

Modifications in the Stress Field of a Long Inclined Fault Caused by the Welded-Contact Boundary Conditions across the Interface between Two Elastic Half-Spaces

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Abstract

In welded-contact boundary conditions, some stress components are not required to be continuous across the boundary between two elastic half-spaces. The purpose of this note is to study the modifications in the stress field of a long inclined strike-slip, dip-slip or tensile fault caused by the welded-contact boundary conditions across the interface between two elastic half-spaces. The Poisson's ratios of the two half-spaces do not appear in the stress field of a strike-slip fault. In the case of a dip-slip fault, the Poisson's ratio of the half-space in which the fault lies, has a significant influence on the stress field across the interface. However, for a tensile fault, the modification in the stress field is significantly affected by the Poisson's ratios of both the half-spaces.

Keywords: Dip-Slip Fault, Stress Field, Strike-Slip Fault, Tensile Fault, Two-Dimensional, Welded-Contact

1. Introduction

Elastic dislocation theory has been developed and applied for the mathematical and physical description of mechanics of earthquakes [1-4]. Extensive reviews of the application of the elastic dislocation theory to earthquake faulting problems have been given by Savage [5] and Rybicki [6]. Using this theory, stress field around earthquake faults can be calculated in a straightforward manner. Both two- and three-dimensional fault models have been used in the literature. Considering the fact that some of the faults are sufficiently long, the two-dimensional fault model is found to be adequate in many situations. Since the Earth is not homogeneous, it is necessary to consider the effect of internal boundaries on the stress field generated by earthquake faults. The knowledge of the modification of the stress field caused by internal boundaries is useful to study secondary faulting.

Bonafede and Rivalta [7] obtained a plane strain analytic solution for the displacement and stress fields produced by a long vertical tensile dislocation in the proximity of the interface between two elastic half-spaces in welded contact. After detailed numerical computations, they observed that the welded-contact boundary conditions between a hard half-space and a soft half-space are

responsible for major changes in the stress field. The stress component, which is not involved in the boundary conditions, is significantly higher along the hard side of the interface. In a subsequent paper, Bonafede and Rivalta [8] derived the corresponding solution for a long vertical tensile crack. They noted that the discontinuities in the elastic parameters across the boundary act as stress concentrators for the stress component not involved in the boundary conditions.

Rivalta *et al.* [9] provided a plane strain analytic solution for the displacement and stress fields induced by an edge dislocation in an elastic half-space in welded contact with another elastic half-space. They found unexpected differences in the normal component of stress parallel to the interface which is not required to be continuous at the interface by the boundary conditions. This stress component shows wide regions of high stress in the harder side of the interface. Stress concentration along the interface is particularly high when the rigidity contrast is high.

In a recent paper, Rybicki and Yamashita [10] derived formulas for two-dimensional anti-plane and in-plane problems relating stresses across a plane boundary between two elastic half-spaces in welded contact, assuming a homogeneous shear stress in one of the two half-spaces. They concluded that the mechanical conditions

related to faulting within the Earth's crust are expected to be favourable in the high rigidity media.

The purpose of this note is to study the modifications in the stress field of a two-dimensional strike-slip, dip-slip or tensile fault caused by the welded-contact boundary conditions across the interface between two elastic half-spaces.

2. Long Inclined Strike-Slip Fault

For a two-dimensional anti-plane strain problem with reference to the x_2x_3 -plane, the non-zero displacement and stress components are of the form

$$\begin{aligned} u_1 &= u_1(x_2, x_3) \\ \tau_{12} &= \mu \frac{\partial u_1}{\partial x_2} \quad \tau_{13} = \mu \frac{\partial u_1}{\partial x_3} \end{aligned} \quad (1)$$

where μ denotes the rigidity.

Consider two homogeneous, isotropic, elastic half-spaces welded along the plane $x_3 = 0$. Let x_3 -axis be drawn vertically downwards, μ_1 denote the rigidity of the upper half-space ($x_3 < 0$) and μ_2 denote the rigidity of the lower half-space ($x_3 > 0$). Rani and Singh [11] obtained the displacement and stress fields due to a long inclined strike-slip fault located in the lower half-space (**Figure 1**). The stress component τ_{13} is continuous across the interface ($x_3 = 0$) due to the welded-contact boundary conditions. At the interface $x_3 = 0$, the stress component τ_{12} is not continuous. At any point of the interface $x_3 = 0$, we have

$$\tau_{12}^{(1)} = \frac{\mu_1 b \sin \delta}{\pi(1+m)} \left[\frac{s_2}{R_2^2} - \frac{s_1}{R_1^2} \right] \quad (2)$$

for the upper half-space, and

$$\tau_{12}^{(2)} = \frac{\mu_2 b \sin \delta}{\pi(1+m)} \left[\frac{s_2}{R_2^2} - \frac{s_1}{R_1^2} \right] \quad (3)$$

for the lower half-space, where

$$\begin{aligned} m &= \frac{\mu_1}{\mu_2} \\ R_1^2 &= (x_2 - s_1 \cos \delta)^2 + s_1^2 \sin^2 \delta \\ R_2^2 &= (x_2 - s_2 \cos \delta)^2 + s_2^2 \sin^2 \delta \end{aligned} \quad (4)$$

δ is the dip angle, b is the fault-slip and s_1, s_2 are shown in **Figure 1**. Therefore, at any point of the interface,

$$K_1 = \frac{\tau_{12}^{(1)}}{\tau_{12}^{(2)}} = \frac{\mu_1}{\mu_2} \quad (5)$$

Equation (5) is similar to the result obtained by Rybicki and Yamashita [10] for a homogeneous anti-plane shear stress conditions in the lower half-space in welded contact with the upper half-space. We thus note that the stress ratio K_1 for a long inclined strike-slip fault is equal to the rigidity ratio. It is independent of the Poisson's ratios of the two half-spaces as well as of the dip angle δ of the fault.

3. Long Inclined Dip-Slip Fault

For a two-dimensional plane strain problem, the non-zero displacement and stress components are of the form

$$\begin{aligned} u_2 &= u_2(x_2, x_3), \quad u_3 = u_3(x_2, x_3) \\ \tau_{11} &= \lambda \theta \\ \tau_{22} &= \lambda \theta + 2\mu \frac{\partial u_2}{\partial x_2} \\ \tau_{33} &= \lambda \theta + 2\mu \frac{\partial u_3}{\partial x_3} \\ \tau_{23} &= \mu \left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2} \right) \end{aligned} \quad (6)$$

where λ, μ are the Lamè constants and

$$\theta = \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} \quad (7)$$

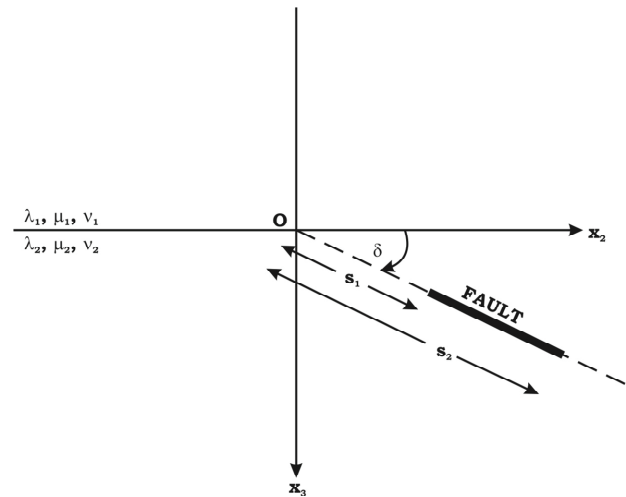


Figure 1. Geometry of a long fault lying in an elastic half-space in welded contact with another elastic half-space. The x_1 -axis is taken parallel to the length of the fault and x_3 -axis normal to the interface between the two half-spaces. δ is the dip angle and s_1, s_2 are the distances of the two edges of the fault from the origin.

Rani and Singh [12] derived closed-form analytic expressions for the displacements and stresses caused by a long inclined dip-slip fault located in an elastic half-space in welded contact with another elastic half-space. The stress components τ_{23} and τ_{33} are continuous across the interface as required by the welded-contact boundary conditions. In **Figure 1**, the origin is taken at the fault trace, that is at the point where the fault, if extended, meets the interface $x_3 = 0$. At the origin, the stress component τ_{22} is given by

$$\tau_{22}^{(1)} = \frac{a_2 \mu_2 b}{\pi} (1 + C_2) \sin 2\delta \left(\frac{1}{s_2} - \frac{1}{s_1} \right) \quad (8)$$

for the upper half-space, and

$$\tau_{22}^{(2)} = \frac{a_2 \mu_2 b}{\pi} (1 - 3C_2) \sin 2\delta \left(\frac{1}{s_2} - \frac{1}{s_1} \right) \quad (9)$$

for the lower half-space, where

$$a_2 = \frac{1}{2(1 - \nu_2)}$$

$$C_2 = \frac{m - 1}{1 - m + 2m / a_2} \quad (10)$$

ν_2 being the Poisson's ratio of the lower half-space. Therefore, at the origin,

$$K_2 = \frac{\tau_{22}^{(1)}}{\tau_{22}^{(2)}} = \frac{1 + C_2}{1 - 3C_2} = \frac{m(1 - \nu_2)}{1 - m\nu_2} = 1 - \frac{1 - m}{1 - m\nu_2} \quad (11)$$

We observe that the stress ratio K_2 for a long inclined dip-slip fault depends upon the rigidity contrast $m = \mu_1/\mu_2$ and the Poisson's ratio ν_2 of the lower half-space. It is independent of the Poisson's ratio ν_1 of the upper half-space as well as of the dip angle δ . From (11), if $m < 1$, then $K_2 < 1$. This implies that the stress near the interface generated in the upper half-space is less than the stress near the interface in the lower half-space in which the fault lies. On the other hand, if $1 < m < 1/\nu_2$, then $K_2 > 1$, implying that the stress generated near the interface in the upper half-space is more than the stress near the interface in the lower half-space. For $m = \nu_2^{-1}$, the stress $\tau_{22}^{(2)}$ at the origin vanishes.

It is interesting to compare (11) with the corresponding result of Rybicki and Yamashita [10] which, in our notation, is reproduced below:

$$K_{2R} = \frac{\tau_{22}^{(1)}}{\tau_{22}^{(2)}} = m + \nu_1 (m - 1)$$

where ν_1 is the Poisson's ratio of the upper half-space. The stress ratio K_{2R} derived by Rybicki and Yamashita

[10] depends upon the rigidity contrast m and the Poisson's ratio of the upper half-space. It is independent of the Poisson's ratio of the lower half-space. It may be noted that while Rybicki and Yamashita [10] considered a homogeneous shear stress acting on an inclined plane in the lower medium, we have considered the stress field of a long inclined dip-slip fault in the lower half-space.

Figure 2 shows the variation of K_2 with m for $\nu_2 = 0.1, 0.2, 0.3, 0.4$. Since K_2 is independent of ν_1 , for $m = 1$, we have $K_2 = 1$ corresponding to the case when the two half-spaces have identical elastic parameters. In the range $0 < m < 1$ (**Figure 2(a)**), we note that $K_2 < 1$ and, for a given m , K_2 decreases as ν_2 increases. The compressibility is given by $\frac{3(1 - 2\nu)}{2(1 + \nu)\mu}$. Therefore, in the

range $0 < m < 1$, for a given m , K_2 increases as the compressibility of the lower half-space increases. In contrast, in the range $1 < m < 2$ (**Figure 2(b)**), $K_2 > 1$ and, for a

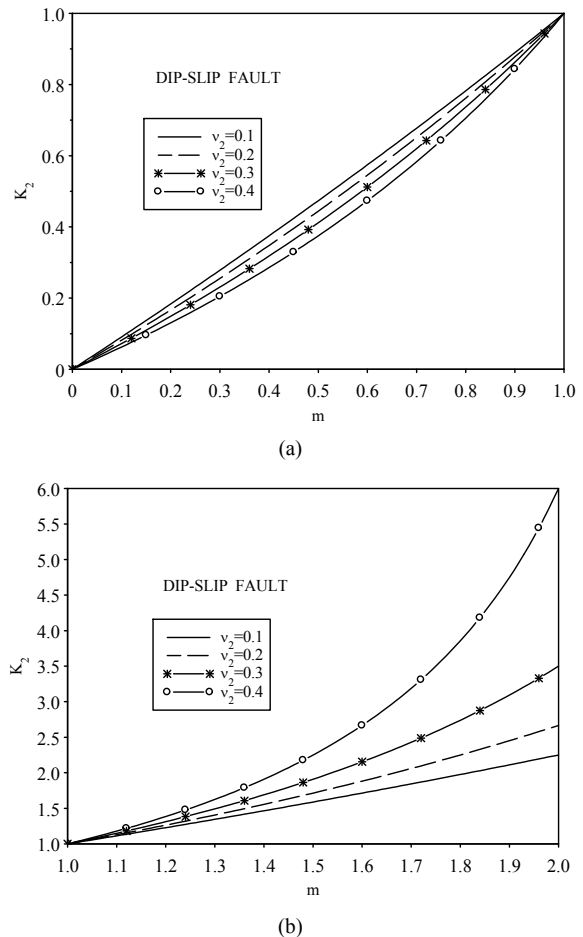


Figure 2. Variation of the stress ratio K_2 for a long dip-slip fault with the rigidity ratio $m = \mu_1/\mu_2$ for four values of the Poisson's ratio ν_2 of the lower half-space for (a) $0 \leq m \leq 1$; (b) $1 \leq m \leq 2$.

given m , K_2 decreases as the compressibility of the lower half-space increases. Thus the most favourable elastic conditions for the generation of higher stresses in the upper half-space (near the interface) occur when the fault lies in the softer half-space with small compressibility.

4. Long Inclined Tensile Fault

This is also a plane strain problem. Kumar *et al.* [13] obtained closed-form, analytic expressions for the displacements and stresses for a long inclined tensile fault situated in an elastic half-space in welded contact with another elastic half-space. At the origin, the stress component τ_{22} is given by

$$\tau_{22}^{(1)} = \frac{a_2 \mu_2 b}{2\pi} (1 + 3C_1 + C_2) \left(\frac{1}{s_2} - \frac{1}{s_1} \right) \quad (12)$$

for the upper half-space, and

$$\tau_{22}^{(2)} = -\frac{a_2 \mu_2 b}{2\pi} (3 + C_1 + 3C_2) \left(\frac{1}{s_2} - \frac{1}{s_1} \right) \quad (13)$$

for the lower half-space, where

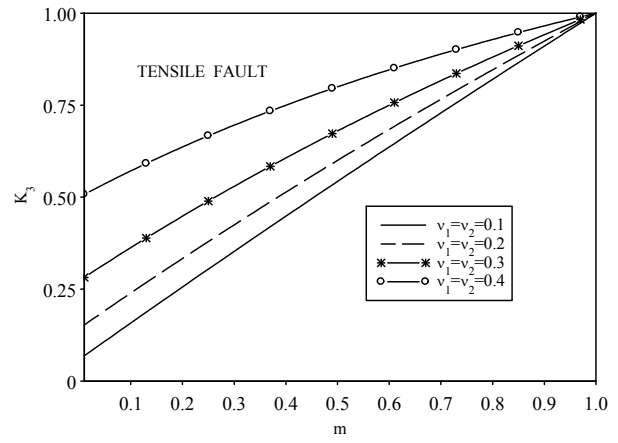
$$C_1 = \frac{2m}{a_2 (1 - m - 2/a_1)}, \quad a_1 = \frac{1}{2(1 - \nu_1)} \quad (14)$$

ν_1 being the Poisson's ratio in the upper half-space. Therefore, at the origin,

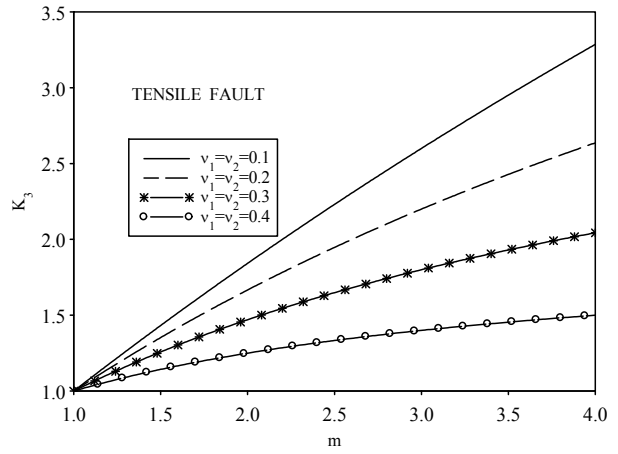
$$K_3 = \frac{\tau_{22}^{(1)}}{\tau_{22}^{(2)}} = -\frac{1 + 3C_1 + C_2}{3 + C_1 + 3C_2} = \frac{2m + \nu_1 - 3m\nu_2}{2 - 3\nu_1 + m\nu_2} \quad (15)$$

It may be noted that the stress ratio K_3 for a long inclined tensile fault depends upon the rigidity ratio m and the Poisson's ratios ν_1, ν_2 . It is independent of the dip angle δ .

Figure 3 shows the variation of K_3 with m for $\nu_1 = \nu_2 = 0.1, 0.2, 0.3, 0.4$. We note that, $K_3 < 1$ for $0 < m < 1$ (**Figure 3(a)**) and $K_3 > 1$ for $1 < m < 4$ (**Figure 3(b)**). When $m = 1$, the two half-spaces have identical properties and $K_3 = 1$. For a given ν_1 , the variation of K_3 with m for $\nu_2 = 0.1, 0.2, 0.3, 0.4$ is shown in **Figure 4**. In this case, $m = 1$ does not imply that the two half-spaces are identical because of the difference in the values of ν_1 and ν_2 . Here, we note that for a given m , K_3 increases as the compressibility of the lower half-space increases. For a given ν_2 , the variation of K_3 with m for $\nu_1 = 0.1, 0.2, 0.3, 0.4$ is shown in **Figure 5**. For a given m , K_3 increases as the compressibility of the upper half-space decreases. **Figure 6** shows the variation of K_3 with ν_2 for a given value of ν_1 . In contrast, **Figure 7** shows the variation of K_3 with ν_1 for a given value of ν_2 . Since, for an elastic material of given rigidity, the compressibility decreases as the Poisson's ratio increases, **Figures 6 and 7** taken together indicate that the most favourable elastic condi-



(a)



(b)

Figure 3. Variation of the stress ratio K_3 for a long tensile fault with the rigidity ratio m for (a) $0 \leq m \leq 1$; (b) $1 \leq m \leq 4$.

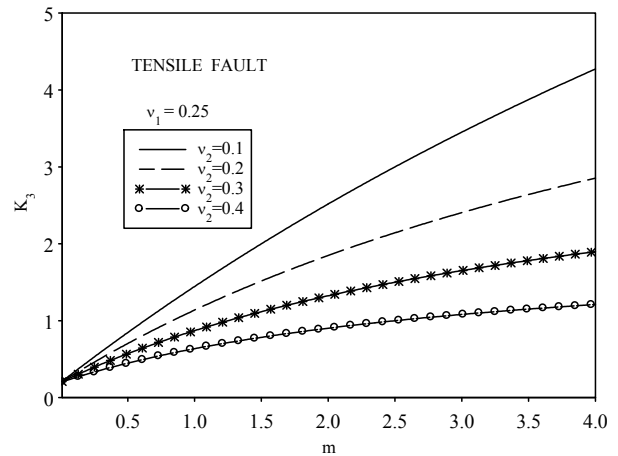


Figure 4. Showing the effect of the Poisson's ratio ν_2 of the lower half-space on the variation of the stress ratio K_3 for a tensile fault with the rigidity ratio m .

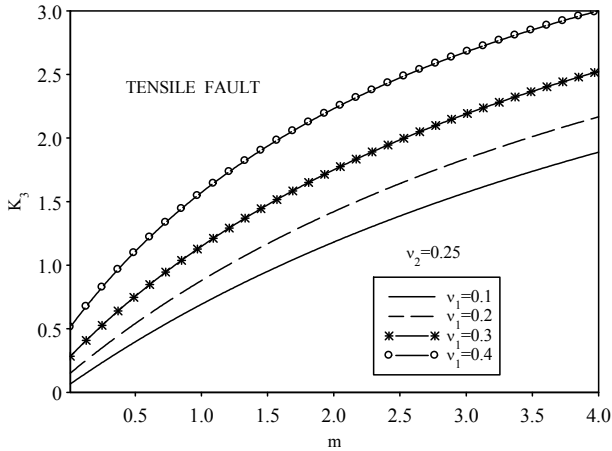


Figure 5. Showing the effect of the Poisson's ratio v_1 of the upper half-space on the variation of the stress ratio K_3 for a tensile fault with the rigidity ratio m .

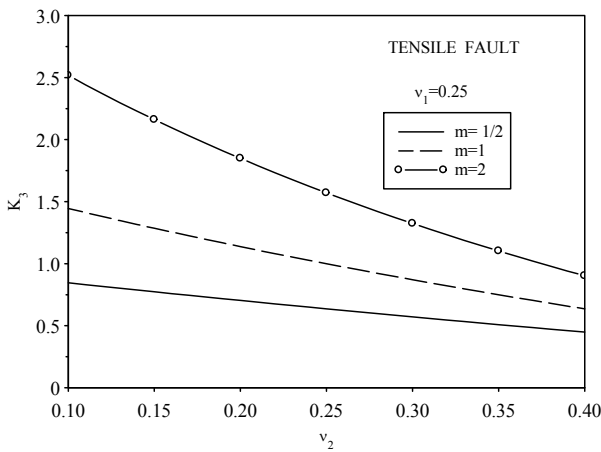


Figure 6. Variation of the stress ratio K_3 for a tensile fault with the Poisson's ratio v_2 of the lower half-space.

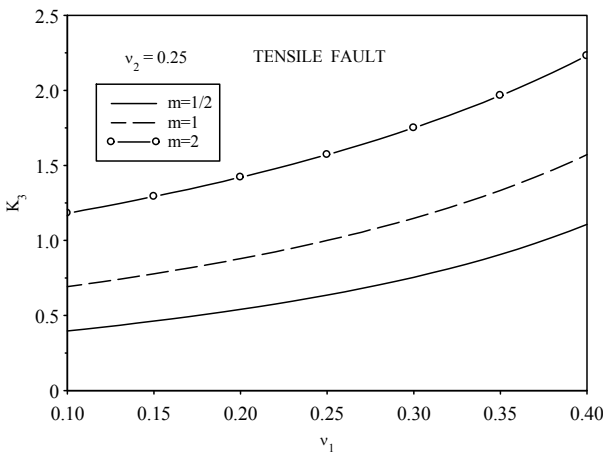


Figure 7. Variation of the stress ratio K_3 for a tensile fault with the Poisson's ratio v_1 of the upper half-space.

tions for generation of high stresses in the upper half-space (near the interface) occur when the tensile fault lies in the softer half-space, the compressibility of the upper half is least and the compressibility of the lower half-space (in which the fault lies) is greatest.

5. Conclusions

In general, when studying the static field in two elastic half-spaces in contact with source in one of the half-spaces, it is assumed that the half-spaces are in welded contact. In welded-contact boundary conditions, the displacement vector and the stress vector across the boundary are assumed to be continuous at the boundary. Consequently, some of the stress components are not required to be continuous at the boundary. We have studied the modifications in the stress field of a two-dimensional inclined strike-slip, dip-slip or tensile fault caused by the welded-contact boundary conditions across the boundary between the two elastic half-spaces. The results of the study can be summarised as under. It is assumed that the boundary between the half-spaces is taken as the $x_3 = 0$ plane and the fault is striking in the x_1 -direction.

1) In the case of a long inclined strike-slip fault, the shear stress τ_{13} is required to be continuous across the interface by the boundary conditions. However, there is no restriction on the shear stress τ_{12} . We find that the ratio of the values of the shear stress τ_{12} at the interface when approached from the two half-spaces is equal to the rigidity ratio. It is independent of the dip angle of the fault or the Poisson's ratios of the two half-spaces.

2) In the case of a long inclined dip-slip fault, the normal stress τ_{33} and the shear stress τ_{13} are required to be continuous. There is no restriction on the normal stresses τ_{11} and τ_{22} . We find that the ratio of the normal stress τ_{22} at the interface when approached from the two half-spaces depends on the rigidity ratio and the Poisson's ratio of the half-space in which the fault lies. It is independent of the Poisson's ratio of the other half-space as well as of the dip angle of the fault. The most favourable elastic conditions for generation of large stresses near the interface in the other half-space occur when the dip-slip fault lies in the softer half-space with small compressibility.

3) In the case of a long inclined tensile fault also, τ_{33} and τ_{13} are continuous. We find that the ratio of the normal stress τ_{22} at the interface depends upon the rigidity ratio as well as upon the Poisson's ratios of the two half-spaces. It is independent of the dip angle. The most favourable elastic conditions for generation of large stresses near the interface in the half-space, in which the fault does not lie, occur when the tensile fault lies in the

softer half-space, the compressibility of the half-space with fault is large and the compressibility of the other half-space is small.

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