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Theory on Orthokinetic Flocculation of Cohesive Sediment: A Review

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Abstract

Investigation on flocculation phenomenon of cohesive fine-grained sediment has been a important part of sediment dynamics. During all of three dynamical factors (i.e., Brownian motion, flow shear and differential settling) that have been verified to play important roles in promoting flocculation of cohesive sediment, the influence of flow shear on sediment flocculation has been paid great attention by many researchers (this flocculation pattern has been termed as "orthokinetic flocculation" in most of published literatures). Among many researches regarding orthokinetic flocculation, the dynamical equation developed by Smoluchowski in 1917 (we called it as Smoluchowski equation hereafter) has been widely adopted as an origin and basement for theoretically analyzing sediment flocculation under a shear flow. Meanwhile, many researchers have also pointed out the deficiencies of Smoluchowski equation (this is because the derivation of Smoluchowski equation was based on six different assumptions), and correspondingly have amended this equation from different aspects. In this paper, we attempt to summarize these results, hopefully providing the theoretical research of sediment orthokinetic flocculation with some references.

Keywords

Sediment, Orthokinetic Flocculation, Review

1. Introduction

Flocculation phenomenon of cohesive sediment can widely be observed in most of estuaries, reservoirs and channels. Sediment flocculation has been confirmed to play an important role in influencing development of estuary delta, geomorphologic variation of sand bar and dredging of mud layer channel. Mechanisms of flocculation are complex, and there are many factors that have been verified to play some roles in influencing the flocculation process, including physical and chemical factors (such as mineral composition of sediment particle, sediment concentration, particle size, type of electrolyte in the suspension, concentration of electrolyte, temperature, pH value), as well as dynamical factors (flow shear, Brownian motion and differential settling). At present, most of researches have been carried out to demonstrate the mechanism and dynamic characteristic of sediment flocculation (Yang & Qian, 1986; Liu, 1994; Guan & Chen, 1995; Jiang & Zhang, 1995; Guan et al., 1996; Zhang, 1996; Jin et al., 1998; Shi, 2000; Chen et al., 2001; Jin et al., 2002; Jiang et al., 2002).

In particular, in the case of estuary and coastal area, flocculation of sediment is inseparable with a flow shear

due to inland runoff, tidal current from open sea, as well as interaction of salty water and fresh water. The kind of flocculation due to a flow shear has been termed as orthokinetic flocculation, and it is of great interest for many researchers. The theoretical basement of most of researches regarding orthokinetic flocculation can be found to originate from a pioneering dynamical equation put forward by Smoluchowski in 1917 (we shall call it Smoluchowski equation hereafter). Smoluchowski equation can simply describe concentration variation of discrete particle during flocculation due to a flow shear. The term in this equation that can describe the influence of a flow shear on flocculation is simply velocity gradient of a laminar flow. Later, Camp and Stein (1943) introduced the root-mean-square (RMS) of velocity gradient representing the turbulent intensity into Smoluchowski equation, which makes the discussion on effect of a turbulent flow on flocculation become possible. Although Smoluchowski equation has been widely as a origin for theoretically analyzing flocculation due to a shear flow, derivation of this equation was based on some assumptions, which inevitably simplifies the real engineering situations and thereby limiting its applicability, as many researches shown. Plenty of researches have been performed to amend Smoluchowski equation from different aspects, and in this paper we attempt to summarize these research results scatted in different literatures, hopefully providing the theoretical study regarding sediment orthokinetic flocculation with some references.

2. Theoretical Analysis of Flocculation Dynamics

In 1917, Smoluchowski derived a dynamical equation to describe orthokinetic flocculation of discrete particle (Smoluchowski, 1917)

$$\frac{\partial n_k}{\partial t} = \frac{1}{2} \sum_{i=1,i+j=k}^{k-1} \frac{4}{3} \left(\frac{du}{dz} \right) n_i n_j R_{ij}^3 - n_k \sum_{i=1}^{max} \frac{4}{3} \left(\frac{du}{dz} \right) n_i R_{ik}^3 \tag{1}$$

where i, j, k are grades of sizes of the flocs, max is that of the maximum floc, n_i, n_j, n_k are number concentrations of i, j, k -grade flocs respectively (here $k = i + j$, meaning a k -grade floc results from collision and subsequent adhesion between i and j -grade flocs), du/dz is velocity gradient in a laminar flow(here u is fluid velocity along the mentioned direction, and z is the direction perpendicular to the mentioned direction in a simple two-dimensional right-handed coordinate system), $R_{ij} = r_i + r_j$, $R_{ik} = r_i + r_k$ are radii of collisions between i, j -grade flocs and between i, k -grade flocs, respectively, r_i, r_j, r_k are radii of i, j, k -grade flocs respectively.

As Thomas et al. (1999) introduced, there exists six assumptions in the derivation of Equation (1), and they are summarized as follows: 1) flow is assumed to be laminar; 2) all particles are special-shaped, and they are composed of solid spheres. The moving trajectory of one particle are assumed completely independent from others, and this trajectory should be rectilinear; 3) no breakup or sedimentation during process of random collision and subsequent adhesion can happen; 4) all of inter-particle collisions can result in corresponding adhesion, indicating the so-called collision efficiency coefficient is equal to be unity; 5) sizes of all particles are the same at the beginning of flocculation (this state can be regarded to be in the mono-disperse condition, in contrast to the poly-disperse condition); 6) collisions between particles only happen between two particles, and those of three or more particles are never mentioned. These assumptions have been found to oversimplify the real engineering situation. As a result, this limits the applicability of Equation (1). Here, we attempt to summarize some results regarding amendments of Smoluchowski equation according to every assumption mentioned above.

2.1. About the First Assumption

2.1.1. Laminar Non-Uniform Shear Flow

Realizing the fact that Equation (1) can only be applied in a laminar uniform flow, Camp and Stein (1943) introduced the root-mean-square (RMS) of velocity gradient, G, in a non-uniform flow into Equation (1) instead of du/dz as follows

$$G = \left(\frac{\Phi}{\mu} \right)^{\frac{1}{2}}, \tag{2}$$

where Φ is dissipate work of flow (per volume per time) due to viscous shear. For the entire system, the statistically-averaged velocity gradient, G, should be

$$\overline{G} = \left(\frac{\overline{\Phi}}{\mu}\right)^{\frac{1}{2}}, \tag{3}$$

where $\overline{\Phi}$ is the averaged value of Φ over the system.

Kramer et al. (1997) and Kramer (1997) pointed out that there may be two problems in the study of Camp and Stein. Tangential parts of rate of strain tensor were only mentioned in their study, whereas normal parts were neglected, as long as Smoluchowski equation is extended from two-dimensional form to three-dimensional form. Statistically-averaged velocity gradient, $\overline{G}$, was simply adopted as a unique parameter to evaluate flocculation process in the study of Camp and Stein, and this will cause some errors. Experiments have demonstrated that flocculation process and resultant effect may be favorably different, although a same $\overline{G}$ can be found in different flocculation reactors. Furthermore, by transforming normal rate of strain tensor into a special rate of strain tensor with only major stress rate remaining, Kramer (1997) derived the flocculation dynamical equation with the following new form:

$$\frac{\partial n_k}{\partial t} = \frac{1}{2}\sum_{i=1,i+j=k}^{k-1}\frac{4\pi}{3}\left|a'_{max}\right|n_i n_j \left(r_i + r_j\right)^3 - n_k\sum_{i=1}^{max}\frac{4\pi}{3}\left|a'_{max}\right|n_i \left(r_i + r_k\right)^3, \tag{4}$$

where all of parameters have the same meaning as introduced above, except for $\left|a'_{max}\right|$ being a absolute value of maximum value of rate of strain tensor after coordinate system transformation. Moreover, Pedocchi and Piedra-Cueva (2005) clarified that rotation of flow field cannot cause the inter-particle collision, and put forward the following equation to describe the collision between particles by using some mathematical methods:

$$N_{ij} = \frac{1}{2}\int_0^{2\pi}\int_0^{\pi}\left|b\sin^2\theta\cos^2\varphi + (1-b)\sin^2\theta\sin^2\varphi - \cos^2\theta\right|\sin\theta d\theta d\varphi * \left(r_i + r_j\right)^3 \lambda' n_i n_j, \tag{5}$$

where N_{ij} is the number of collisions between i, j-grade flocs (per volume per time); θ, φ are basic parameters in the spherical coordinate system ($0 \leq \varphi \leq 2\pi$, $0 \leq \theta \leq \pi$); b is a coefficient related to three eigenvalues of deformation rate tensor after coordinate system transformation, λ' is a "common part" of these eigenvalues, thus the eigenvalues are $b\lambda'$, $(1-b)\lambda'$, $-\lambda'$. For a laminar shear flow with velocity gradient being a constant ($du/dz = C > 0$), three eigenvalues become $C/2$, 0, $-C/2$ ($b=1, \lambda' = C/2$ can also be obtained correspondingly), and thus Equation (5) reduces to $N_{ij} = 4/3\left(r_i + r_j\right)^3 Cn_i n_j$, which is consistent with the expression describing number of collisions between i, j-grade flocs in Smoluchowski equation.

2.1.2. Turbulent Flow

Turbulent flow is accompanied by a series of eddy motions. Those large eddies which contain the energy of system have been termed as "containing-energy eddies". Energy can be transported from those large eddies to smaller eddies, until all of energy can be dissipated by viscous force of a particular eddy with a minimum size. This kind of eddy with the minimum size has been termed as "Kolmogorovf micro-scale eddy" (Xia, 1992), and its size, η, can be expressed to be

$$\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{\frac{1}{4}}, \tag{6}$$

where ν is kinematic viscosity of flow, and ε is energy dissipate rate of the turbulent flow.

Camp and Stein (1943) stated that it will be reasonable to substitute velocity gradient in the turbulent flow, G, with the following form for du/dz in Equation (1)

$$G = \left(\frac{\varepsilon}{\nu}\right)^{\frac{1}{2}}. \tag{7}$$

Since the system is mentioned, average velocity gradient, $\overline{G}$, with the following form should be used correspondingly

$$\overline{G} = \left(\frac{\overline{\varepsilon}}{\nu}\right)^{\frac{1}{2}}, \tag{8}$$

where $\overline{\varepsilon}$ is the averaged value of ε over the system.

Saffman and Turner (1956) also analyzed aggregation phenomenon of particle in a turbulent flow. By assuming that floc size is smaller than Kolmogorovf micro-scale η and that the turbulent flow is homogeneous, they presented the following expression to describe inter-particle collision

$$N_{ij} = \sqrt{\frac{8\pi}{15}} (r_i + r_j)^3 \sqrt{\frac{\varepsilon}{\nu}} n_i n_j, \tag{9}$$

whereas Dilichatsios and Probstein (1974) studied the aggregation of particles whose sizes are between Kolmogorovf micro-scale η and size of containing-energy eddy, and put forward a new mathematical expressions as follows:

$$N_{ij} = 1.37\pi\varepsilon^{\frac{1}{3}} (r_i + r_j)^{\frac{7}{3}} n_i n_j. \tag{10}$$

2.2. About the Second Assumption

2.2.1. Fractal Structure of Floc

The second assumption behind Smoluchowski equation has been regarded to be incompatible with some experimental observations of porous flocs, and experiments have shown that flocs have the fractal structures(Burban et al., 1989; Li et al., 1998; Ma & Pierre, 1999; Lartiges et al., 2001; Li et al., 2004). For a single floc having the fractal structure, there is a simple mathematical relation to represent the relation between its mass and size as follows(Meakin, 1988):

$$M_f \sim d_f^{D_f}, \tag{11}$$

where M_f is mass of the floc, d_f characteristic length of the floc (this characteristic length can be commonly regarded to be equal to size of the floc), and D_f is mass fractal dimension of the floc, representing compactness degree of the floc (here $1 < D_f \le 3$). The expression between characteristic length of the floc and number of single primary particle contained in the floc can be expressed as follows (Feder, 1988):

$$i = \left(\frac{d_f}{d_0}\right)^{D_f}, \tag{12}$$

where i is number of single primary particle contained in the fractal floc, and d_0 is size of single primary particle. On substituting Equation (12) into Equation (1), the dynamical equation to describe the process of flocculation of fractal floc can be rewritten (Kramer & Clark, 1999):

$$\frac{\partial n_k}{\partial t} = \frac{1}{2} \sum_{i=1, i+j=k}^{k-1} \frac{4}{3} \left(\frac{du}{dz}\right) n_i n_j r_0^3 \left(i_i^{\frac{1}{D_f}} + i_j^{\frac{1}{D_f}}\right)^3 - n_k \sum_{i=1}^{max} \frac{4}{3} \left(\frac{du}{dz}\right) n_i r_0^3 \left(i_i^{\frac{1}{D_f}} + i_k^{\frac{1}{D_f}}\right)^3, \tag{13}$$

where r_0 is radius of single primary particle, and i_i, i_j, i_k are numbers of single primary particle contained in i, j, k -grade flocs respectively.

2.2.2. Moving Trajectory of Particle

1) Rectilinear trajectory, curvilinear trajectory and others Each of Smoluchowski equation, Camp and Stein method, and Saffman and Turner equation neglects influences of hydrodynamic force and short-range interactive force, when one particle touches and collides with another particle, and considered moving trajectory of the particle to be rectilinear. This understanding may be not in accordance with real situations. When two mentioned particles get into contact, surrounding fluid between these particles would be squeezed out, consequently leading to the rotation of particle relative to another particle and finally resulting in deviation of moving trajectory of particle from the rectilinear one; meanwhile, short-range interactive force between particles should

also play some role in the collision of particle. Han and Lawler (1992) pointed out that moving trajectory of particle may be curvilinear, and that frequency function of collision between particles can be expressed as follows:

$$\beta_{cur} = e_{cur}\beta_{rec}, \tag{14}$$

where β_{cur} is a frequency function of collision between particles (or collision kernel in some literatures) under the assumption of a curvilinear moving trajectory(its mathematical expression such as frequency function, β_{ij}, of collision between i, j-grade flocs is: $\beta_{ij} = N_{ij}/n_i n_j$), β_{rec} being a frequency function of collision under the assumption of a rectilinear trajectory (from Smoluchowski equation, expression of frequency function, $\beta_{rec}(i,j)$, of collision between i, j-grade flocs can be found to be $\beta_{rec}(i,j) = 4/3 G(r_i + r_j)^3$), and e_{cur} is a correction coefficient(this coefficient can be gotten by numerical simulation). For a curvilinear trajectory model, it can be inferred that flocculation rate is no longer proportional to velocity gradient, G.

Neither the rectilinear trajectory model nor the curvilinear trajectory model considers the porous and loose properties of floc, which means that a simple usage of the rectilinear or curvilinear trajectory model may bring some deviations from real situation. There are some researches that attempt to tackle this problem. Veerapaneni and Wiesner (1996) and Thill et al. (2001) proposed a new formula to calculate frequency function of collision, β_{ij}, between i, j-grade flocs by incorporating the considerations of fractal structure and permeability of floc as follows

$$\beta_{ij} = \frac{4}{3}G\left(\sqrt{E_{fi}}\,r_i + \sqrt{E_{fj}}\,r_j\right)^3, \tag{15}$$

where E_{fi}, E_{fj} are efficiency coefficients of collection of fluid surrounding i, j-grade flocs, respectively, and they are closely related to permeability of floc (their values can be determined using some empirical formulae). On the other hand, Kusters et al. (1997) have developed a simple shell-core efficiency model to calculate frequency function of collision as expressed as follows

$$\beta_{ij} \cong 1.294\alpha_{ij}G(r_i + r_j)^3, \tag{16}$$

where α_{ij} is a collision efficiency coefficient, $\alpha_{ij} \sim Fl^{-0.18}$ (van de ven & Mason, 1977), and Fl is a ratio of the hydrodynamic force and Van der Waals force between primary particles contained in i, j-grade flocs (when calculating the ratio Fl, it needs to be noticed that core is not penetrative in the shell-core model, whereas shell is penetrative). In particular, α_{ij} can be calculated by analyzing the moving trajectory of particle. Furthermore, Li and Logan (1997) paid special attention to the collision between a fractal floc and a small particle, and put forward the following formula to calculate frequency function, β_{frac}, of collision: $\beta_{frac} = e_{frac}\beta_{rec}$, where β_{frac} is a frequency function of collision between a fractal floc and a small particle, the meaning of β_{rec} has been explained as before, and e_{frac} is a correction coefficient. Some experiments have shown that there is a empirical relation $\beta_{frac} \sim G^{1-0.33D_f}$. Thus this expression will reduce to $\beta_{frac} \sim G$ when the floc is infinitely loose(that is $D_f = 0$). In this case, collision trajectory between the floc and the small particle should be rectilinear, and the model put forward by Li and Logan becomes to agree with the rectilinear trajectory model; while the floc is a compact sphere (that is $D_f \approx 3$), the expression reduces to the relation indicating β_{frac} has little relationship with G. In this case, collision trajectory between the floc and the small particle should be curvilinear, and thus Li and Logan's model presents to be in agreement with the curvilinear trajectory model. As an addition, some calculations have shown that β_{frac} is larger than the value from a curvilinear trajectory model by five orders of magnitude and smaller than that from the rectilinear one by two orders of magnitude, respectively.

2) Two expression forms of frequency function of collision In Smoluchowski equation, all of particles that can collide with the mentioned particle is regarded simply to be in a cylinder with its radius being equal to the collision radius $R_{collision}$ (a algebraic sum of radii of colliding particles), so this kind of expression form of frequency function of collision has been termed as cylinder formula. However, Saffman and Turner equation (1956) mentioned that all of particles that can collide with the mentioned particle should be on the surface of the sphere with its centre being the centroid of the mentioned particle and its radius being equal to the collision radius $R_{collision}$, and in contrast this kind of expression form has been termed as spherical formula. Two kinds of

expression forms can be shown as follows:

$$\beta_{ij} = \pi R^2_{collision} \langle u \rangle = \pi \left(r_i + r_j \right)^2 \langle u \rangle, \tag{17}$$

$$\beta_{ij}^{sph} = 2\pi R^2_{collision} \langle u_r \rangle = 2\pi \left(r_i + r_j \right)^2 \langle u_r \rangle, \tag{18}$$

where β_{ij}^{sph} is frequency function of collision between i, j-grade flocs corresponding to the spherical model, $\langle u \rangle$ is the statistically-averaged value of the relative velocity of two arbitrary points in the flow filed, whereas $\langle u_r \rangle$ is counterpart along the radial direction. Comparison of Equation (17) and Equation (18) can find that Equation (17) was larger than Equation (18) by more than twenty-five percents when the mentioned flow is homogeneously turbulent and by twenty percents when the flow is a laminar one, and that these equations were the same when a random motion or a simple gravity sedimentation is mentioned (Wang et al., 1998).

2.3. About the Third Assumption

2.3.1. Breakup of Floc

The porous and loose floc is prone to undergo breakup in a shear flow. It may be understood that breakup of the floc will happen as long as a shear stress, τ, exerted by the fluid on the floc exceeds strength of the floc, σ_T. Two kinds of models to describe breakup of the floc have been widely accepted (Muhle, 1993; Thomas et al., 1999). When size of the floc exceeds Kolmogorovf micro-scale, η, the floc should be in the inertial sub-range in a turbulent flow. In this case, the floc will undertake the action of fluctuating pressure of the turbulent flow, which may cause a large-scale fragmentation of the floc, and this kind of breakup has been termed as large-scale fragmentation. While for those flocs with their sizes being smaller than Kolmogorovf micro-scale, η, they should be in the viscous sub-range of the turbulent flow. In this case, the flocs bear the action of surface disruption of the turbulent flow, consequently making some small particles in the floc be easy to separate them from main part of the floc, and in contrast this kind of floc breakup has been termed as surface erosion.

2.3.2. Maximum Size of Floc (or Critical Size)

When size of the floc reaches the maximum, $d_{\max}$, the floc should be on the verge of its breakup. Many works have been carried out to discuss this critical condition corresponding to breakup of the floc, and three kinds of viewpoints can simply be summarized. (1)When a hydrodynamic stress, τ, exerted by fluid on the floc is equal to strength of the floc, σ_T, the mentioned floc is in the critical condition of its breakup; (2)When the force due to the flow shear, F_τ, is equal to comprehensive interactive force between primary particles in the floc, F, the floc will be in the critical condition of breakup; (3)When the external kinetic energy is equal to bonding energy of the floc, the floc is in the critical condition. On the first viewpoint, Muhle and Domasch (1990) and Lu et al. (1998) presented different formulae for calculating size of the maximum floc depending on different sub-ranges where the floc as expressed as follows:

$$\text{Viscous range:} \quad d_{\max} = 2\rho_w^{-\frac{1}{2}} \left(\varepsilon v \right)^{-\frac{1}{4}} F^{\frac{1}{2}};$$

$$\text{Transition range:} \quad d_{\max} = 1.5\rho_w^{-\frac{1}{2}} d_0^{-1} \varepsilon^{-\frac{1}{2}} v^{\frac{1}{2}} F^{\frac{1}{2}};$$

$$\text{Inertial range:} \quad d_{\max} = 0.68\rho_w^{-\frac{3}{5}} d_0^{-\frac{6}{5}} \varepsilon^{-\frac{11}{20}} v^{\frac{9}{20}} F^{\frac{3}{5}} \tag{19}$$

where F is the comprehensive interactive force between primary particles contained in the floc as explained above, and ρ_w is density of fluid. From the second viewpoint, Coufort et al. (2005; 2008) concluded that there was a simple expression representing the relation between size of the maximum floc and the turbulent intensity as follows

$$d_{\max} \sim \varepsilon^{-\frac{1}{4}}, \tag{20}$$

regardless of the fact that the floc may be in different sub-ranges. Based on the third viewpoint, Bache (2004) presented a more general mathematical expression between size of the maximum floc and the turbulent intensity

as follows

$$d_{\max} = c\varepsilon^{-\frac{m}{2}}, \tag{21}$$

where c, m are two positive constants. Similar works can also be found in Bouyer and Line (2004). On subs tituting Equation (7) into Equation (21), we can have $d_{\max} = c'G^{-m}$ or $\log d_{\max} = \log c' - m\log G$, where c' is a constant ($c' = cv^{-\frac{m}{2}}$). In this logarithmic mathematical expression, we can infer that the larger m coupled with the same G corresponds to the small $d_{\max}$. This means m may be a parameter to represent floc strength. Additionally, c' is also a parameter related to floc strength, and it may be closely related to the method for measuring size of the floc.

2.4. About the Fourth Assumption

Smoluchowski equation assumed that each collision between two particles can result in a subsequent adhesion between particles. This assumption means the so-called collision efficiency coefficient is equal to unity. The va-lidity of this assumption has been questioned by many researchers (e.g., Elimelech & O'Melia, 1990; Kim & Kramer, 2007). They stated that whether the collision between particles can lead to subsequent adhesion or not depended on many factors (such as the repulsive force of double-electrical layer on the surface of particle, spa-tial hindering interaction or hydrodynamic interaction between one particle and another particle, as well as viscous effect of fluid). These factors can be characterized by the collision efficiency coefficient, but at present the determination of this coefficient seems a difficult work.

It should be pointed out that hydrodynamic interaction between colliding particles may play a important role in influencing adhesion between particles, in particular when particles are in the turbulent flow. In fact, the in-fluence of hydrodynamic interaction on the collision efficiency coefficient can be analyzed in terms of the mov-ing trajectory of particle, and a detailed summary can be found in Section 2.2.2.

2.5. About the Fifth and Sixth Assumptions

Although the establishment of Smoluchowski equation is strictly based on the assumption of a mono-disperse flocculation system, it is also applicable to a poly-disperse flocculation system after introducing the size group method (Chang et al., 1992).

Regarding the sixth assumption behind Smoluchowski equation (i.e., the mutual collision is only between two particles), if number concentration of single particle in a flocculation system is low, the error caused by this as-sumption may be negligible; whereas for those flocculation systems in which number concentration of single particle is so high that collisions among three or more particles are not easily neglected, collision patterns of particles will be complex, and a simple usage of Smoluchowski equation will lead to some unexpected errors. At present, the research regarding this aspect have been rarely found (Chang et al., 1992).

As a summary, a complete dynamical equation describing the flocculation of particle incorporating breakup of the floc can be expressed as follows (Kim & Kramer, 2007):

$$\frac{dn_k}{dt} = \frac{1}{2}\sum_{i=1,i+j=k}^{k-1}\alpha_{ij}\beta_{ij}n_in_j - n_k\sum_{i=1}^{\max}\alpha_{ik}\beta_{ik}n_i - S_kn_k + \sum_{i=k+1}^{\max}\gamma_{ik}S_in_i, \tag{22}$$

where α_{ik}, β_{ik} are collision efficiency coefficient and collision frequency function of i, k-grade flocs re-spectively, S_k is a function of breakup of k-grade floc, γ_{ik} is distribution percent of k-grade floc in all of the flocs due to breakup of i-grade floc with its size being larger than k-grade floc. On the right-handed side of Equation (22), the first term represents the rate of increase of number concentration of k-grade floc due to flocculation of small flocs with their sizes being smaller than k-grade floc, the second term denotes the rate of decrease of number concentration of k-grade floc due to flocculation of k-grade floc with other flocs with different sizes, the third term showing the rate of decrease of number concentration of k-grade floc due to its breakup, and the last term presents the rate of increase of number concentration of k-grade floc due to breakup of those big flocs with their sizes being larger than k-grade floc. There are many research results regarding mathematical expressions of S_k, γ_{ik}, and two summaries can be found in studies of Han et al. (2003) and Kim and Kramer (2007). In fact, it will be difficult to obtain the analytical solution to Equation (22). Thus, some nu-

merical methods have commonly been adopted to deal with this equation, and some physical insights regarding flocculation of particle have been demonstrated in many researches(Higashitani & Iimura, 1998; Kramer & Clark, 1999; Runkana, 2003; Selomulya et al., 2003; Zhang & Li, 2003; Li et al., 2004; Prat & Ducoste, 2006; Coufort et al., 2007; Kim & Kramer, 2007).

3. Some Remarks

Most of researches on orthokinetic flocculation of particle have been carried out in the fields of colloidal chemistry, marine engineering, water treatment and the study of water environment. In contrast, some theoretical studies regarding orthokinetic flocculation of sediment particle have not been commonly found. Whether Smoluchowski equation can be directly applied to investigate flocculation phenomenon of sediment particle or not is worthy being studied. Here we attempt to put forward some viewpoints, hopefully providing the theoretical study of sediment orthokinetic flocculation with a few references.

1) **Particularity of flocculation of sediment** Smoluchowski equation dealt with the particle with single mineral components, a spherical structure, a single charge distribution on its surface and a uniform size. However, in real situations, properties of sediment particle are indeed complex, and sediment particle has diverse mineral components (for example, Illite, Kaoline, Montmorillonite), an irregular shape, and a complex distribution of surface charges (Charges distributed on the surface and edge of sediment particle are opposite). It will be questionable to apply Smoluchowski equation into the research field of sediment orthokinetic flocculation without any modifications (Yang et al., 2003).

2) **Difficulty of method for determining maximum size of the floc** It is widely recognized that size of the floc reaches the maximum only if the floc is in the critical condition of its breakup. However, it may be difficult to determine maximum size of the floc because of a lack of elaborate knowledge on the interactions among eddies in the turbulent flow and the flocs, which needs to be paid much attention in further research.

3) **Complexity of structure of floc** Introduction of fractal theory has tackled the problem that it is difficult to describe complex structure of the floc mathematically to some degree. But as Maggi et al. (2007) and Son and Hsu (2009) have pointed out by experiment and numerical simulation, fractal properties of sediment floc were not in a good agreement with fractal theory(for example, experiments shown that the fractal dimension of sediment floc was variable, which is contradictory to the conclusion from fractal theory that the fractal dimension should be constant for a given type of sediment). More detailed analysis on floc structure may be helpful to understand the process of orthokinetic flocculation of sediment particle.

4) **Difficulty of determination of collision efficiency coefficient** Determination of collision efficiency coefficient is of theoretical and practical importance in the discipline of particle flocculation. In the context of colloidal chemistry, this coefficient has been regarded as the reciprocal of stability ratio of coagulation, and hence it should closely be related to physicochemical characteristics of particle surface(Chang et al., 1992). In some published literatures, collision efficiency coefficient has been simply dealt with as a empirical constant, may due to a lack of effective methods to determine this coefficient. Furthermore, when DLVO theory in the discipline of colloidal chemistry (a detailed introduction can be found in Chang et al. (1992) is adopted to analyze the collision efficiency coefficient in the sediment flocculation, applicability of this theory for a micro-sized sediment particle needs to be taken into account.

Acknowledgements

References

Bache, D. H. (2004). Floc Rupture and Turbulence: A Framework for Analysis. *Chemical Engineering Science, 59,* 2521-2534. http://dx.doi.org/10.1016/j.ces.2004.01.055

Bouyer, D., & Line, A. (2004). Experimental Analysis of Floc Size Distribution under Different Hydrodynamics in a Mixing Tank. *AIChE Journal, 50,* 2064-2081. http://dx.doi.org/10.1002/aic.10242

Burban, P. Y., Lick, W, & Lick, J. (1989). The Flocculation of Fine-Grained Sediments in Estuarine Waters. *Journal of Geophysical Research, 94,* 8323-8330. http://dx.doi.org/10.1029/JC094iC06p08323

Camp, T. R., & Stein, P. C. (1943). Velocity Gradients and Internal Work in Fluid Motion. *Journal of Boston Society of Civil Engineering, 30,* 219-237.

Chang, Q., Fu, J., & Li, Z.(1992). *The Principal of Flocculation.* Lanzhou: Lanzhou University Press. (In Chinese)

Chen, H., Shao, M., & Li, Z. (2001). Preliminary Study on the Effect of NaCl on Fine Sediment Flocculation and Settling in Still Water. *Acta Pedologica Sinica, 1,* 131-134. (In Chinese)

Coufort, C., Bouyer, D., & Line, A. (2005). Flocculation Related to Local Hydrodynamics in a Taylor-Couette Reactor and in a Jar. *Chemical Engineering Science, 60,* 2179-2192. http://dx.doi.org/10.1016/j.ces.2004.10.038

Coufort, C., Bouyer, D., Line, A., & Haut, B. (2007). Modelling of Flocculation Using a Population Balance Equation. *Chemical Engineering and Processing, 46,* 1264-1273. http://dx.doi.org/10.1016/j.cep.2006.10.012

Coufort, C., Dumas, C., Bouyer, D., & Line, A. (2008). Analysis of Floc Size Distribution in a Mixing Tank. *Chemical Engineering and Processing: Process Intensification, 47,* 287-294. http://dx.doi.org/10.1016/j.cep.2007.01.009

Delichatsios, M. A., & Probstein, R. F. (1974). Coagulation in Turbulent Flow: Theory and Experiment. *Journal of Colloid and Interface Science, 51,* 394-405. http://dx.doi.org/10.1016/0021-9797(75)90135-6

Elimelech, M., & O'Melia, C. R. (1990). Effect of Partilce Size on Collision Efficiency in the Deposition of Brownian Particles with Electrostaic Energy Barriers. *Langmuir, 6,* 1153-1163. http://dx.doi.org/10.1021/la00096a023

Feder, J. (1988). *Fractals.* New York: Plenum. http://dx.doi.org/10.1007/978-1-4899-2124-6

Guan, X., & Chen, Y. (1995). Experimental Study on Dynamic Formula of Sand Coagulation Sinking in Stationary Water in Yangtze Estuary. *Ocean Engineering, 1,* 46-50. (In Chinese)

Guan, X., Chen, Y., & Du, X. (1996). Experimental Study on Mechanism of Flocculation in Yangtze Estuary. *Journal of Hydraulic Engineering, 6,* 70-80. (In Chinese)

Han, B., Akeprathumchai, S., Wickramasinghe, S. R., & Qian, S. (2003). Flocculation of Piological Cells: Experiment vs Theory. *AIChE Journal, 49,* 1687-1701. http://dx.doi.org/10.1002/aic.690490709

Han, M. Y., & Lawler, D. F. (1992). The Relative Insignificance of G in Flocculation. *American Water Works Association Journal, 84,* 79-91.

Higashitani, K., & Iimura, K. (1998). Two-Dimensional Simulation of Breakup Process of Aggregates in Shear and Elongational Flows. *Journal of Colloid and Interface Science, 204,* 320-327. http://dx.doi.org/10.1006/jcis.1998.5561

Jiang, G., & Zhang, Z. (1995). Flocculation Deposition of Fin-Grained Sediment and the Concentration of Cation in Yangtze Estuary. *Acta Oceanologica Sinica, 17,* 76-82. (In Chinese)

Jiang, G., Yao, Y., & Tang, Z. (2002). The Analysis of Factors of Flocculation of Fine-Grained Sediment in Yangtze Estuary. *Acta Oceanologica Sinica, 24,* 51-56. (In Chinese)

Jin, D., Wu, H., & Shi, F. (1998). Analysis of Sediment Flocculation. *Water Conservancy and Hydropower in Northeast China, 11,* 25-37. (in Chinese)

Jin, Y., Wang, Y., & Li, Y. (2002). Experimental Study on Flocculation of Cohesive Fine Grain Sediment in Yangtze River Estuary. *Journal of Hohai University (Natural Sciences), 30,* 61-63. (In Chinese)

Kim, J., & Kramer, T. A. (2007). Adjustable Discretized Population Balance Equations: Numerical Simulation and Parameters Estimation for Fractal Aggregation and Breakup. *Colloids and Surfaces A: Physicochemical and Engineering Aspects, 27,* 173-188. http://dx.doi.org/10.1016/j.colsurfa.2006.06.020

Kramer, T. A. (1997). *The Modeling of Coagulation Kinetics in Complex Laminar Flow.* Ph.D. Thesis, Illinois: University of Illinois at Urbana-Champaign.

Kramer, T. A., & Clark, M. A. (1997). Influence of Strain Rate on Coagulation Kinetics. *Journal of Environmental Engineering, 123,* 444-452. http://dx.doi.org/10.1061/(ASCE)0733-9372(1997)123:5(444)

Kramer, T. A., & Clark, M. M. (1999). Incorporation of Aggregate Breakup in the Simulation of Orthokinetic Coagulation. *Journal of Colloid and Interface Science, 216,* 116-126. http://dx.doi.org/10.1006/jcis.1999.6305

Kusters, K. A., Wijers, J. G., & Thoenes, D. (1997). Aggregation Kinetics of Small Particles in Agitated Vessels. *Chemical Engineering Science, 5,* 107-121. http://dx.doi.org/10.1016/S0009-2509(96)00375-2

Lartiges, B. S., Deneux-Mustin, S., Villemin, G., Mustin, C., Barrès, O., Chamerois, M., et al. (2001). Composition Structure and Size Distribution of Suspended Particulates from Rhine River. *Water Research, 135,* 808-816. http://dx.doi.org/10.1016/S0043-1354(00)00293-1

Li, D., Tan, W., & Huang, M. (2004). Study on Fractal Properties of Flocs. *Water and Wastewater Engineering, 30,* 5-9. (In Chinese)

Li, X. Y., Passow, U., & Logan, B. E. (1998). Fractal Dimensions of Small (15-200 μm) Particles in Eastern Pacific Coastal Waters. *Deep-Sea Research, 45,* 115-131. http://dx.doi.org/10.1016/S0967-0637(97)00058-7

Li, X., & Logan, B. (1997). Collision Frequencies between Fractal Aggregates and Small Particles in a Turbulently Sheared Fluid. *Environmental Science and Technology, 31,* 1237-1242. http://dx.doi.org/10.1021/es960772o

Li, X., Zhang, J., & Joseph Lee, H. W. (2004). Modelling Particle Size Distribution Dynamics in Marine Waters. *Water Research, 38,* 1305-1317. http://dx.doi.org/10.1016/j.watres.2003.11.010

Liu, Y. (1994). The Influence of Temperature on Setting Velocity and Siltation of Cohesive Sediment. *Express Water Resources & Hydropower Information, 13,* 21-24. (In Chinese)

Lu, S., Ding, Y., & Guo, J. (1998). Kinetics of Fine Particle Aggregation in Turbulence. *Advances in Colloid and Interface Science, 78,* 197-235. http://dx.doi.org/10.1016/S0001-8686(98)00062-1

Ma, K. S., & Pierre, A. C. (1995). Colloidal Behavior of Montmorillonite in the Presence of Fe^{3+} Ions. *Colloids and Surfaces A: Physicochemical and Engineering Aspects, 155,* 359-372. http://dx.doi.org/10.1016/S0927-7757(99)00032-1

Maggi, F., Mietta, F., & Winterwerp, J. C. (2007). Effect of Variable Fractal Dimension on the Floc Size Distribution of Suspended Cohesive Sediment. *Journal of Hydrology, 343,* 43-55. http://dx.doi.org/10.1016/j.jhydrol.2007.05.035

Meakin, P. (1988). Fractal Aggregation. *Advances in the Colloid Interface, 28,* 249-331. http://dx.doi.org/10.1016/0001-8686(87)80016-7

Muhle, E. K. (1993). Floc Stability in Laminar and Turbulent Floc. In B. Dobias (Ed.), *Coagulation and Flocculation* (pp. 355-390). New York: Marcel Dekker.

Muhle, E. K., & Domasch, K. (1990). Floc Strength in Bridging Flocculation. In H. H. Hahn, & R. Klute (Eds.), *Chemical water and Wastewater Treatment* (pp.106-115). Heidelberg: Springer-Verlag Berlin Heidelberg. http://dx.doi.org/10.1007/978-3-642-76093-8_8

Pedocchi, F., & Piedra-Cueva, I. (2005). Camp and Stein's Velocity Gradient Formalization. *Journal of Environmental Engineering, 131,* 1369-1376. http://dx.doi.org/10.1061/(ASCE)0733-9372(2005)131:10(1369)

Prat, O. P., & Ducoste, J. J. (2006). Modeling Spatial Distribution of Floc Size in Turbulent Processes Using the Quadrature Method of Moment and Computational Fluid Dynamics. *Chemical Engineering Science, 61,* 75-86. http://dx.doi.org/10.1016/j.ces.2004.11.070

Runkana, V. (2003). *Mathematical modeling of flocculation and dispersion of colloidal suspensions*. Ph.D. Thesis, New York: Columbia University.

Saffman, P. G., & Turner, J. S. (1956). On the Collisions of Drops in Turbulent Clouds. *Journal of Fluid Mechanics, 1,* 16-30. http://dx.doi.org/10.1017/S0022112056000020

Selomulya, C., Bushell, G., Amal, R., & Waite, T. D. (2003). Understanding the Role of Restructuring in Flocculation: The Application of a Population Balance Model. *Chemical Engineering Science, 58,* 327-338. http://dx.doi.org/10.1016/S0009-2509(02)00523-7

Shi, Z. (2000). Fine Sediment Processes in Yangtze River Estuary. *Journal of Sediment Research, 6,* 72-80. (in Chinese)

Smoluchowski, M. V. (1917). Versuch Einer Mathematischen Theorie der Koagulationskinetik kolloider Losungen. *Zeitschrift f. Physik. Chemie. XCII, 92,* 129-168.

Son, M., & Hsu, T. J. (2009). The Effect of Variable Yield Strength and Variable Fractal Dimension on Flocculation of Cohesive Sediment. *Water Research, 43,* 3582-3592. http://dx.doi.org/10.1016/j.watres.2009.05.016

Thill, A., Moustier, S., Aziz, J., Wiesner, M. R., & Bottero, J. Y. (2001). Flocs Restructuring during Aggregation: Experimental Evidence and Numerical Simulation. *Journal of Colloid and Interface Science, 243,* 171-182. http://dx.doi.org/10.1006/jcis.2001.7801

Thomas, D. N., Judd, S. J., & Fawcett, N. (1999). Flocculation Modeling: A Review. *Water Research, 33,* 1579-1592. http://dx.doi.org/10.1016/S0043-1354(98)00392-3

van de Ven, T. G. M., & Maso, S. G. (1977). The Microrheology of Colloidal Dispersions. VII. Orthokinetic Doublet Formation of Spheres. *Colloid and Polymer Science, 255,* 468-479. http://dx.doi.org/10.1007/BF01536463

Veerapaneni, S., & Wiesner, M. R. (1996). Hydrodynamics of Fractal Aggregates with Radially Varying Permeability. *Journal of Colloid and Interface Science, 177,* 45-57. http://dx.doi.org/10.1006/jcis.1996.0005

Wang, L., Wexler, A. S., & Zhou, Y. (1998). Statistical Mechanical Descriptions of Turbulent Coagulation. *Physic of Fluids, 10,* 2647-2651. http://dx.doi.org/10.1063/1.869777

Xia, Z. (1992). *Modern Hydraulics (Volume 3)*. Beijing: High Education Press. (In Chinese)

Yang, M., & Qian, N. (1986). The Effect of Turbulence on the Flocculation Structure of the Slurry of Fine-Grained Sediment. *Journal of Hydraulic Engineering, 8,* 21-30. (In Chinese)

Yang, T., Xiong X., Zhan X., & Yang, M. (2003). On Flocculation of Cohesive Fine Sediment. *Hydro-Science and Engineering, 2,* 65-77. (In Chinese)

Zhang, J., & Li, X. (2003). Modelling Particle-Size Distribution Dynamics in a Flooculation System. *AIChE Journal, 49,* 1870-1882. http://dx.doi.org/10.1002/aic.690490723

Zhang, Z. (1996). Studies on Basic Characteristics of Fine Sediment in Yangtze Estuary. *Journal of Sediment Research, 1,* 67-73. (In Chinese)