

Simulation on Calculation Accuracy of Three Methods for Live Line Measuring the Parameters of Transmission Lines with Mutual Inductance

Jianjun Su¹, Ronghua Zhang¹, Demin Cui¹, Yongqiang Chai¹, Xiaobo Li¹,
Chengxue Zhang², Peiyan Li², Zhijian Hu², Yingying Hu²

¹Dezhou Power Supply Company, Shandong Electric Power Group Co., Dezhou, China

²School of Electrical Engineering, Wuhan University, Wuhan, China

Email: cxzhang@whu.edu.cn

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ABSTRACT

Live line measurement methods can reduce the loss of power outages and eliminate interference. There are three live line measurement methods including integral method, differential method and algebraic method. A simulation model of two coupled parallel transmission lines spanning on the same towers is built in PSCAD and the calculation errors of these three methods are compared with different sampling frequencies by using of Matlab. The effect of harmonic on calculation is also involved. The simulation results indicate that harmonic has the least effect on the algebraic method which provides stable result and small error.

Keywords: Lines with Mutual Inductance; Zero Sequence Parameters; Live Line Measurement; Algebraic Method; Differential Method; Integral Method

1. Introduction

With the development of power system and limitation of transmission line corridor, the number of lines with mutual inductance increases. Zero sequence parameters of the lines, which include zero sequence self-impedance and mutual impedance, are important basis of relay setting so that the parameters' precision has a significant effect on power system's safe operation [1]. These parameters are mainly influenced by earthing resistance rate. Chinese relay rules specify that zero sequence parameters of lines belong to 110kV and higher voltage levels must be measured [2, 3]. In the methods of live line measurement [2], there are two approaches to avoid the disadvantage that all the lines to be measured should be shut down? First, shut down one of the lines and add an external power source. Second, generate big zero sequence current in the way that open one phase breaker of an operating line (about 0.5 seconds) by the protective relay, re-close the phase breaker automatically to restore normal operation. An over determined equation set used for calculating the parameters are obtained under different measurement modes. The set is solved by using least square method. There are three live line measurement methods, including integral method, differential method and algebraic method [4-8]. Data that algebraic method needs is sampled in a period, while several successive

sampling points are needed by integral method and differential method [9-11]. This paper simulates all these three methods in different sampling frequencies, with and without harmonic, and analyses the measurement errors. The conclusion can help to choose a proper measurement method.

2. The Three Measurement Methods

2.1. Algebraic Method

The model of n transmission lines with mutual inductance is shown in **Figure 1**.

Where Z_{ii} are the zero sequence self-impedances of the lines, and Z_{ij} ($i \neq j$) are the mutual impedances. While the zero sequence current increment is generated on a line, the other lines coupled with it will induct zero

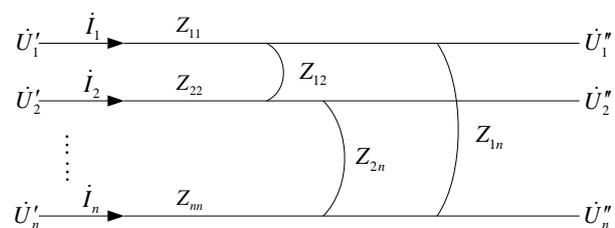


Figure 1. The model of transmission lines with mutual inductances.

sequence current increment $\Delta \dot{I}_i$ and zero sequence voltage increment $\Delta \dot{U}_i$. The voltage-current characteristic of the lines with mutual inductance is described in Equation (1).

$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1i} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2i} & \cdots & Z_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ Z_{i1} & Z_{i2} & \cdots & Z_{ii} & \cdots & Z_{in} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ Z_{n1} & Z_{n2} & \cdots & Z_{ni} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} \Delta \dot{I}_1 \\ \Delta \dot{I}_2 \\ \cdots \\ \Delta \dot{I}_i \\ \cdots \\ \Delta \dot{I}_n \end{bmatrix} = \begin{bmatrix} \Delta \dot{U}_1 \\ \Delta \dot{U}_2 \\ \cdots \\ \Delta \dot{U}_i \\ \cdots \\ \Delta \dot{U}_n \end{bmatrix} \quad (1)$$

Simplify Equation (1) as:

$$Z \Delta \dot{I} = \Delta \dot{U} \quad (2)$$

where Z is the zero sequence impedance matrix, $\Delta \dot{I}$ and $\Delta \dot{U}$ are the increment vector of zero sequence currents and voltages of all lines.

The increments can be produced by adding large enough current on a shutdown line while the other lines are on operation. Different equations produced by different measurement modes form the over determined equation set. The set is solved through least square method.

The algebraic method excludes the influence of zero sequence voltage and current existed in the lines by using increment of voltage and current. The algebraic method needs at least half period sampling points. The algebraic method's accuracy increases by eliminating harmonic through the Fourier method.

2.2. Differential Method

The model of n transmission lines with mutual inductance is shown in **Figure 2**.

Where R_{ii} and L_{ii} are the zero sequence self-resistance and self-inductance of the i -th line, R_{ij} and L_{ij} are the zero sequence mutual resistance and inductance between the i -th and the j -th line ($i, j = 1, 2, \dots, n, i \neq j$), i_i is the instantaneous value of the i -th line's zero sequence current, u_i' and u_i'' are the instantaneous values of zero sequence voltage of the i -th line's head and end separately, u_i is The instantaneous value of the i -th

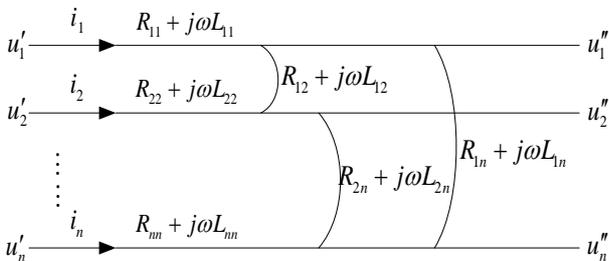


Figure 2. The model of transmission lines with mutual inductances by differential method.

line's zero sequence voltage difference, which $u_i = u_i' - u_i''$. Equation set of the differential method is described in Equation (3).

$$\begin{cases} i_1 R_{11} + L_{11} \frac{di_1}{dt} + i_2 R_{12} + L_{12} \frac{di_2}{dt} \\ + \cdots + i_n R_{1n} + L_{1n} \frac{di_n}{dt} = u_1 \\ i_1 R_{12} + L_{12} \frac{di_1}{dt} + i_2 R_{22} + L_{22} \frac{di_2}{dt} \\ + \cdots + i_n R_{2n} + L_{2n} \frac{di_n}{dt} = u_2 \\ \cdots \\ i_1 R_{1n} + L_{1n} \frac{di_1}{dt} + i_2 R_{2n} + L_{2n} \frac{di_2}{dt} \\ + \cdots + i_n R_{nn} + L_{nn} \frac{di_n}{dt} = u_n \end{cases} \quad (3)$$

where $i_i(k-1)$, $i_i(k)$, $i_i(k+1)$ and $u_i(k-1)$, $u_i(k)$, $u_i(k+1)$ are separately the zero sequence current and voltage of three successive sampling points. Equation (4) is the matrix form of Equation (3), and is discretized in the way of replacing the derivative terms di_i/dt by $[i_i(k+1) - i_i(k-1)]/2T_s$.

$$\begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1n} \\ R_{12} & R_{22} & \cdots & R_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ R_{1n} & R_{2n} & \cdots & R_{nn} \end{bmatrix} \begin{bmatrix} i_1(k) \\ i_2(k) \\ \vdots \\ i_n(k) \end{bmatrix} + \begin{bmatrix} L_{11} & L_{12} & \cdots & L_{1n} \\ L_{12} & L_{22} & \cdots & L_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ L_{1n} & L_{2n} & \cdots & L_{nn} \end{bmatrix} \times \begin{bmatrix} [i_1(k+1) - i_1(k-1)]/2T \\ [i_2(k+1) - i_2(k-1)]/2T \\ \vdots \\ [i_n(k+1) - i_n(k-1)]/2T \end{bmatrix} = \begin{bmatrix} u_1(k) \\ u_2(k) \\ \vdots \\ u_n(k) \end{bmatrix} \quad (4)$$

An equation can be achieved with any three successive sampling points. Parameters of the lines can be solved from the over determined equation set obtained through different measurement modes.

For only three sampling points needed in the differential method, much more equations can be obtained by sampling a series successive points. Different equation sets can be obtained by sampling different series of points. The accuracy of differential method can be enhanced by averaging the results solved from these sets.

2.3. Integral Method

Equation (5) is the integral equation set of the live line measurement. It is formed in the way of replacing the

derivative terms in Equation (3) by integral terms.

$$\left\{ \begin{aligned} &R_{11} \int_{t_k}^{t_{k+1}} i_1 dt + L_{11} [i_1(t_{k+1}) - i_1(t_k)] + \dots \\ &+ R_{1n} \int_{t_k}^{t_{k+1}} i_n dt + L_{1n} [i_n(t_{k+1}) - i_n(t_k)] = \int_{t_k}^{t_{k+1}} u_1 \\ &R_{12} \int_{t_k}^{t_{k+1}} i_1 dt + L_{12} [i_1(t_{k+1}) - i_1(t_k)] + \dots \\ &+ R_{2n} \int_{t_k}^{t_{k+1}} i_n dt + L_{2n} [i_n(t_{k+1}) - i_n(t_k)] = \int_{t_k}^{t_{k+1}} u_2 \\ &\dots \\ &R_{1n} \int_{t_k}^{t_{k+1}} i_1 dt + L_{1n} [i_1(t_{k+1}) - i_1(t_k)] + \dots \\ &+ R_{mn} \int_{t_k}^{t_{k+1}} i_n dt + L_{mn} [i_n(t_{k+1}) - i_n(t_k)] = \int_{t_k}^{t_{k+1}} u_n \end{aligned} \right. \quad (5)$$

Trapezoidal rule is used to calculate the integral value approximately. Therefore, Equation (5) is transformed into Equation (6).

Where T_s is the sampling period? Only two sampling points are needed. Much more equations can be obtained by sampling a series of successive points. Therefore, the accuracy of integral method will be enhanced.

$$\begin{aligned} &\begin{bmatrix} R_{11} & R_{12} & \dots & R_{1n} \\ R_{12} & R_{22} & \dots & R_{2n} \\ \vdots & \vdots & \dots & \vdots \\ R_{1n} & R_{2n} & \dots & R_{nn} \end{bmatrix} \begin{bmatrix} [i_1(k+1) + i_1(k)]T_s/2 \\ [i_2(k+1) + i_2(k)]T_s/2 \\ \vdots \\ [i_n(k+1) + i_n(k)]T_s/2 \end{bmatrix} \\ &+ \begin{bmatrix} L_{11} & L_{12} & \dots & L_{1n} \\ L_{12} & L_{22} & \dots & L_{2n} \\ \vdots & \vdots & \dots & \vdots \\ L_{1n} & L_{2n} & \dots & L_{nn} \end{bmatrix} \times \begin{bmatrix} i_1(k+1) - i_1(k) \\ i_2(k+1) - i_2(k) \\ \vdots \\ i_n(k+1) - i_n(k) \end{bmatrix} \quad (6) \\ &= \begin{bmatrix} [u_1(k+1) + u_1(k)]T_s/2 \\ [u_2(k+1) + u_2(k)]T_s/2 \\ \vdots \\ [u_n(k+1) + u_n(k)]T_s/2 \end{bmatrix} \end{aligned}$$

The influence of distributed capacitance is ignored in all these three methods.

3. PSCAD Simulation Model

A simulation model built in PSCAD is shown in **Figure 3**. In the model, there are two coupled parallel transmission lines spanning on the same towers. The lengths of the lines are both 50 km. All the lines are shut down and connected with an external zero sequence power sources in turn where L1 is the line that operates normally. The head end is connected with a 110 kV three-phase power source. The tail end is connected with 50 MW active load and 10 Mvar reactive load. Tail end of L2 is three-phase connected and grounding. L2's head end is three-phase

connected and an external voltage source is applied with.

PSCAD describes the line's characteristic in RLC mode. Transmission line is represented by the Bergeron model which separates the line into several distributed π type modules. This model assumes that the line's self-impedance and mutual impedance per unit length is constant and frequency-independent. The parameters per unit length in RLC mode are shown in **Figure 4**.

The reference values of zero sequence impedances are obtained according to the input parameters. The self-impedances of L1 and L2 are $8.479 + j66.385 \Omega$, and their module values are 66.920Ω . The mutual impedance between L1 and L2 is $6.750 + j34.500 \Omega$, and its module value is 35.154Ω .

4. Calculation Result and Error Analysis

In the PSCAD model, an external zero sequence power source is connected to the shutdown line. There are two types of the source. Type 1 only outputs fundamental voltage while type 2 outputs both fundamental and harmonic voltage. This section illustrates the influence and analyzes the errors of the both types.

4.1. Type 1

The output voltage of type 1 is 1 kV. Simulation lasts 0.5 s. Data sampling begins at 0.4 s. Data of a whole period is used by algebraic method. Several successive sampling points are used by differential method and integral method separately. The results of differential method and integral method are achieved in the way of averaging the measurement results. **Table 1** shows the lines' self-impedance and mutual impedance calculated through three

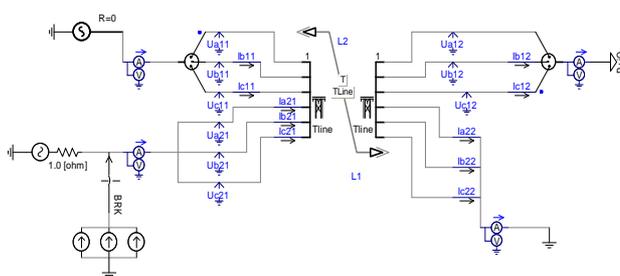


Figure 3. PSCAD simulation model.

Manual Entry of Y,Z

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+ve Sequence R: 0.36294e-4 [ohm/m]
+ve Sequence XL: 0.5031e-3 [ohm/m]
+ve Sequence XC: 302.151 [Mohm*m]
0 Sequence R: 0.16958e-3 [ohm/m]
0 Sequence XL: 0.13277e-2 [ohm/m]
0 Sequence XC: 1590.33 [Mohm*m]
0 Sequence Mutual R: 0.135e-3 [ohm/m]
0 Sequence Mutual XL: 0.069e-2 [ohm/m]
0 Sequence Mutual XC: 5056.0 [Mohm*m]
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Figure 4. The reference values of the lines' parameters.

methods and their errors under different sampling frequencies. Errors are calculated by

$$error = \frac{Z_c - Z_r}{Z_r} \times 100\%$$

where Z_c is the calculated value and Z_r is the reference value.

The effect of distribution capacitance is included in errors. **Table 1** indicates that the errors of algebraic method are the smallest. Errors of the other two methods get bigger as the sampling frequency decreases. Curves of errors changing with sampling frequency are shown in **Figure 5**.

4.2. Type 2

The voltage output by Type 2 contains 8% 3rd, 5% 5th and 5% 7th harmonic. **Table 2** shows the lines' zero sequence parameters calculated with the three methods and their errors under different sampling frequencies.

Table 2 indicates that harmonic has no influence on

algebraic method. Errors of the other two methods deteriorate apparently compared with the ones without harmonic, and get bigger as the sampling frequency decreases. Curves of errors changing with sampling frequency are shown in **Figure 5**.

4.3. Error Analysis

The simulation model contains two 50 km lines. All the three methods ignore the influence of distributed capacitance. Therefore, the errors caused by distributed capacitance are contained in the results [8].

The algebraic method utilizes the data of a period. Calculated after Fourier filtering, the algebraic method is not affected by harmonic. Therefore, it has the highest accuracy. And its error gets bigger as the sampling frequency decreases. In differential method, principle error exits due to using $[i_i(k+1) - i_i(k-1)]/2T_s$ to approximate di_i/dt . In integral method, principle error exits due to using trapezoid area to approximate integral value. Errors of the two methods both increase as the sampling frequency decreases.

Table 1. The calculated values with power source type 1.

Measurement Method	Zero Sequence Impedance(Ω)	Sampling Frequency							
		1 kHz		2 kHz		5 kHz		10 kHz	
		Z	Error (%)	Z	Error (%)	Z	Error (%)	Z	Error (%)
Algebraic Method	Self-Impedance	66.972	0.071	66.972	0.071	66.970	0.068	66.970	0.068
	Mutual Impedance	35.179	0.072	35.180	0.072	35.180	0.072	35.180	0.072
Differential Method	Self-Impedance	68.171	1.725	67.237	0.468	67.016	0.138	66.985	0.090
	Mutual Impedance	35.734	1.650	35.333	0.510	35.212	0.168	35.185	0.075
Integral Method	Self-Impedance	66.427	-0.742	66.832	-0.137	66.975	0.076	66.974	0.074
	Mutual Impedance	34.900	-0.718	35.1175	-0.104	35.181	0.076	35.181	0.075

Table 2. The calculated values with power source type 2.

Measurement Method	Zero Sequence Impedance(Ω)	Sampling Frequency							
		1 kHz		2 kHz		5 kHz		10 kHz	
		Z	Error (%)	Z	Error (%)	Z	Error (%)	Z	Error (%)
Algebraic Method	Self-Impedance	66.972	0.072	66.972	0.071	66.970	0.068	66.970	0.068
	Mutual Impedance	35.179	0.072	35.180	0.072	35.180	0.072	35.180	0.072
Differential Method	Self-Impedance	68.548	2.427	67.536	0.914	67.116	0.286	67.018	0.141
	Mutual Impedance	36.008	2.430	35.438	0.809	35.247	0.265	35.213	0.168
Integral Method	Self-Impedance	66.164	-1.136	66.733	-0.285	66.987	0.094	66.986	0.092
	Mutual Impedance	34.800	-1.006	35.083	-0.204	35.183	0.081	35.182	0.080

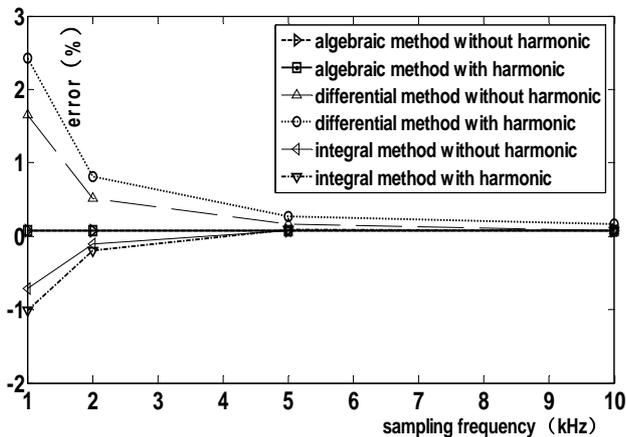


Figure 5. Curves of errors changing with sampling frequency.

As the lines is short and the results of both differential and integral methods are achieved in the way of averaging three results of measurement, the errors of all the three methods are less than 0.5% in 5 kHz sampling frequency. The algebraic method is the most accurate one.

5. Conclusions

A simulation model of two double-circuit lines spanning on the same towers is built in PSCAD. The zero sequence self-impedance and mutual impedance of the lines are calculated through the algebraic method, differential method and integral method. Errors of these methods are analyzed in two conditions that the external power source with or without harmonics. Principle errors exist due to the approximate calculation in the differential and integral methods. Therefore, errors increase along with the decreasing of sampling frequency. Moreover, errors rise when harmonic is involved. Owing to the filter characteristic of Fourier algorithm, the algebraic method has the highest accuracy; the algebraic method is preferred in actual engineering application.

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