controllers for a class of nonlinear networked control systems was considered in [26-28] by solving approximate uncertain linear networked Takagi-Sugeno (T-S) models with both network induced-delay and packet dropout. However, they do not quantize the signals. The foregoing facts motivate the present study.

In this research work, we address the robust $\mathcal{H}_\infty$ state feedback control problem for discrete-time networked systems with state quantization and disturbances. The T-S fuzzy systems with norm-bounded uncertainties are utilized to characterize the nonlinear NCSs. Since the computation available is often limited, the quantized feedback controller is designed under consideration of effect of network-induced delay and data dropout, the employed quantizer is time-varying. By using a new fuzzy Lyapunov-Krasovskii functional (LKF), we provide a sufficient LMI-based condition for the existence of a fuzzy controller. A numerical example shows the feasibility of the developed technique.

Notations and facts: In the sequel, the Euclidean norm is used for vectors. We use W^t and W^{-1} to denote the transpose and the inverse of any square matrix W, respectively. We use $W > 0$ $(\geq, <, \leq 0)$ to denote a symmetric positive definite (positive semi-definite, negative, negative semi-definite matrix W and I to denote the $n \times n$ identity matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. In symmetric block matrices or complex matrix expressions, we use the symbol ($\bullet$) to represent a term that is induced by symmetry.

Fact 1: For any real matrices $\sum_1, \sum_2$ and $\sum_3$ with appropriate dimensions and $\sum_3^t \sum_3 \leq I$, it follows that

$$\sum\nolimits_1 \sum\nolimits_3 \sum\nolimits_2 + \sum\nolimits_2^t \sum\nolimits_3^t \sum\nolimits_1^t \leq \upsilon^{-1} \sum\nolimits_1 \sum\nolimits_1^t + \upsilon^{-1} \sum\nolimits_2^t \sum\nolimits_2, \forall \upsilon > 0$$

Sometimes, the arguments of a function will be omitted when no confusion can arise.

2. Problem Description

A typical networked control system typically has a clock-driven sampler and a quantizer, controller, a zero-order hold (ZOH) which is event-driven. The sampling period is assumed to be $T \geq 0$ with the sampling instants as $S_k, k = 1, \cdots, \infty$. The plant belongs to class of uncertain discrete-time systems where the parametric uncertainties are norm-bounded.

In what follows, we consider that this class is represented by Takagi-Sugeno fuzzy model composed of a set of fuzzy implications, and each implication is expressed by a linear system model. The jth rule of this Takagi-Sugeno model has the following form:

Rule j: If $\theta_1(k)$ is $M_{j1}, \cdots$ and $\theta_n(k)$ is M_{jn}, then

$$x(k+1) = A_{j\Delta} x(k) + B_{j\Delta} u(k) + \Gamma_{j\Delta} w(k),$$

$$y(k) = C_{j\Delta} x(k) + D_{j\Delta} u(k) + \Phi_{j\Delta} w(k),$$

$$x(k) = \psi(k), k \in [-\tau_M, 0], j = 1, 2, \cdots, r \tag{1}$$

where $\theta_1(k), \theta_2(k), \cdots, \theta_n(k)$ are the premise variables, each M_{jm} $(m = 1, 2, \cdots n)$ are the fuzzy sets, r is the number of if-then rules and $x(k) \in \Re^n$ is the state vector, $u(k) \in \Re^m$ is the control input, $y(k) \in \Re^q$ is the output, $w(k) \in \Re^p$ is the disturbance input which belongs to $\ell_2[0, \infty)$ and τ_M indicates the maximum allowable signal transmission delay. The uncertain matrices $A_{j\Delta}, \cdots, \Gamma_{j\Delta}$ are represented by:

$$\begin{bmatrix} A_{j\Delta} & B_{j\Delta} & \Gamma_{j\Delta} \\ C_{j\Delta} & D_{j\Delta} & \Phi_{j\Delta} \end{bmatrix} = \begin{bmatrix} A_j & B_j & \Gamma_j \\ C_j & D_j & \Phi_j \end{bmatrix} + \begin{bmatrix} \Delta A_j & \Delta B_j & \Delta \Gamma_j \\ \Delta C_j & \Delta D_j & \Delta \Phi_j \end{bmatrix}$$

$$\begin{bmatrix} \Delta A_j & \Delta B_j & \Delta \Gamma_j \\ \Delta C_j & \Delta D_j & \Delta \Phi_j \end{bmatrix} = \begin{bmatrix} M_{1j} \\ M_{2j} \end{bmatrix} F_j(k) \begin{bmatrix} N_{1j} & N_{2j} & N_{3j} \end{bmatrix} \tag{2}$$

where the matrices $A_j, B_j, \cdots, \Phi_j$ describe the nominal dynamics and $M_{1j}, M_{2j}, N_{1j}, N_{2j}, N_{3j}$ are known constant real matrices with appropriate dimensions. The matrices $F_j(k)$ are unknown time-varying and satisfying $F_j^t(k) F_j(k), \leq I, \forall k \geq 0$.

Using a center average defuzzifier [1], product inference, and incorporating fuzzy "blending", the fuzzy system under consideration can be cast into the form

$$\begin{aligned} x(k+1) &= \sum_{j=1}^{r} f_j(\theta(k)) \left[A_{j\Delta} x(k) + B_{j\Delta} u(k) + \Gamma_{j\Delta} w(k) \right] \\ y(k) &= \sum_{j=1}^{r} f_j(\theta(k)) \left[C_{j\Delta} x(k) + D_{j\Delta} u(k) + \Phi_{j\Delta} w(k) \right] \end{aligned} \tag{3}$$

where

$$\begin{aligned} f_j(\theta(k)) &= \frac{F_j(\theta(k))}{\sum_{j=1}^{r} F_j(\theta(k))}, \\ F_j(\theta(k)) &= \prod_{m=1}^{n} M_{jm}(\theta_m(k)) \end{aligned} \tag{4}$$

where $M_{jm}(\theta_m(k))$ is the grade of membership of $\theta_m(k)$ in M_{jm}. In the sequel, we assume that

$$F_j(\theta(k)) \geq 0,\ j = 1, 2, \cdots, r,\ \sum_{j=1}^{r} F_j(\theta(k)) > 0,\ \forall k \geq 0$$

and therefore

$$f_j(\theta(k)) \geq 0, \sum_{j=1}^{r} f_j(\theta(k)) = 1, \forall k \geq 0$$

Our objective in this paper is to design a fuzzy $\mathcal{H}_\infty$ state feedback controller with state quantization.

3. Controlled Fuzzy System

In what follows, we proceed to consider establish the main result for the uncertain discrete-time fuzzy networked control systems described by (3) and design the quantized fuzzy $\mathcal{H}_\infty$ state feedback controller. We consider a limited capacity communication channel and for reducing the amount of data rate of transmitting in the network, which led to the increase quality of service of the network, we assume that the state vector $x(k)$ is measurable. The state signal from sensor to the controller is quantized via a quantizer, and then transmitted with a single packet. To reflect realty, network-induced time delay is modeled as an input delay and the packet dropout will be considered.

3.1. State-Feedback Control

In effect, we seek to design the state-feedback controller:

$$u(k) = g(\nu(k)), \quad \nu(k) = K\, x(d_k T) \tag{5}$$

where $g(\nu(k))$ is the feedback law to be defined in the sequel and $d_k, (k = 1, 2, 3, \cdots)$ are some integers such that $\{d_1, d_2, d_3, \cdots\} \subset \{1, 2, 3, \cdots\}$. Introduce $\tau(k) = k - d_k T$ which contains the information of packet dropouts and improper packet sequence in the control signal. Note that $d_k T = k - (k - d_k T) = k - \tau(k)$.

It has been pointed out in [19] that when $d_{k+1} - d_k = 1$, there would be no packets dropout and the case $d_{k+1} - d_k > 1$ represents continuous packets lost. In addition, when $d_{k+1} < d_k$, the new packet reaches the destination before the old one. This case might lead to a less conservative result. In the sequel, we assume that $d_{k+1} > d_k$ and it is readily seen that

$$\tau(k) \le (d_{k+1} - d_k)\, T + \tau(k+1),$$
$$k \in \left[(d_k T + \tau(k)),\ (d_{k+1} T + \tau(k+1)) \right]$$

It should be observed that $\tau(k)$ accounts for the time from the instant $d_k T$ when sensor nodes sample the sensor data from the plant to the instant when actuator transfer data to the plant. Extending on this, we remark that

$$\bigcup_{k=1}^{\infty} \left[(d_k T + \tau(k)), (d_{k+1} T + \tau(k+1)) \right] = [k_0, \infty),\ k_0 \ge 0$$

Consequently, we define

$$\tau_m \le \tau(k) \le \tau_M,\ \tau_m = \eta_m T,\ \tau_M = \eta_M T$$

where $\eta_m > 0, \eta_M > 0$ are known finite integers.

3.2. Quantizer

Let the quantizer be described as

$$q(\nu) = \left[q_1(\nu_1)\, q_2(\nu_2) \cdots q_m(\nu_m) \right]^t,\ q_j(-\nu_j) = -q_j(\nu_j)$$

where $q(\cdot),\ j = 1, 2, \cdots, m$ is a symmetric, static and and time-invariant quantizer and the associated set of quantization levels is expressed as

$$Q = \left\{ \pm\sigma_j,\ j = \pm 1, \pm 2, \cdots \right\} \cup \{0\} \tag{6}$$

Note that the quantization regions are quite arbitrary. In case of logarithmic quantizer, the set of quantization levels Q becomes

$$Q_\alpha = \left\{ \pm\sigma_j, \sigma_j = \alpha_j \sigma_0,\ j = \pm 1, \pm 2, \cdots \right\} \cup \{\pm\sigma_0\} \cup \{0\}$$

where σ_0 is the initial state of the quantizer and $0 < \alpha_j < 1$ is a parameter associated with the quantizer $f(\cdot)$. In this regard, a particular characterization of the quantizer is given by

$$q(x) = \begin{cases} \sigma_j & \text{if } \dfrac{1}{1+\delta_j}\sigma_j < x \le \dfrac{1}{1-\delta_j}\sigma_j, x > 0 \\ 0 & \text{if } x = 0 \\ -q(-x) & \text{if } x < 0 \end{cases}$$

where $\delta_j = \dfrac{1-\alpha_j}{1+\alpha_j}$. It follows from [19] that, for any $q(\cdot)$, a sector bound expression can be expressed as:

$$q_j(x_j) = \left(1 + D_{q_j}(x_j)\right) x_j,\ \left| D_{q_j}(x_j) \right| \le \delta_j$$

For simplicity in exposition, we use D_q to denote $D_{q_j}(x_j)$. Thus, $q(\cdot)$ can be written as

$$q(x) = (I + D_q)\, x$$

We assume henceforth that the updating signal at the instant k has experienced signal transmission delay $\tau(k)$, however the delay between the sensor and quantizer is neglected. In view of the limited capacity in communication channel, the state signal from sensor to the controller is quantized via a logarithmic quantizer $q(\cdot)$ for reducing the amount of data rate of transmitting in the network. When the static and time-invariant quantizer $q(x) = x$, the state feedback controller would be in the form of $u(k) = K\, x(d_k T)$, which is the same as a traditional one.

Incorporating the notion of parallel distributed compensation, the following fuzzy state-feedback stabilizing control law is used:

Rule j: If $\theta_1(k)$ is $M_{j1}, \cdots$ and $\theta_n(k)$ is M_{jn}, then

$$u(k) = K_j\, (I + D_q)\, x(k - \tau(k)) \tag{7}$$

where K_j is the control gain for rule $j, j = 1, 2, \cdots, r$. Accordingly, the overall fuzzy control law is expressed by

$$u(k) = \sum_{j=1}^{r} f_j\left(\theta(k - \tau(k))\right) K_j\, (I + D_q)\, x(k - \tau(k)) \tag{8}$$

Applying controller (8) to system (3) with some mathematical manipulations, the resulting closed-loop system can be cast into the form:

$$\begin{aligned} x(k+1) &= \sum_{j=1}^{r} f_j\left(\theta\left(k-\tau(k)\right)\right)\left[\hat{A}_{j\Delta}(k)x(k)\right. \\ &\left. +\hat{B}_{j\Delta}(k)K_j(I+D_q)x\left(k-\tau(k)\right)+\hat{\Gamma}_{j\Delta}(k)w(k)\right], \\ y(k) &= \sum_{j=1}^{r} f_j\left(\theta\left(k-\tau(k)\right)\right)\left[\hat{C}_{j\Delta}(k)x(k)\right. \\ &\left. +\hat{D}_{j\Delta}(k)K_j\left(I+D_q\right)x\left(k-\tau(k)\right)+\hat{\Phi}_{j\Delta}(k)w(k)\right] \end{aligned} \tag{9}$$

which belongs to the class of switched time-delay system [15], where

$$\begin{aligned} &\hat{A}_{j\Delta}(k) = \sum_{j=1}^{r} f_j\left(\theta(k)\right)A_{j\Delta}, \hat{B}_{j\Delta}(k) = \sum_{j=1}^{r} f_j\left(\theta(k)\right)B_{j\Delta}, \\ &\hat{C}_{j\Delta}(k) = \sum_{j=1}^{r} f_j\left(\theta(k)\right)C_{j\Delta}, \hat{D}_{j\Delta}(k) = \sum_{j=1}^{r} f_j\left(\theta(k)\right)D_{j\Delta}, \\ &\hat{\Gamma}_{j\Delta}(k) = \sum_{j=1}^{r} f_j\left(\theta(k)\right)\Gamma_{j\Delta}, \hat{\Phi}_{j\Delta}(k) = \sum_{j=1}^{r} f_j\left(\theta(k)\right)\Phi_{j\Delta} \end{aligned} \tag{10}$$

4. Quantized Fuzzy Control Design

In this section, we seek to establish a sufficient condition for the solvability of the robust $\mathcal{H}_\infty$ control problem. This condition will be expressed in an LMI framework to facilitate the design of the desired fuzzy state feedback controllers. Based on the so-called parallel distributed compensation scheme, the following theorem establishes a delay-dependent stabilization condition for the closed-loop fuzzy networked control system (9):

Theorem 4.1 *Consider system* (9). *Given the bounds* τ_m, τ_M *and a scalar constant* $\gamma > 0$, *there exists a fuzzy controller in the form of* (8), *such that the uncertain closed-loop fuzzy system* (9) *with an* $\mathcal{H}_\infty$ *disturbance attention level* γ *is asymptotically stable, if there exist matrices* $0 < P_j, 0 < Q_j, 0 < Z_j, 0 < S_j, 0 < R_{aj}, 0 < R_{cj}, K_j$, *matrices* $\Psi_a, \Psi_c, \Upsilon_a, \Upsilon_c, \Theta_a, \Theta_c$ *and scalars* $\varepsilon_{1j} > 0, \varepsilon_{2j} > 0, \varepsilon_{3j} > 0, \varepsilon_{4j}$, *satisfying*

$$\tilde{\Xi}_j = \begin{bmatrix} \tilde{\Xi}_{tj} & \tilde{\Xi}_{sj} \\ \bullet & -\tilde{\Xi}_{rj} \end{bmatrix} < 0, \ 1 \le j \le r \tag{11}$$

$$\begin{aligned} \tilde{\Xi}_{tj} &= \hat{\Xi}_{oj} + \left(\sigma_{1j}+\sigma_{2j}+\sigma_{3j}+\sigma_{4j}\right)\Lambda_j\Lambda_j^t \\ \tilde{\Xi}_{rj} &= diag\left[\varepsilon_{1j}I \quad \varepsilon_{2j}I \quad \varepsilon_{3j}I \quad \varepsilon_{4j}I\right], \\ \tilde{\Xi}_{sj} &= \left[\sigma_{1j} \quad \sigma_{2j} \quad \sigma_{3j} \quad \sigma_{4j}\right], \end{aligned} \tag{12}$$

$$\begin{aligned} \Lambda_j^t &= \begin{bmatrix} N_{1j} & N_{2j}K_j & 0 & 0 & N_{3j} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \sigma_{1j} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & M_{1j}^t & 0 & 0 & 0 & 0 \end{bmatrix} \\ \sigma_{2j} &= \sqrt{\tau_s}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\tau_s}M_{1j}^tR_{aj} & 0 & 0 & 0 \end{bmatrix} \\ \sigma_{3j} &= \sqrt{\tau_M}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\tau_M}M_{1j}^tR_{cj} & 0 & 0 \end{bmatrix} \\ \sigma_{4j} &= \sqrt{\tau_M}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{2j}^t & 0 \end{bmatrix} \end{aligned} \tag{13}$$

Proof: In what follows, we adopt a parameter-dependent approach [15]. Consider system (9) with $1 \le j, m \le r$ and define

$$\begin{aligned} &P(k) = \sum_{j=1}^{r} f_j\left(\theta(k)\right)P_j, Q(k) = \sum_{j=1}^{r} f_j\left(\theta(k)\right)Q_j, \\ &Z(k) = \sum_{j=1}^{r} f_j\left(\theta(k)\right)Z_j, S(k) = \sum_{j=1}^{r} f_j\left(\theta(k)\right)S_j, \\ &R_a(k) = \sum_{j=1}^{r} f_j\left(\theta(k)\right)R_{aj}, R_c(k) = \sum_{j=1}^{r} f_j\left(\theta(k)\right)R_{cj} \end{aligned} \tag{16}$$

$$\hat{\Xi}_{oj} = \begin{bmatrix} \Xi_{ooj} & \Xi_{oaj} & \Psi_a & -\Phi_a & 0 & A_j^t & \sqrt{\tau_s}\left(A_j-I\right)^t & \sqrt{\tau_M}\left(A_j-I\right)^t & C_j^t & 0 \\ \bullet & -\Xi_{aa} & \Psi_c & -\Phi_c & 0 & K_j^tB_j^t & \sqrt{\tau_s}K_j^tB_j^t & \sqrt{\tau_M}K_j^tB_j^t & K_j^tD_j^t & 0 \\ \bullet & \bullet & -Z_j & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & -S_j & 0 & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\gamma^2I & \Gamma_j^t & 0 & 0 & \Phi_j^t & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -P_j+2I & 0 & 0 & 0 & B_jK_j \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -R_{aj}+2I & 0 & 0 & B_jK_j \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -R_{cj}+2I & 0 & B_jK_j \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -I & D_jK_j \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -I \end{bmatrix} \tag{14}$$

$$\begin{aligned} &\Xi_{ooj} = -P_j + \left(\tau_s+1\right)Q_j + Z_j + S_j + \Theta_a + \Theta_a^t, \Xi_{oaj} = -\Theta_a + \Theta_c^t + \Upsilon_a - \Psi_a \\ &\Xi_{aaj} = Q_j + \Theta_c + \Theta_c^t - \Upsilon_c - \Upsilon_c^t + \Psi_c + \Psi_c^t \end{aligned} \tag{15}$$

where $0 < P_j, 0 < Q_j, 0 < Z_j, 0 < S_j, 0 < R_{aj}, 0 < R_{cj}$ are matrices of appropriate dimensions and $P(k), Q(k), Z(k), S(k), R_a(k), R_c(k)$ are fuzzy weighting matrices that are directly include the membership functions instead of a single matrix, a fact that aims at relaxing the conservatism. For simplicity in notation, we let

$$\begin{aligned}
A_\Delta(k) &= \sum_{j=1}^{r} f_j\left(\theta\left(k-\tau(k)\right)\right)\hat{A}_{j\Delta}(k),\\
B_\Delta(k) &= \sum_{j=1}^{r} f_j\left(\theta\left(k-\tau(k)\right)\right)\hat{B}_{j\Delta}(k)K_j\left(I+D_q\right),\\
C_\Delta(k) &= \sum_{j=1}^{r} f_j\left(\theta\left(k-\tau(k)\right)\right)\hat{C}_{j\Delta}(k),\\
D_\Delta(k) &= \sum_{j=1}^{r} f_j\left(\theta\left(k-\tau(k)\right)\right)\hat{D}_{j\Delta}(k)K_j\left(I+D_q\right),\\
\tilde{\Gamma}_\Delta(k) &= \sum_{j=1}^{r} f_j\left(\theta\left(k-\tau(k)\right)\right)\hat{\Gamma}_{j\Delta}(k),\\
\tilde{\Phi}_\Delta(k) &= \sum_{j=1}^{r} f_j\left(\theta\left(k-\tau(k)\right)\right)\hat{\Phi}_{j\Delta}(k)
\end{aligned} \tag{17}$$

In terms of the state increment $\delta x(k) = x(k+1) - x(k)$ and the time-span $\tau_s = \tau_M - \tau_m$, we consider the Lyapunov-Krasovskii functional (LKF):

$$\begin{aligned}
V(k) &= V_o(k) + V_a(k) + V_c(k) + V_m(k) + V_n(k)\\
V_o(k) &= x^t(k)P(k)x(k),\\
V_a(k) &= \sum_{\ell=k-\tau(k)}^{k-1} x^t(\ell)Q(\ell)x(\ell),\\
V_c(k) &= \sum_{\ell=k-\tau_m}^{k-1} x^t(\ell)Z\ell x(\ell) + \sum_{\ell=k-\tau_M}^{k-1} x^t(\ell)S(\ell)x(\ell),\\
V_m(k) &= \sum_{j=-\tau_M+1}^{-\tau_m}\sum_{m=k+j}^{k-1} x^t(m)Q(m)x(m)\\
V_n(k) &= \sum_{j=-\tau_M}^{-\tau_m-1}\sum_{m=k+j}^{k-1} \delta x^t(m)R_a(m)\delta x(m)\\
&\quad + \sum_{j=-\tau_M}^{-1}\sum_{m=k+j}^{k-1} \delta x^t(m)R_c(m)\delta x(m)
\end{aligned} \tag{18}$$

We focus initially on the case $1 \le j \le r$. A straightforward computation gives the first-difference of $\Delta V(k) = V(k+1) - V(k)$ along the solutions of (17) with the help of (9) and (10) as:

$$\begin{aligned}
\Delta V_o(k) &= x^t(k+1)P(k+1)x(k+1) - x^t(k)P(k)x(k)\\
&= \left[A_\Delta(k)x(k) + B_\Delta(k)x(k-\tau(k)) + \tilde{\Gamma}_\Delta(k)w(k)\right]^t P(k)\left[A_\Delta(k)x(k) + B_\Delta(k)x(k-\tau(k)) + \tilde{\Gamma}_\Delta(k)w(k)\right]\\
&\quad - x^t(k)P(k)x(k),
\end{aligned}$$

$$\Delta V_a(k) \le x^t(k)Q(k)x(k) - x^t(k-\tau(k))Q(k)x(k-\tau(k)) + \sum_{j=k-\tau_M+1}^{k-\tau_m} x_j^t Q(k)x_j$$

$$\Delta V_c(k) = x^t(k)Z(k)x(k) - x^t(k-\tau_m)Z(k)x(k-\tau_m) + x^t(k)Sx(k) - x^t(k-\tau_M)S(k)x(k-d\tau_M)$$

$$\Delta V_m(k) = (\tau_M - \tau_m)x^t(k)Q(k)x(k) - \sum_{j=k-\tau_M+1}^{k-\tau_m} x^t(k)Q(k)x(k)$$

$$\Delta V_n(k) = (\tau_M - \tau_m)\delta x^t(k)R_a(k)\delta x(k) + \tau_M\delta x^t(k)R_c(k)\delta x(k) - \sum_{j=k-\tau_M}^{k-\tau_m-1}\delta x_j^t R_a(k)\delta x_j - \sum_{j=k-\tau_M}^{k-1}\delta x_j^t R_c(k)\delta x_j \tag{19}$$

To facilitate the delay-dependence analysis, we invoke the following identities

$$\left[2x^t(k)\Theta_a + 2x^t(k-d(k))\Theta_c\right]\left[x(k) - x(k-\tau(k)) - \sum_{j=k-\tau(k)}^{k-1}\delta x_j\right] = 0$$

$$\left[2x^t(k)\Upsilon_a + 2x^t(k-\tau(k))\Upsilon_c][x(k-\tau(k)) - x(k-\tau_M) - \sum_{j=k-\tau_M}^{k-\tau(k)-1}\delta x_j\right] = 0$$

$$\left[2x^t(k)\Psi_a + 2x^t(k-\tau(k))\Psi_c][x(k-\tau_m) - x(k-\tau(k)) - \sum_{j=k-\tau(k)}^{k-\tau_m-1}\delta x_j\right] = 0 \quad (20)$$

for some matrices $\Theta_a,\cdots,\Psi_c$, and proceed to get

$$\begin{aligned}\Delta V(k) &= \Delta V_o(k) + \Delta V_a(k) + \Delta V_c(k) + \Delta V_m(k) + \Delta V_n(k)\\&+\left[2x^t(k)\Theta_a + 2x^t(k-\tau(k))\Theta_c][x(k) - x(k-\tau(k)) - \sum_{j=k-\tau(k)}^{k-1}\delta x_j\right]\\&+\left[2x^t(k)\Upsilon_a + 2x^t(k-\tau(k))\Upsilon_c][x(k-\tau(k)) - x(k-\tau_M) - \sum_{j=k-\tau_M}^{k-\tau(k)-1}\delta x_j\right]\\&+\left[2x^t(k)\Psi_a + 2x^t(k-\tau(k))\Psi_c][x(k-\tau_m) - x(k-\tau(k)) - \sum_{j=k-\tau(k)}^{k-\tau_m-1}\delta x_j\right]\end{aligned} \quad (21)$$

In terms of

$$\xi^t(k) = \left[x^t(k)\ \ x^t(k-\tau(k))\,x^t(k-\tau_m)\,x^t(k-\tau_M)\right]$$

we cast (31) with $w(k)\equiv 0$ into the form:

$$\Delta V(k) \le \xi^t(k)\Pi_{\Delta j}\,\xi(k)$$

$$\Pi_{\Delta j} = \Pi_{oj} + \Xi_{1j}^t P_j \Xi_{1j} + \Xi_{2j}^t R_{aj}\Xi_{2j} + \Xi_{3j}^t R_{cj}\Xi_{3j}$$

$$\Pi_{oj} = \begin{bmatrix}\Xi_{ooj} & \Xi_{oaj} & \Psi_a & -\Upsilon_a\\ \bullet & -\Xi_{aaj} & \Psi_c & -\Upsilon_c\\ \bullet & \bullet & -Z & 0\\ \bullet & \bullet & \bullet & -S\end{bmatrix},$$

$$\Xi_{1j}^t = \begin{bmatrix}A_\Delta^t\\ B_\Delta^t\\ 0\\ 0\end{bmatrix}, \Xi_{3j}^t = \begin{bmatrix}\sqrt{\tau_M}(A_\Delta - I)^t\\ \sqrt{\tau_M}B_\Delta\\ 0\\ 0\end{bmatrix}, \quad \Xi_{2j}^t = \begin{bmatrix}\sqrt{\tau_s}(A_\Delta - I)^t\\ \sqrt{\tau_s}B_\Delta\\ 0\\ 0\end{bmatrix} \quad (22)$$

where $\Xi_{oo},\Xi_{aa},\Xi_{oa}$ are given by (15). If $\Pi_{\Delta j} < 0$ for all admissible uncertainties satisfying (2), then by Schur complements it follows from (32) that $\Delta V(k) < 0$, for any $\xi(k)\neq 0$ guaranteeing the internal stability. Proceeding further and to assure the closed-loop stability with γ-disturbance attenuation, we follow [15] to get:

$$\begin{aligned}\Delta V(k) + y^t(k)y(k) - \gamma^2 w^t(k)w(k) &= \Delta V(k) - \gamma^2 w^t(k)w(k)\\&+\left[C_\Delta(k)x(k) + D_\Delta(k)x(k-\tau(k)) + \tilde{\Phi}_\Delta(k)w(k)\right]^t\\&\left[C_\Delta(k)x(k) + D_\Delta(k)x(k-\tau(k)) + \tilde{\Phi}_\Delta(k)w(k)\right]\Delta V(k)\\&\le \left[\xi(k)\ w^t(k)\right]\Sigma_{\Delta j}(k)\left[\xi^t(k)\ w^t(k)\right]^t < 0\end{aligned} \quad (23)$$

when $\sum(k) < 0$ where

$$\sum\nolimits_{\Delta j}(k) = \Xi_{oj} + \Xi_{aj}^t P_j \Xi_{aj} + \Xi_{cj}^t R_{aj}\Xi_{cj} + \Xi_{ej}^t R_{cj}\Xi_{ej} + \Xi_{mj}^t \Xi_{mj}$$

$$\Xi_{oj} = \begin{bmatrix}\Pi_{oj} & 0\\ \bullet & -\gamma^2 I\end{bmatrix}, \Xi_{aj}^t = \begin{bmatrix}\Xi_{1j}^t\\ \tilde{\Gamma}_\Delta^t(k)\end{bmatrix}, \Xi_{cj}^t = \begin{bmatrix}\Xi_{2j}^t\\ 0\end{bmatrix},$$

$$\Xi_{mj} = \left[C_\Delta^t(k)\quad D_\Delta^t(k)\quad 0\quad 0\quad \tilde{\Phi}_\Delta^t(k)\right]^t,$$

$$\Xi_{ej}^t = \begin{bmatrix}\Xi_{3j}^t\\ 0\end{bmatrix} \quad (24)$$

Next, by applying Fact 1, we obtain

$$\begin{aligned}\Xi_{\Delta j} &\le \breve{\Xi}_{oj} + \varepsilon_{1j}^{-1}\hat{\sigma}_{1j}\hat{\sigma}_{1j}^t + \varepsilon_{2j}^{-1}\hat{\sigma}_{2j}\hat{\sigma}_{2j}^t\\&+\varepsilon_{3j}^{-1}\hat{\sigma}_{3j}\hat{\sigma}_{3j}^t + \varepsilon_{4j}^{-1}\hat{\sigma}_{4j}\hat{\sigma}_{4j}^t\\&< 0\end{aligned} \quad (25)$$

for some scalars $\varepsilon_{1j} > 0, \varepsilon_{2j} > 0, \varepsilon_{3j} > 0, \varepsilon_{4j} > 0$. Note that $f_j(\theta(k)) \ge 0,\ f_j(\theta(k-\tau(k))) \ge 0, 1\le j\le r$. The quantities $\hat{\sigma}_{1j},\hat{\sigma}_{2j},\hat{\sigma}_{3j},\hat{\sigma}_{4j}$ correspond to $\sigma_{1j},\sigma_{2j},\sigma_{3j},\sigma_{4j}$ given by (13) after deleting the last element, and

$$\tilde{\Xi}_{oj} = \begin{bmatrix} \Xi_{ooj} & \Xi_{oaj} & \Psi_a & -\Phi_a & 0 & A_j^t & \sqrt{\tau_s}\left(A_j - I\right)^t & \sqrt{\tau_M}\left(A_j - I\right)^t & C_j^t \\ \bullet & -\Xi_{aaj} & \Psi_c & -\Phi_c & 0 & \Omega_{aj}^t & \sqrt{\tau_s}\Omega_{aj}^t & \sqrt{\tau_M}\Omega_{aj}^t & \Omega_{cj}^t \\ \bullet & \bullet & -Z & 0 & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & -S & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\gamma^2 I & \Gamma_j^t & 0 & 0 & \Phi_j^t \\ \bullet & \bullet & \bullet & \bullet & \bullet & -P_j^{-1} & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -R_{aj}^{-1} & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -R_{cj}^{-1} & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -I \end{bmatrix},$$

$$\Omega_{aj} = \left(B_j K_j \left(I + D_q\right)\right),\ \Omega_{cj} = \left(D_j K_j \left(I + D_q\right)\right) \quad (26)$$

where $\Xi_{ooj}, \Xi_{aaj}, \Xi_{oaj}$ are given by (15). Further convexification of $\Xi_{\Delta j}$ in (35) yields

$$\Xi_{\Delta j} \le \hat{\tilde{\Xi}}_{oj} + \varepsilon_{1j}^{-1}\sigma_{1j}\sigma_{1j}^t + \varepsilon_{2j}^{-1}\sigma_{2j}\sigma_{2j}^t + \varepsilon_{3j}^{-1}\sigma_{3j}\sigma_{3j}^t + \varepsilon_{4j}^{-1}\sigma_{4j}\sigma_{4j}^t < 0 \quad (27)$$

By Schur complements using the algebraic inequality $\left(X - I\right)X^{-1}\left(X - I\right) \ge 0$ for any matrix $X > 0$, the desired stability condition can then be cast into the LMI (11), which concludes the proof.

Remark 4.1 *It is significant to observe that* Theorem 4.1 *provides a delay-dependent condition for the design of robust* $\mathcal{H}_\infty$ *for fuzzy NCS in terms of feasibility testing of a family of strict LMIs with a total number of LMI-variables as* $6\left(r+1\right)+r$. *The key feature is that the matrix gain* K_j *is treated as a direct LMI variable. This will eventually lessen the conservatism in robust fuzzy control design.*

Remark 4.2 *It is worthy to note that the number of LMIs increases linearly with the number of rules* r *which limits the applicability of the method for very large values of* r. *Had we used*

$$P\left(k\right) = \sum_{j=1}^{r} f_j\left(\theta\left(k\right)\right)P_j := P,\ Q\left(k\right) = \sum_{j=1}^{r} f_j\left(\theta\left(k\right)\right)P_j := Q,$$

$$Z\left(k\right) = \sum_{j=1}^{r} f_j\left(\theta\left(k\right)\right)P_j := Z, S\left(k\right) = \sum_{j=1}^{r} f_j\left(\theta\left(k\right)\right)P_j := S,$$

$$R_a\left(k\right) = \sum_{j=1}^{r} f_j\left(\theta\left(k\right)\right)R_{aj} := R_a, R_c\left(k\right) = \sum_{j=1}^{r} f_j\left(\theta\left(k\right)\right)R_{cj} := R_a$$

then Theorem 4.1 reduces to the following corollary:

Corollary 4.1 *Given the bounds* τ_m, τ_M *and a scalar constants* $\gamma > 0$, *there exists a fuzzy controller in the form of* (8), *such that the uncertain closed-loop fuzzy system* (9) *with an* $\mathcal{H}_\infty$ *disturbance attention level* γ *is asymptotically stable, if there exist matrices* $0 < P$, $0 < Q, 0 < Z, 0 < S, 0 < R_a, 0 < R_c$, *matrices* K_j, Ψ_a, Ψ_c, $\Upsilon_a, \Upsilon_c, \Theta_a, \Theta_c$ *and scalars* $\varepsilon_{1j} > 0, \varepsilon_{2j} > 0, \varepsilon_{3j} > 0, \varepsilon_{4j}$, *satisfying*

$$\tilde{\tilde{\Xi}} = \begin{bmatrix} \tilde{\tilde{\Xi}}_t & \tilde{\tilde{\Xi}}_s \\ \bullet & -\tilde{\tilde{\Xi}}_r \end{bmatrix} < 0, \quad (28)$$

$$\tilde{\tilde{\Xi}}_t = \tilde{\tilde{\Xi}}_o + \left(\sigma_{1j} + \sigma_{2j} + \sigma_{3j} + \sigma_{4j}\right)\Lambda\Lambda^t$$

$$\tilde{\tilde{\Xi}}_r = diag\left[\varepsilon_{1j}I \quad \varepsilon_{2j}I \quad \varepsilon_{3j}I \quad \varepsilon_{4j}I\right],$$

$$\tilde{\Pi}_s = \left[\sigma_1 \quad \sigma_2 \quad \sigma_3 \quad \sigma_4\right],$$

$$\Lambda_j^t = \left[N_{1j} \quad N_{2j}K_j \quad 0 \quad 0 \quad N_{3j} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\right]$$

$$\sigma_{1j} = \left[0 \quad 0 \quad 0 \quad 0 \quad 0 \quad M_{1j}^t \quad 0 \quad 0 \quad 0 \quad 0\right]$$

$$\sigma_{2j} = \sqrt{\tau_s}\left[0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \sqrt{\tau_s}M_{1j}^t R_{aj} \quad 0 \quad 0 \quad 0\right]$$

$$\sigma_{3j} = \sqrt{\tau_M}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\tau_M} M_{1j}^t R_{cj} & 0 & 0 \end{bmatrix},$$

$$\sigma_{4j} = \sqrt{\tau_M}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{2j}^t & 0 \end{bmatrix} \quad (29)$$

$$\hat{\Xi}_o = \begin{bmatrix} \Xi_{oo} & \Xi_{oa} & \Psi_a & -\Phi_a & 0 & A_j^t & \sqrt{\tau_s}(A_j - I)^t & \sqrt{\tau_M}(A_j - I)^t & C_j^t & 0 \\ \bullet & -\Xi_{aa} & \Psi_c & -\Phi_c & 0 & K_j^t B_j^t & \sqrt{\tau_s} K_j^t B_j^t & \sqrt{\tau_M} K_j^t B_j^t & K_j^t D_j^t & 0 \\ \bullet & \bullet & -Z & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & -S & 0 & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\gamma^2 I & \Gamma_j^t & 0 & 0 & \Phi_j^t & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -P + 2I & 0 & 0 & 0 & B_j K_j \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -R_a + 2I & 0 & 0 & B_j K_j \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -R_c + 2I & 0 & B_j K_j \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -I & D_j K_j \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -I \end{bmatrix} \quad (30)$$

$$\Xi_{ooj} = -P + (\tau_s + 1)Q + Z + S + \Theta_a + \Theta_a^t$$

$$\Xi_{aaj} = Q + \Theta_c + \Theta_c^t - \Upsilon_c - \Upsilon_c^t + \Psi_c + \Psi_c^t,$$

$$\Xi_{oaj} = -\Theta_a + \Theta_c^t + \Upsilon_a - \Psi_a \quad (31)$$

and the number of LMI variables would be $12 + r$. The price paid is that the LKF becomes non-fuzzy.

5. Special Cases

In this section, we seek to derive a sufficient condition for the solvability of the robust $\mathcal{H}_\infty$ control problem for NCS without quantizer.

NCS without Quantizer

In this case, the resulting closed-loop fuzzy system can be expressed as:

$$x(k+1) = \sum_{j=1}^{r} f_j(\theta(k))\, f_j(\theta(k))\big[A_{j\Delta}(k)x(k) + B_{j\Delta}(k)K_j(I + D_q)x(k - \tau(k)) + \Gamma_{j\Delta}(k)w(k)\big],$$

$$y(k) = \sum_{j=1}^{r} f_j(\theta(k))\, f_j(\theta(k))\big[C_{j\Delta}(k)x(k) + D_{j\Delta}(k)K_j(I + D_q)x(k - \tau(k)) + \Phi_{j\Delta}(k)w(k)\big] \quad (32)$$

The corresponding control design is given by the following corollary:

Corollary 5.1 *Given the bounds* τ_m, τ_M *and a scalar constants* $\gamma > 0$, *there exists a fuzzy controller in the form of* (8), *such that the uncertain closed-loop fuzzy system* (23) *with an* $\mathcal{H}_\infty$ *disturbance attention level* γ *is asymptotically stable*, *if there exist matrices* $0 < P_j, 0 < Q_j, 0 < Z_j$, $0 < S_j, 0 < R_{aj}, 0 < R_{cj}$, *matrices* $K_j, \Psi_a, \Psi_c, \Upsilon_a, \Upsilon_c, \Theta_a$, Θ_c *and scalars* $\varepsilon_{1j} > 0, \varepsilon_{2j} > 0, \varepsilon_{3j} > 0, \varepsilon_{4j}$, *satisfying*

$$\tilde{\Xi}_j = \begin{bmatrix} \Xi_{tj} & \tilde{\Xi}_{sj} \\ \bullet & -\tilde{\Xi}_{rj} \end{bmatrix} < 0, \; 1 \le j \le r \quad (33)$$

$$\Xi_{tj} = \Xi_{oj} + (\sigma_{1j} + \sigma_{2j} + \sigma_{3j} + \sigma_{4j})\Lambda_j \Lambda_j^t$$

$$\Xi_{oj} = \begin{bmatrix} \Xi_{ooj} & \Xi_{oaj} & \Psi_a & -\Phi_a & 0 & A_j^t & \sqrt{\tau_s}(A_j - I)^t & \sqrt{\tau_M}(A_j - I)^t & C_j^t \\ \bullet & -\Xi_{aa} & \Psi_c & -\Phi_c & 0 & K_j^t B_j^t & \sqrt{\tau_s} K_j^t B_j^t & \sqrt{\tau_M} K_j^t B_j^t & K_j^t D_j^t \\ \bullet & \bullet & -Z_j & 0 & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & -S_j & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\gamma^2 I & \Gamma_j^t & 0 & 0 & \Phi_j^t \\ \bullet & \bullet & \bullet & \bullet & \bullet & -P_j + 2I & -R_{aj} + 2I & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -R_{aj} + 2I & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -R_{cj} + 2I & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -I \end{bmatrix} \quad (34)$$

where the various terms are as in (13)-(15).

6. Example

In what follows, a typical simulation example is considered to illustrate the fuzzy controller design procedure developed in Theorem 4.1. A class of discrete-time fuzzy networked control systems model with state quantization is described by:

Rule 1: If $x_1(k)$ is M_1, then

$$x(k+1) = A_{1\Delta}x(k) + B_{1\Delta}u(k) + \Gamma_{1\Delta}w(k),$$

$$y(k) = C_{1\Delta}x(k) + D_{1\Delta}u(k) + \Phi_{1\Delta}w(k),$$

$$A_1 = \begin{bmatrix} -0.05 & -0.8 \\ 0.7 & -0.04 \end{bmatrix}, B_1 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, D_1 = 0.2$$

$$\Gamma_1 = \begin{bmatrix} -0.1 & -0.3 \\ 0.2 & 0.5 \end{bmatrix}, C_1^t = \begin{bmatrix} 0.2 \\ 0.5 \end{bmatrix}, \Phi_1^t = \begin{bmatrix} 0.1 \\ 0.4 \end{bmatrix},$$

$$M_{11} = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}, N_{11}^t = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, N_{31}^t = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}$$

$$M_{21} = 0.2, N_{21} = 0.05$$

Rule 2: If $x_2(k)$ is M_2, then

$$x(k+1) = A_{2\Delta}x(k) + B_{2\Delta}u(k) + \Gamma_{2\Delta}w(k),$$

$$y(k) = C_{2\Delta}x(k) + D_{2\Delta}u(k) + \Phi_{2\Delta}w(k),$$

$$A_2 = \begin{bmatrix} -0.03 & 0.5 \\ -1.2 & -0.04 \end{bmatrix}, B_2 = \begin{bmatrix} 0.09 \\ 0.06 \end{bmatrix}, D_2 = 0.3$$

$$\Gamma_2 = \begin{bmatrix} 0.2 & -0.6 \\ 0.1 & -0.4 \end{bmatrix}, C_2^t = \begin{bmatrix} 0.3 \\ 0.05 \end{bmatrix}, \Phi_2^t = \begin{bmatrix} 0.1 \\ 0.4 \end{bmatrix},$$

$$M_{12} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, N_{12}^t = \begin{bmatrix} 0.05 \\ 0.08 \end{bmatrix}, N_{32}^t = \begin{bmatrix} 0.03 \\ 0.06 \end{bmatrix},$$

$$M_{22} = 0.3, N_{22} = 0.04$$

Rule 3: If $x_3(k)$ is M_3, then

$$x(k+1) = A_{3\Delta}x(k) + B_{3\Delta}u(k) + \Gamma_{3\Delta}w(k),$$

$$y(k) = C_{3\Delta}x(k) + D_{3\Delta}u(k) + \Phi_{3\Delta}w(k),$$

$$A_3 = \begin{bmatrix} -0.05 & -0.8 \\ 0.7 & -0.04 \end{bmatrix}, B_3 = \begin{bmatrix} 0.07 \\ 0.05 \end{bmatrix}, D_3 = 0.4$$

$$\Gamma_3 = \begin{bmatrix} -0.1 & -0.3 \\ 0.2 & 0.5 \end{bmatrix}, C_3^t = \begin{bmatrix} 0.5 \\ 0.04 \end{bmatrix}, \Phi_3^t = \begin{bmatrix} 0.08 \\ 0.06 \end{bmatrix},$$

$$M_{13} = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, N_{13}^t = \begin{bmatrix} 0.06 \\ 0.04 \end{bmatrix}, N_{33}^t = \begin{bmatrix} 0.04 \\ 0.04 \end{bmatrix},$$

$$M_{23} = 0.4, N_{23} = 0.05$$

The membership functions for the rules 1, 2, 3 are

$$M_1(x_1(k)) = \frac{1}{1+exp(-3x_1(k))},$$

$$M_2(x_2(k)) = \frac{1}{1+exp(-2x_2(k))},$$

$$M_3(x_3(k)) = 1 - M_1(x_1(k)) - M_2(x_2(k))$$

For the purpose of implementation, we consider the fuzzy system to be controlled through a network. A quantizer $q(\cdot)$ is selected to be of of logarithmic type with $\alpha_1 = 0.8$, $\alpha_2 = 0.7$, $\alpha_3 = 0.9$, leading to $\delta_1 = 0.1111$, $\delta_2 = 0.1765$, $\delta_3 = 0.0526$. The bounds on data packet dropout are selected as $\tau_m = 0.6$, $\tau_M = 3.2$, respectively. Using the Matlab LMI solver, the feasible solution of

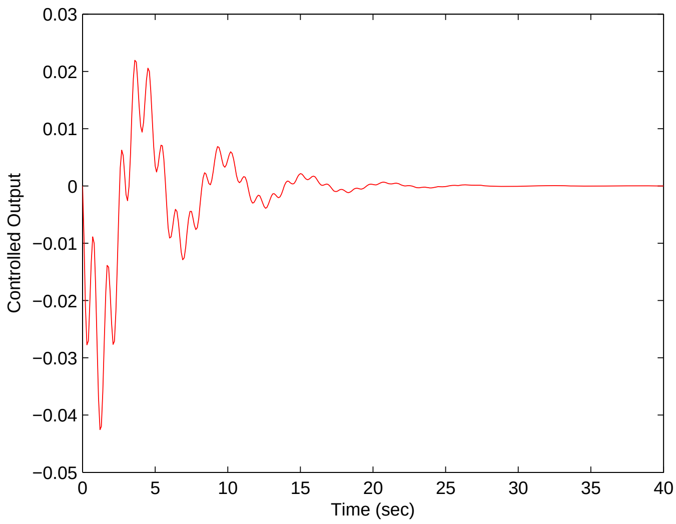

Figure 1. Controlled-output trajectory.

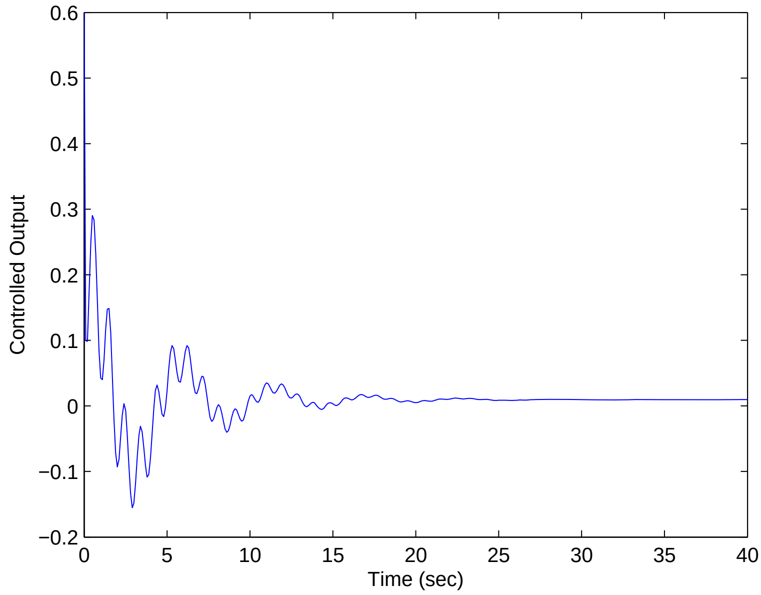

Figure 2. First state trajectory.

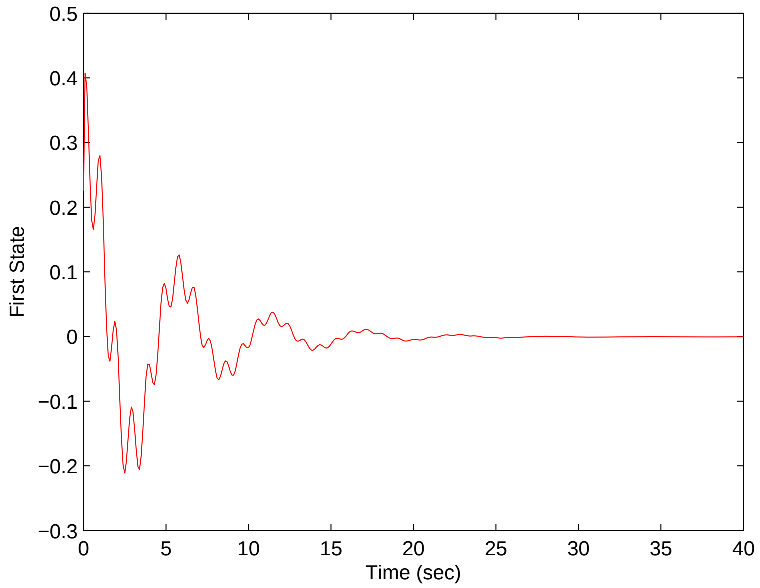

Figure 3. Second state trajectory.

Theorem 4.1 yields the fuzzy $\mathcal{H}_\infty$ state-feedback controller gains of the form:

$$\gamma = 0.25,$$
$$K_1 = [18.5403 \quad 9.3657],$$
$$K_2 = [17.7234 \quad 8.8945]$$
$$K_3 = [15.7715 \quad 8.2863]$$

The simulation results of the state and controlled-output trajectories are plotted in **Figures 1-3**. It is quite evident that all the state and output variables of the fuzzy system settle at the equilibrium level within 20 sec.

7. Conclusion

We have addressed the problem of robust $\mathcal{H}_\infty$ state-feedback controller design for discrete-time Takagi-Sugeno (T-S) fuzzy networked control systems including state quantization. A quantized feedback fuzzy controller has been designed under consideration of effect of network-induced delay and data dropout, and the time-varying quantizer has been selected to be logarithmic. By employing a fuzzy Lyapunov-Krasovskii functional, we have derived some LMI-based sufficient conditions for the existence of fuzzy controller. A numerical example has been given to illustrate the efficiency of the theoretic results.

8. Acknowledgements

The author would like to thank the deanship of scientific research (DSR) at KFUPM for financial support through research group project RG1105-1.

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