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Dynamic Neural Network Based Nonlinear Control of a Distillation Column

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Abstract

Taking advantage of the knowledge of top and bottom compositions of a distillation column, a dynamic neural network (DNN) is designed to identify the input-output relationship of the column. The weight-training algorithm is derived from a Lyapunov function. Based on this empirical model, a nonlinear H_∞ controller is synthesized. The effectiveness of the control strategy is demonstrated using simulation results.

1. Introduction

A distillation column is a strongly nonlinear process with multivariate interactions among outputs and some uncertainty often exists in the system, which renders the analysis and control of a distillation column very difficult [1,2]. In practice, the single-point control is commonly used which is sample and easy to tune. However, the consumption of energy in single-point control is large and the product of the other end is not guaranteed. For high purity distillation column, the two-point control or other strategies have been investigated [1,2]. Considering the intrinsic nonlinearity of a distillation column, nonlinear controller has been designed based on the rigorous mathematical model [3,4]. Although a nonlinear controller may be effective in simulation, its implementation in practical plants is complex because of the lack of measurement of some key variables. Furthermore, a controller based on the accurate mathematical model may lead to poor performance in case of large perturbation. For this reason, most controllers for distillation columns are synthesized based on the input-output relations such as transfer function or a model obtained by system identification [1,5-8].

Because of its capability to approximate arbitrary nonlinear mapping, neural network has been actively used in nonlinear system identification and control [9-12]. The multilayer feed-forward neural network (MFNN) is one of the most widely used neural networks as a system model in the design of a model-based controller. A dynamic neural network (DNN) is more suitable to approximate a nonlinear dynamic process. Therefore, dynamic recurrent neural network has been used to learn the input-output relationship of a column and a local optimal controller based on the neural network model was given in [8]. Non-linear adaptive controllers based on MFNN and RBFNN have also been studied [13,14]. A recent review [15] for the past 28 years showed that most of the implementations of advanced control like internal model control were based on linear models. Many recently published papers [16,17] on neural control for distillation columns are some extensions of previous research and supported by simulations. Although a neural network can be trained in simulation to approximate any nonlinear process, the control law must be designed to be robust for the modeling error. H_∞ control has been a very popular for robust control of nonlinear system [18-20]. The application of H_∞ control to distillation columns has not been widely discussed.

In this paper, a dynamic neural network is designed to identify the input-output relation of a distillation column online. The training algorithm of the network is derived from a Lyapunov function so that the convergence of weights and the finite bound of the identification error are guaranteed. To cope with model error, a nonlinear H_∞ controller based on the trained network model is employed to enhance the robustness of the control system. Simulation results demonstrate the effectiveness of the online identifier and the control algorithm.

2. DNN Based On-Line Identifier of a Binary Distillation Column

Assume the distillation column is with (L, V) control structure where L and V are reflux flow in condenser and boil up flow in reboiler (kmol/min) respectively. Then, let $u1$, $u2$ be L, V respectively. The outputs are the light compositions of the top and bottom products of a distillation column.

In some set points, assume the nonlinear input-output relationship of the distillation column can be expressed approximately as below:

$$\dot{Y} = f(Y) + \begin{pmatrix} g_1(Y) & g_2(Y) \end{pmatrix} \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} \quad (1)$$

where $f(Y), g_i(Y) \in R^{2 \times 1}$, $g_i(Y) = \begin{pmatrix} g_{1i}(Y) & g_{2i}(Y) \end{pmatrix}^T$, and according to some mechanistic knowledge, $g_{11} > 0$, $g_{22} < 0$, $g_{12} < 0$, $g_{21} > 0$ for any $Y_i \in [0,1]$, $i = 1, 2$.

Assume the outputs Y_1, Y_2 are known, a dynamic neural network based identifier is designed to approximate (1) as below:

$$\dot{\hat{Y}} = A\hat{Y} + \hat{f}(Y, W_f) - AY$$
$$+ \begin{pmatrix} \hat{g}_1(Y, W_{g_1}) & \hat{g}_2(Y, W_{g_2}) \end{pmatrix} \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} \quad (2)$$

where A is any stable matrix. Y and $\hat{Y}$ are the output vectors of the column and the identifier respectively. $\hat{f}$ and $\hat{g}_i$ ($i = 1,2$) are all MFNN, which are the estimates of f and g_i in (1) respectively. The active function of the output-layer of MFNN $\hat{f}$ is linear. The active function of $\hat{g}_1$ is a sigmoid function $k/(1 + \exp(-x))$, and the active function of $\hat{g}_2$ is $-k/(1 + \exp(-x))$, where k is a tunable parameter satisfying $k > 0$. W_f and W_{g_i} are the weight matrixes.

From [9], for $\forall \varepsilon > 0$ and any continuous function $F(x)$ there exist a three-layer feedforward network satisfing:

$$\sup_x \left\| F(x) - \hat{F}(x, W^*) \right\| < \varepsilon$$

where W^* denotes the optimal weight matrix and $\hat{F}(x, W^*)$ denotes the outputs of the network.

Let W_f^* and $W_{g_i}^*$ be the corresponding optimal weight matrix, system (1) can be rewritten as：

$$\dot{Y} = AY + \hat{f}(Y, W_f^*) - AY + \sum_{i=1}^{2} \hat{g}_i(Y, W_{g_i}^*) u_i(t) + \xi(Y) \quad (3)$$

where $\xi(Y)$ is the modeling error of the optimal identifer satisfing:

$$\sup_{Y \in \Omega_Y} \left\| \xi(Y) \right\| \leq \bar{\xi}, \quad \Omega_Y = \left\{ Y | 0 < Y_1, Y_2 < 1 \right\}$$

which can be made as small as possible by adjusting the neural network structure.

Let θ be an extended column vector of all the ele-

ments of a weight matrix W. θ^* denotes the corresponding extended vector of the optimal weight matrix. Then the extended column vector of W_f and W_{g_i} are θ_f and θ_{g_i}. Define identification error as $e = Y - \hat{Y}$ and parameter estimation error as $\tilde{\theta}_j = \theta_j^* - \theta_j$ ($j = f, g_i$). From (2) and (3) we have

$$\dot{e} = Ae + \tilde{f} + \sum_{i=1}^{2} \tilde{g}_i u_i(t) + \xi(Y) \quad (4)$$

where

$$\tilde{f} = \hat{f}(Y, \theta_f^*) - \hat{f}(Y, \theta_f) = \frac{\partial \hat{f}}{\partial \theta_f} \tilde{\theta}_f + O_{\tilde{f}}\left(\left\| \tilde{\theta}_f \right\|^{1+\alpha} \right),$$

$$0 < \alpha < 1,$$

$$\tilde{g}_i = \hat{g}_i(Y, \theta_{g_i}^*) - \hat{g}_i(Y, \theta_{g_i}) = \frac{\partial \hat{g}_i}{\partial \theta_{g_i}} \tilde{\theta}_{g_i} + O_{\tilde{g}_i}\left(\left\| \tilde{\theta}_{g_i} \right\|^{1+\alpha} \right)$$

Define

$$d(t) = O_{\tilde{f}} + \sum_{i=1}^{2} O_{\tilde{g}_i} u_i(t) + \xi(Y) \quad (5)$$

Assume:

$$\sup \left\| d(t) \right\| \leq \varepsilon_f + \varepsilon_g \sup u \quad (6)$$

where ε_j ($j = f, g$) is a finite constant.

The updating algorithms of the parameter vectors θ_f and θ_{g_i} are as followings：

$$\dot{\theta}_f = \left(\frac{\partial \hat{f}}{\partial \theta_f} \right)^{\tau} Pe - k_f \|P\| \cdot \|e\| \theta_f,$$

$$\dot{\theta}_{g_i} = \left(\frac{\partial \hat{g}_i}{\partial \theta_{g_i}} \right)^{\tau} Pe u_i(t) - k_{g_i} \|P\| \cdot \|e\| \theta_{g_i} \quad (7)$$

where $k_j > 0$, $j = f, g_i$, and P is the positive defined solution of the following Lyapunov equation:

$$PA + A^T P = -Q$$

Q is a positive defined matrix and the minimal eigenvalue of Q is λ_0.

Theorem 1: If $u(t)$ is bounded and the weight updating alogrithm is (7), then the identification error e and parameter estimation error $\tilde{\theta}_j$ ($j = f, g_i$) are all ultimately uniformly bounded (UUB)

Proof: Consider the Lyapunov function candidate：

$$V = \frac{1}{2} e^{\tau} Pe + \frac{1}{2}\left(\tilde{\theta}_f^{\ \tau} \tilde{\theta}_f + \sum_{i=1}^{2} \tilde{\theta}_{g_i}^{\ \tau} \tilde{\theta}_{g_i} \right).$$

Differentiating it along (4) and (5) yields

$$\dot{V} = -\frac{1}{2} e^{\tau} Qe + e^{\tau} P \frac{\partial \hat{f}}{\partial \theta_f} \tilde{\theta}_f + e^{\tau} P \sum_{i=1}^{2} \frac{\partial \hat{g}_i}{\partial \theta_{g_i}} u_i(t) \tilde{\theta}_{g_i}$$

$$+ e^{\tau} Pd(t) - \dot{\theta}_f^{\ \tau} \tilde{\theta}_f - \sum_{i=1}^{2} \dot{\theta}_{g_i}^{\ \tau} \tilde{\theta}_{g_i}$$

Substituting (10) into the above equation leads to

$$\dot{V} = -\frac{1}{2}e^{\tau}Qe + e^{\tau}Pd(t) + \|e\| \cdot \|P\|(k_f \tilde{\theta}_f{}^{\tau}(\theta_f^* - \tilde{\theta}_f)$$

$$+ \sum_{i=1}^{2} k_{g_i} \tilde{\theta}_{g_i}{}^{\tau}(\theta_{g_i}^* - \tilde{\theta}_{g_i}))$$

$$\dot{V} \le -\frac{\lambda_0}{2}\|e\|^2 + \|e\| \cdot \|P\| \cdot \|d\| + \|e\| \cdot \|P\| \cdot (-k_f \|\tilde{\theta}_f\|^2$$

$$- \sum_{i=1}^{2} k_{g_i} \|\tilde{\theta}_{g_i}\|^2 + k_f \|\tilde{\theta}_f\| \cdot \|\theta_f^*\| + \sum_{i=1}^{2} k_{g_i} \|\tilde{\theta}_{g_i}\| \cdot \|\theta_{g_i}^*\|)$$

We can further have:

$$\dot{V} \le -\frac{\lambda_0}{2}\|e\|^2 - \|P\| \cdot \|e\| \left(k_f \left(\|\tilde{\theta}_f\| - \frac{1}{2}\|\theta_f^*\| \right)^2 - \frac{k_f}{4}\|\theta_f^*\|^2 \right.$$

$$\left. + \sum_{i=1}^{2} k_{g_i} \left(\left(\|\tilde{\theta}_{g_i}\| - \frac{1}{2}\|\theta_{g_i}^*\| \right)^2 - \frac{1}{4}\|\theta_{g_i}^*\|^2 \right) - \|d\| \right)$$

Assuming $u(t)$ is bounded and combining it with (6), we have

$$\|d(t)\| \le \varepsilon_f + \varepsilon_g \bar{u} \le \varepsilon_m .$$

So ε_m is bounded.

Define $\varepsilon_d = k_f \frac{1}{4}\|\theta_f^*\|^2 + \sum_{i=1}^{2} k_{g_i} \frac{1}{4}\|\theta_{g_i}^*\|^2 + \varepsilon_m .$

It is easy to verify that

if $\|e\| \ge \frac{2}{\lambda_0 \|P\|}\varepsilon_d$

or: $\|\tilde{\theta}_f\| \ge \frac{1}{2}\|\theta_f^*\| + \sqrt{\varepsilon_d/k_f}$,

or: $\|\tilde{\theta}_{g_i}\| \ge \frac{1}{2}\|\theta_{g_i}^*\| + \sqrt{\varepsilon_d/k_{g_i}}$

then $\dot{V} < 0$.

For θ_i^* being bounded, when $u_i(t) \in L_\infty$, we have $\varepsilon_d \in L_\infty$. According to Barbalat's theorem [21], it can be seen that:

$$e, \tilde{\theta}_f, \tilde{\theta}_{g_i} \in L_\infty .$$

Considering (5), we further have $\dot{e} \in L_\infty$.

3. Nonlinear H∞ Control of the Binary Distillation Column

3.1. Nonlinear H∞ Control

Consider an affine nonlinear controller system

$$\dot{x} = \overline{f}(x) + \overline{g}_1(x)\omega + \overline{g}_2(x)u$$

$$y = h(x) + k(x)u \qquad (8)$$

where $x \in R^n$, $u \in R^m$, $\omega \in R^s$, $y \in R^p$; $\overline{f}(x)$, $\overline{g}_1(x)$,

$\overline{g}_2(x)$, $h(x)$ and $k(x)$ are smooth functions; u is control input ω is uncertainty noise and $\overline{f}(0) = 0$, $h(0) = 0$.

For system (8), the design objective of nonlinear H∞ control is to make the following conditions guaranteed [19,20].

1) The closed-loop system is asymptotically stable; and,

2) for a given $\gamma > 0$, the following inequality holds

$$\int_0^t \|y(\tau)\|^2 d\tau \le \gamma^2 \int_0^t \|\omega(\tau)\|^2 d\tau \quad \forall t \ge 0 .$$

According to [19], defining $V_x = \frac{\partial V}{\partial x}$ for $\forall x \in R^n$

and $R(x) = k^T(x)k(x)$, we have:

Proposition: If $R(x)$ is nonsingular, nonlinear H∞ control law is expressed as

$$u = -R^{-1}(x)\left(\frac{1}{2}\overline{g}_2^{\tau}(x)V_x^{\tau} + k^{\tau}(x)h(x) \right) \qquad (9)$$

where $V(x)$ $(V(0) = 0)$ satisfies

$$V_x \hat{\overline{f}}(x) + \hat{h}^{\tau}(x)\hat{h}(x) + \frac{1}{4}V_x \hat{R}(x)V_x^{\tau} \le 0$$

where

$$\hat{\overline{f}}(x) = \overline{f}(x) - \overline{g}_2(x)R^{-1}(x)k^{\tau}(x)h(x)$$

$$\hat{h}(x) = (I - k(x)R^{-1}(x)k^{\tau}(x))h(x)$$

$$\hat{R}(x) = \frac{1}{\gamma^2}\overline{g}_1(x)\overline{g}_1^{\tau}(x) - \overline{g}_2(x)R^{-1}(x)\overline{g}_2^{\tau} .$$

3.2. Nonlinear H∞ Control of the Binary Distillation Column

Considering the existence of the modeling error, system (3) can be rewritten as:

$$\dot{Y} = AY + \hat{f} - AY + \begin{pmatrix} \hat{g}_1 & \hat{g}_2 \end{pmatrix} u(t) + \omega \qquad (10)$$

where ω is the uncertainty variable denoting the modeling error.

Assuming the desired tracking trajectory is $\dot{Y}_d = AY_d + s$, where s is a reference variable.

Define $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = Y - Y_d = \begin{pmatrix} Y_1 - Y_{1d} \\ Y_2 - Y_{2d} \end{pmatrix} .$

From (10), the tracking error equation can be derived as below:

$$\dot{x} = a(x) + Ax + G_1\omega + G_2u \qquad (11)$$

where

$$a(x) = \hat{f} - AY - s , G_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, G_2 = \begin{pmatrix} \hat{g}_{11} & \hat{g}_{12} \\ \hat{g}_{21} & \hat{g}_{22} \end{pmatrix} .$$

For the binary distillation column, $G_2(Y)$ is invertible. If the penalty variables are chosen to be

$$z = h(x) + k(x)u = a(x) + G_2u .\qquad(12)$$

Substituting (9) into (12), we have:

$$u = -R^{-1}(x)\left[G_2^T(x)Px + k^T(x)h(x)\right]$$
$$= -G_2^{-1}\left(Px + a(x)\right)\qquad(13)$$

where P is the positive defined solution of the following Riccati inequality:

$$PA + A^TP + P\left(\frac{1}{\gamma^2}-1\right)P \le 0 .\qquad(14)$$

Remark: P can be obtained by solving a Riccati equation as below:

$$PA + A^TP + P\left(\frac{1}{\gamma^2}-1\right)P + C^TC = 0$$

where $[A,C]$ is observable.

4. Simulation

After The binary distillation model used in the simulation is the same as the one developed by [2]. The nominal values of outputs are $Y_{10} = 0.98(\text{mol}\%)$ and $Y_{20} = 0.02\,(\text{mol}\%)$. The nominal values of reflux flow and boil-up flow are $L = 2.28625$ (kmol/min) and $V = 2.78625$ (kmol/min). The feed flow and feed composition are $F = 1$ (kmol/min) and $Z_f = 0.5$ (mol%).

The distillation column is controlled using the control strategy (13) developed in this paper. The tacking property and the disturbance attenuation property of the control system are demonstrated through simulation. To make the transient response be more elegant, the dynamic neural network based identifier is trained for some time offline.

In the simulation, we choose

$$A = \begin{pmatrix} -9 & 0 \\ 0 & -9 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \gamma = 1.1$$

and the positive definite P we obtained is

$$P = \begin{pmatrix} 0.0555 & 0 \\ 0 & 0.0555 \end{pmatrix}.$$

1) Servo properties

Let the setpoint of the top light component change from nominal value 0.98 to 0.995 and the bottom composition remain nominal value 0.02. The augmentations of system output are demonstrated in **Figure 1**. The curves of the control inputs are in **Figure 2**.

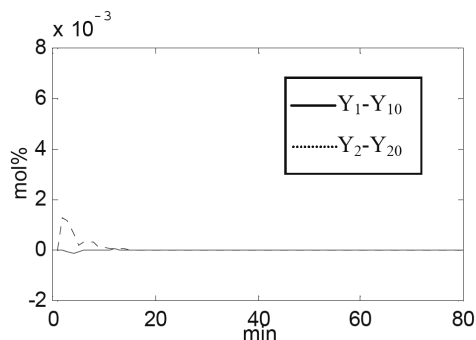2) Robustness

The responses of the closed-loop system are illustrated in **Figures 3** and **4** when feed flow F and feed composition Z_f increase 10% respectively.

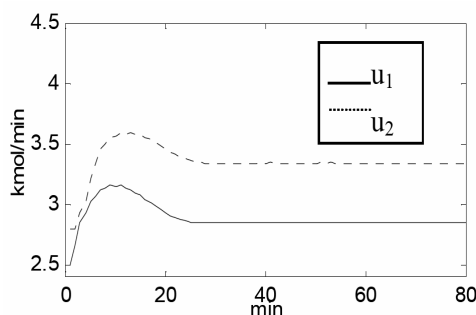**Figure 1. The augmentations of outputs.**

Figure 2. The curves of control inputs.

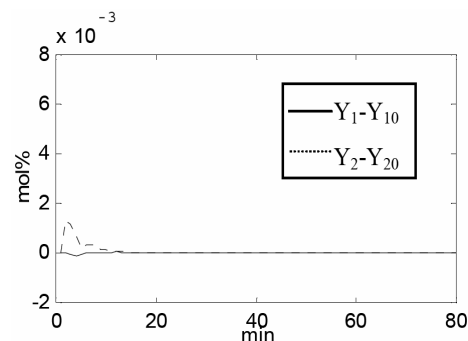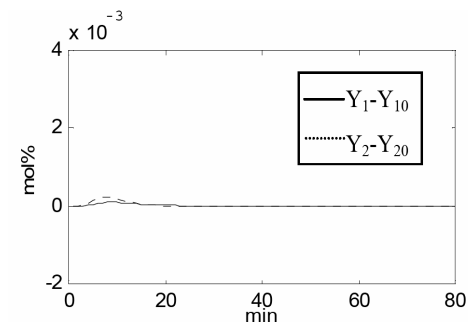**Figure 3. The augmentations of outputs (Feed increases 10%)**.

Figure 4. The augmentations of outputs (Feed composition increases 10%).

5. Conclusions

A dynamic neural network based online nonlinear identifier for a binary distillation column is designed. The learning algorithm of the network weights is established in detail, which can guarantee the boundedness of the identification error. To deal with the modeling error, a nonlinear H_∞ controller based on the identifier is given by choosing the penalty variables for the column. The effectiveness of the proposed strategy is demonstrated in simulation results. The algorithm developed in this paper can be applied to other chemical processes as well.

6. Acknowledgements

7. References

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