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Optimization of Tracking Error for Robust Portfolio of Risk Assets with Transaction Cost

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ABSTRACT

Based on the optimization of robust portfolio with tracking error, a robust mean-variance portfolio selection model of tracking error with transaction cost is presented for the case that only risky assets exist and expected returns of assets are uncertain and belong to a convex polyhedron. This paper aims to solve the problem of the portfolio with the selection of the ratio on the condition of maximumimum fluctuation of the tracking error, making the expectation of the return to be the maximumimum. It also makes the portfolio's practical choice by the function of the linear transaction cost as the same time of construction and application of the model. Empirical analysis with five real stocks is performed by the method of LMI (Linear Matrix Inequality) to show the efficiency of the model.

Keywords: Transaction Cost; Tracking Error; Robust; Risky Assets; Portfolio Optimization

1. Introduction

The Mean-Variance model developed by Markowitz [1] has been playing an important role in the field of asset allocation. Based on this theory, some robust portfolio optimization methods later has made up it effectively [2-11]. Ben has researched the topic of the robust optimization method and applications [2]. Pinara has studied the issue about the robust profit opportunities in risky financial portfolios [7]. On the condition of the separation of ownership of the funds and the rights of investment and management in the recent financial trading market, the model of tracking error robust portfolio optimization has been applied in the practice of financial investment: Assuming the situation that both the rate of expected returns and the covariance matrix is uncertain which belong to the known convex combinations, Costa has studied the issue about tracking error robust portfolio with the method of the linear-matrix inequalities [12]. Moreover, robust portfolio selection with transaction cost has been given high priority: Bertsimas[13] has researched about robust multiperiod portfolio management in the presence of transaction cost. Xue [14] has studied about mean-variance portfolio optimal problem under concave transaction cost and Erdogany [15] has studied the issue about robust active portfolio management.

In this paper, we have researched the topic of tracking error robust portfolio of risky assets with transaction cost on the condition of the uncertainty of both the expected return rate and the covariance matrix belonging to the known convex sets. In the end of this paper, an empirical analysis is given through the method of LMI (Linear Matrix Inequality) to show the efficiency of the model.

2. Introdution to Parameters

It is assumed that there are only n risky assets in the market and which can be written as $A=\left(a_1,a_2,\cdots a_n\right)$.

a) The weight vector of the risky portfolio is

$$z=\left(z_1,z_2,\ldots,z_n\right)'$$

where $\left(z_1,z_2,\ldots,z_n\right)'$ is the transpose of $\left(z_1,z_2,\ldots,z_n\right)$. z_i is the weight of portfolio in the $i-th$ assets, $0\le z_i\le 1,\ i=1,2,\cdots,n$, and it satisfied with $z'\cdot I=1$ in which I is the n-dimensional unit column vector.

b) The rate of the return vector of the risky assets can be expressed as $r=\left(r_1,r_2,\ldots,r_n\right)'$, where the transpose of $\left(r_1,r_2,\ldots,r_n\right)$ is $\left(r_1,r_2,\ldots,r_n\right)'$.

c) The rate of the expected return vector of the risky assets can be written as:

$$E(r)=\mu=\left(\mu_1,\mu_2,\ldots,\mu_n\right)'.$$

while the covariance matrix is noted as $G\in R^{n\times n}$.

d) The vector of the transaction cost can be noted as:

$$P(z)=(P_1(z_1),P_2(z_1),\ldots,P_n(z_n))',$$

where $P_i(z_i)$ is the individual transaction cost on the $i-th$ risky asset.

e) The net weight vector of portfolio is

$$\widehat{z}=z-P(z)=\left(\widehat{z}_1,\widehat{z}_2,\ldots,\widehat{z}_n\right)',$$

where $\widehat{z}_i = z_i - P_i(z_i)$ is denoted for the weights of the $i-th$ risky assets with transaction cost.

f) The sum of the returns can be expressed as:

$$R_\mu(z) = z'\mu = \sum_{i=1}^{n} u_i z_i .$$

g) The net of the returns of the risky assets can be denoted as

$$NR_u(z) = \sum_{i=1}^{n} u_i \widehat{z}_i = \sum_{i=1}^{n} u_i (z_i - P_i(z_i)) \tag{1}$$

h) The variance of the net returns of the portfolio can be written as:

$$\sigma^2(\widehat{z}) = \widehat{z}' G \widehat{z} \tag{2}$$

i) The function of linear transaction cost of the risky assets can be expressed as:

$$P_i(z_i) = a_i z_i + b_i,\ i = 1,2,\cdots,n$$

As the fixed cost can be omitted, the above formula can be simplified for

$$P_i(z_i) = a_i z_i,\ \ 0 < a_i < 1 \tag{3}$$

3. Tracking Error Robust Portfolio Optimization under Certain Condition

Tracking error can be actually called for the difference between the return of the investor's portfolio and benchmark return portfolio. It can be assumed that the vector of the predetermined benchmark return is $z_B = \left(z_{B_1}, z_{B_2}, \ldots, z_{B_n}\right)'$ here. Usually, the tracking error can be expressed either as the relative return

$$\beta(z) = \left(z - z_B\right)' u$$

or as the variance $\sigma^2(z) = \left(z - z_B\right)' G \left(z - z_B\right)$.

Actually, it can be written into two situations which are equal with each other in the optimization of the tracking error: the first situation is to minimum the variance in the condition of guaranting the fixed objective of the relative return, the second situation is to maximum the return under guaranting fixed the relative variance. The model of the second situation can be expressed as [12]

$$\begin{cases} & \max_z \beta(z) \\ s.t. & \sigma^2(z) \le \sigma_0^{\ 2} \\ & z \in \Psi \end{cases} \tag{4}$$

4. Tracking Error Robust Portfolio Optimization in Uncertain Market

The uncertain condition here is that both the expected return and the covariance can be variable with the change of the environment of the financial market. Costa assumed that the parameter μ and G belong to two different convex combinations respectively to describe the uncertain[12], it can be written as

$$u \in Con\left\{u_1, u_2, \cdots, u_t\right\},$$
$$G \in Con\left\{G_1, G_2, \cdots, G_t\right\}$$

where $u_k = (u_{k1}, u_{k2}, \cdots, u_{kn})', G_k,\ k = 1,2,\cdots,t$ are stood for the expected return and the covariance rerespectively under the uncertain condition which are obtained by the different methods.

The reference [12] demonstrates the form of the linear matrix inequalities to denote for the optimization in the uncertain condition of the tracking error robust portfolio with *n* risky assets and 1 risk-free asset based on the condition above, so we can get robust portfolio optimization with *n* risky assets through the model above as follows:

$$\begin{cases} & \min_z \ \ -\beta \\ s.t. & \begin{bmatrix} \alpha_k & (z - z_B)' G_k \\ G_k (z - z_B) & G_k \end{bmatrix} \ge 0 \\ & (z - z_B)' u_k \ge \beta,\ k = 1,2,\cdots,t \\ & \sum_{i=1}^{n} z_i = 1 \\ & 0 \le z_i \le 1,\ i = 1,2,\cdots,n \end{cases} \tag{5}$$

where $\beta = \max_k (z - z_B)' u_k, k = 1,2,...t$ has introduced the bounded variable of the deviationist fluctuations, the function of the model is to get the vector of weight of the portfolio *z* to satisfy the condition of the maximum expected return β. The significance of mathematical finance is to find a portfolio with maximum worst case expected performance about a benchmark, with guaranteed fixed maximum tracking error volatility. Actually, the maximum return can be written as:

$$\widehat{\beta} = -\min_z \max_k \left\{-(z - z_B)' u_k;\ z \in \Lambda(\alpha)\right\}$$

where

$$\Lambda(\alpha) = \left\{z \in \Psi;\ \alpha_k - (z - z_B)' G_k (z - z_B) \ge 0,\ k = 1,2,\cdots,t\right\}$$

and $\alpha_k - (z - z_B)' G_k (z - z_B) \ge 0$ can be obtained by the method of Schur complement, $\alpha_k = (\alpha_1, \alpha_2, ..., \alpha_t)'$ is the vector of maximum volatility of the tracking error in uncertain economy of the future[12].

5. Tracking Error Robust Portfolio Optimization of Risky Assets with Transaction Cost

Based on model (5) above, we consider the linear transaction cost to bulid a new model of tracking errof robust portfolio optimization of risky assets with transaction cost. This model is express as following formula.

$$\begin{cases} \min\limits_{z} \quad -\beta \\ s.t. \quad \begin{bmatrix} \alpha_k & (\widehat{z}-\widehat{z}_B)'G_k \\ G_k(\widehat{z}-\widehat{z}_B) & G_k \end{bmatrix} \geq 0 \\ (\widehat{z}-\widehat{z}_B)'u_k \geq \beta, \\ k=1,2,\cdots,t \\ \sum\limits_{i=1}^{n} z_i = 1 \\ 0 \leq z_i \leq 1, i=1,2,\cdots,n \end{cases} \tag{6}$$

Some contents of the model is illustrated as follows.

The known parameter α_k is the maximum volatility vector of the tracking error of the uncertain market in the future. The random parameters G_k (covariance matrix) and u_k (expected return) can be got by the $k-th$ method respectively.

The variables $\widehat{z}$ and $\widehat{z}_B$ are the net weight vector of portfolio and the benchmark of the net weight vector of portfolio respectively. β stands for the expected value of the tracking error portfolio in the different financial markets. The objective of the function is to get the weight $\widehat{z}$ of the portfolio with the maximum β.

The two subscripts i and k stands for the $i-th$ asset and $k-th$ market situation respectively. $i=1,2,\cdots,n$ means that the number of the risky assets is n and $k=1,2,\cdots,t$ means that the number of the market situations is t here.

The first inequality can be written as $\alpha-(\widehat{z}-\widehat{z}_B)'G_k(\widehat{z}-\widehat{z}_B)\geq 0$ which can be obtained by the method of Schur complement. The second inequality is suitable for representing the constraint of the expected return of portfolio. The equality represents for the sum of weights is 1. The last inequality means that z_i is equal or less than 1 and equal or more than 0, and no short sales are permitted.

We get the objective of the robust portfolio optimization which is equal to get the determined weights vector of portfolio with guaranting the maximum expected return in the condition of the maximum volatility at last.

6. Empirical Analysis

The empirical analysis below can be accomplished by Matlab and LMI (Linear Matrix Inequality).

a) Data acquisition

We selected the five stocks: 000407, 000725, 000552, 000045, 000651 from the trading market of Shenzhen securities. According to the daily close prices from August 20, 2012 to September 28, 2012, we can obtain the daily return rate as the following **Table 1**.

b) The function of the transaction cost

We assume $P_i(z_i)=0.003z_i$, $i=1,\cdots,5.$

c) The benchmark of the weight of the portfolio

Table 1. The daily return rate of the five stocks.

date	The daily return rate				
	A-002081	B-002482	C-002375	D-002325	E-002041
1	-0.0164	0.0000	0.0039	-0.0082	-0.0125
2	0.0066	0.0058	0.0213	0.0397	-0.0054
3	-0.0115	0.0000	-0.0246	-0.0111	-0.0309
4	-0.0050	0.0058	0.0312	0.0032	0.0056
5	0.0017	-0.0058	-0.0313	-0.0239	0.0116
6	-0.0370	-0.0174	-0.0243	-0.0356	0.0079
7	-0.0018	0.0237	0.0147	0.0168	0.0148
8	-0.0388	0.0287	-0.0172	0.0626	-0.0034
9	-0.0128	-0.0056	0.0121	-0.0338	0.0035
10	0.0655	0.0113	0.0067	0.0920	0.0109
11	-0.0164	0.0000	0.0039	-0.0082	-0.0125
12	0.0463	0.0000	0.0153	0.0329	0.0201
13	-0.0221	-0.0056	0.0105	-0.0073	0.0010
14	0.0018	0.0000	0.0040	-0.0015	0.0062
15	-0.0018	0.0000	-0.0026	0.0045	-0.0014
16	0.0365	0.0341	0.0368	0.0224	0.0156
17	0.0152	0.0110	0.0069	0.0087	0.0075
18	0.0050	-0.0054	-0.0056	0.0291	-0.0174
19	-0.0250	0.0055	0.0012	-0.0112	-0.0066
20	-0.0251	0.0163	-0.0200	-0.0284	-0.0080
21	0.0051	0.0899	-0.0031	-0.0087	-0.0052
22	-0.0441	-0.0383	-0.0334	-0.0324	-0.0110
23	-0.0106	-0.0251	-0.0225	0.0091	-0.0034
24	0.0179	0.0206	0.0046	0.0107	0.0092
24	-0.0513	-0.0153	-0.0229	-0.0606	-0.0053
26	0.0056	-0.0156	0.0297	0.0113	0.0030
27	0.0074	0.0107	0.0400	0.0195	0.0025
28	-0.0201	0.0053	-0.0149	0.0032	0.0010
29	0.0019	-0.0265	-0.0046	-0.0687	-0.0010
30	0.0018	0.0272	0.0353	0.0034	0.0259
31	0.0360	0.0214	0.0258	0.0200	0.0274

We make the benchmark of the weight of the portfolio is $z_B=(0.20,0.20,0.20,0.20,0.20)'$, so the benchmark of the weight of the portfolio can be written as:

$$\widehat{z}_B=(0.197,0.197,0.197,0.197,0.197)'.$$

d) The uncertainty of the financial market

Let $t=2(k=1,2)$. u_1,u_2 can be got by the different methods. $u_1=(-0.0023,0.0052,0.0024,0.0019,0.0021)'$ which is the arithmetic mean of the rate of return of thrity trading days. u_2 can be written as $u_2=(0.0064,0.0280,0.0020,0.0124,\text{-}0.0022)'$ which is the mean of the one thousand random value between the maximum return's rate and the minimum return's rate.Measwhile G_1,G_2 can be written as:

$$G_1=\begin{bmatrix} 7 & 2 & 3 & 5 & 2 \\ 2 & 5 & 2 & 3 & 1 \\ 3 & 2 & 5 & 3 & 1 \\ 5 & 3 & 3 & 11 & 1 \\ 2 & 1 & 1 & 1 & 2 \end{bmatrix}\times 10^{-4}, \qquad G_2=\begin{bmatrix} 7 & 3 & 4 & 5 & 3 \\ 3 & 5 & 3 & 3 & 2 \\ 4 & 3 & 5 & 4 & 2 \\ 5 & 3 & 4 & 11 & 2 \\ 3 & 2 & 2 & 2 & 2 \end{bmatrix}\times 10^{-4},$$

where G_1 can be got by the rate of return of thrity trading days, G_2 can be obtained by the method that we endued 0.4 and 0.6 to the first 20 trading days and the last 10 trading days respectively accroding to the weights.

e) The maximum volatility of tracking error

The maximum volatility of tracking error can be set to $\alpha_1 = 0.0013, \alpha_2 = 0.0034$, so the solution of the model (6) is able to be calculated as follows:

$$\hat{z} = (0.3217, 0.2078, 0.1153, 0.2065, 0.1487)',$$

$$\hat{\beta} = 0.1689$$

7. Conclusions

In this paper, a tracking error robust portfolio optimization model of risky assets with transaction cost is established for the practice of financial market. The optimization expands the previous theory of the portfolio. Moreover, it can be much more useful and efficient in the application of the practice of portfolio selection. We will have a further discussion on the other types of the function with transaction cost and the issue of portfolio selection in the condition of the uncertain financial market.

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