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Using Universal Line Model (ULM) for Representing Three-phase Lines

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ABSTRACT

The second-order differential equations that describe the transmission line are difficult to solve due to the mutual coupling among phases and the fact that the parameters are distributed along their length. A method for the analysis of polyphase systems is the technique that decouples their phases. Thus, a system that has n phases coupled can be represented by n decoupled single-phase systems which are mathematically identical to the original system. Once obtained the n-phase circuit, it's possible to calculate the voltages and currents at any point on the line using computational methods. The *Universal Line Model* (ULM) transforms the differential equations in the time domain to algebraic equations in the frequency domain, solve them and obtain the solution in the frequency domain using the inverse Laplace transform. This work will analyze the method of modal decomposition in a three-phase transmission line for the calculation of voltages and currents of the line during the energizing process.

Keywords: Electromagnetic Transients; Transmission Lines; Modal Decomposition; Distributed Parameters

1. Introduction

The second-order differential equations describing a polyphase transmission line are difficult to solve due to coupling among the phases. An important method for the analysis of polyphase systems is the technique that decouples the phases of the line. Thus, a system that has n phases coupled can be represented by n decoupled single-phase systems which are mathematically identical to the original system [1, 2]. For a generic polyphase system, the matrix of the eigenvectors of matrix product [Z][Y] decouples the phases of the transmission line. There is for a single product [Z] [Y], several sets of eigenvectors to decouple the line. It´s has known two types of transformation to modal decomposition. The first is a transformation that separates the line in its exact modes, and the second is a transformation that separates the line in its quasi-modes, using the Clarke´s matrix transformation, which can decouple polyphase system in n single-phase systems. The exact modes are completely decoupled from each other and are obtained from the use of matrices [TI] and [TV] as the transformation matrices.

The exact modes are completely decoupled from each other and they are obtained from the use of matrices $[T_I]$ and $[T_V]$ as the transformation matrices. The matrices $[T_I]$ and $[T_V]$ are the eigenvectors associated with the products [Y] [Z] and [Z] [Y], respectively, and, in general, complex matrices, whose elements are frequency dependents. The quasi-modes are obtained from the use of Clarke´s matrix as the only matrix transformation. The Clarke's matrix is a real and constant matrix, whose elements are frequency independents, easy to implement in software that performs simulations directly in the time domain. If the transmission line is ideally transposed Clarke's matrix it decomposes in their exact modes. However, if the line is ideally transposed, but has a vertical symmetry, the Clarke´s matrix separate the line in their quasi-modes can, in some situations, be considered identical to the exact modes. This paper describes the process of decomposition of the line in their quasi-modes.

2. Quasi-modes of Transmission Lines

When the transmission line is ideally transposed Clarke's matrix separates the line in its exact modes. In cases in which a line cannot be considered ideally transposed, but has a vertical symmetry plane, it's possible with some approximations; use the Clarke´s matrix to determine the exact modes. Under these conditions it can obtained the line decomposed into their quasi-modes. For lines represented in theirs quasi-modes, the matrices [Yqm] and [Zqm] have some nonzero elements outside the main diagonal, which will be disregarded. Due to the fact that these matrices are not diagonal matrices, do not get the exact modes of the line, but in their quasi-modes [1,2].

Considering the three-phase line, transposed or not, the exact modes can be considered almost equivalent to the mode-alpha, beta and zero, respectively. The Clarke´s matrix $[T_{clarke}]$ is expressed as according to (1):

$$\left[T_{clarke}\right]=\begin{bmatrix}\frac{2}{\sqrt{6}} & 0 & \frac{1}{\sqrt{3}}\\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}}\\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}}\end{bmatrix} \quad (1)$$

The impedance and admittance's matrix of quasi-line modes are expressed to (2) and (3):

$$\left[Z_{qm}\right]=\left[T_{clarke}\right]^{T}\left[Z\right]\left[T_{clarke}\right] \quad (2)$$

$$\left[Y_{qm}\right]=\left[T_{clarke}\right]^{-1}\left[Y\right]\left[T_{clarke}\right]^{-T} \quad (3)$$

If the transmission line is ideally transposed, the matrices $[Z_{qm}]$ and $[Y_{qm}]$ are identical to the matrix modal impedance $[Z_m]$ and modal admittance $[Y_m]$. Under these conditions the Clarke´s matrix separates the line in their exact modes. If the line has a vertical symmetry plane, but cannot be considered ideally transposed matrices $[Z_{qm}]$ and [Yqm] are written as shown in (4) and (5) [1, 2, 4]:

$$\left[Z_{qm}\right]=\begin{bmatrix}Z_{\alpha} & 0 & Z_{\alpha 0}\\ 0 & Z_{\beta} & 0\\ Z_{0\alpha} & 0 & Z_{0}\end{bmatrix} \quad (4)$$

$$\left[Y_{qm}\right]=\begin{bmatrix}Y_{\alpha} & 0 & Y_{\alpha 0}\\ 0 & Y_{\beta} & 0\\ Y_{0\alpha} & 0 & Y_{0}\end{bmatrix} \quad (5)$$

In the (4) and (5) shows this fact when the line is not ideally transposed, the coupling exists between the alpha and zero modes. However, in certain situations, the coupling between the modes alpha and zero can be disregarded. The matrices $[Z_{qm}]$ and $[Y_{qm}]$ are written as shown in (6) and (7):

$$\left[Z_{qm}\right]\cong\begin{bmatrix}Z_{\alpha} & 0 & 0\\ 0 & Z_{\beta} & 0\\ 0 & 0 & Z_{0}\end{bmatrix} \quad (6)$$

$$\left[Y_{qm}\right]\cong\begin{bmatrix}Y_{\alpha} & 0 & 0\\ 0 & Y_{\beta} & 0\\ 0 & 0 & Y_{0}\end{bmatrix} \quad (7)$$

The voltage and current of quasi-modes are obtained as shown by (8) and (9):

$$\left[V_{qm}\right]=\left[T_{clarke}\right]^{T}\left[V\right] \quad (8)$$

$$\left[I_{qm}\right]=\left[T_{clarke}\right]^{-1}\left[I\right] \quad (9)$$

Equations (8) and (9) can be implemented in computer programs such as MATLAB® that performing simulations directly in the time domain. Using the solution of differential equations mentioned above to represent the line, it can be calculate the currents and voltages of the line in the frequency domain, and the values of currents and voltages in the time domain can be obtained using the transformed inverse Laplace implemented numerically.[3].

To check the performance of this model, it will be used the model called *Universal Line Model* (ULM) [3]. The ULM is one model in which the currents and voltages in the transmission line are written analytically from the differential equations of the line. This model, in which the currents and voltages are calculated in the frequency domain, allows taking into account the distributed nature of the parameters of longitudinal and transverse of the line. The response in the time domain can be obtained by using the Inverse Transform of Laplace [3].

To check the performance of the model, the three-phase line of 100 km in length will be decomposed into its three modes of propagation, where each mode is represented by a single-phase transmission line with an excitation source. The voltages and currents in each mode will be obtained in the frequency domain, and using the inverse Laplace transform implemented numerically, the voltages and currents will be obtained in the time domain. With the values obtained and using the $[T_{clarke}]$ it will be obtained the voltages and the currents in the three-phase transmission line in time domain.

3. Universal Line Model

The *Universal Line Model* (ULM) is a model in which the currents and voltages in the transmission line are written analytically from differential equations of the line [3]. This model the currents and voltages are calculated in the frequency domain, taking into account the distributed nature of parameters of longitudinal and transverse line. The response in the time domain can be obtained by using the Inverse Laplace Transform [3]. A transmission line is characterized by the fact that their parameters are distributed along their length. This fact causes the voltages and currents along line behave like waves, and these are described by partial differential equations. Generally, the differential equations mentioned are difficult to solve in the time domain because the integral convolution but in the frequency domain these equations become simpler and solutions are known. The solution in the frequency domain is generic and can be applied for any condition of the line, whereas the fixed parameters and or variables in function of frequency. The solution in the time domain, this depends on the convolution integral where solutions are not easily obtained.

4. Currents and Voltages in the Single-phase Transmission Line

For analysis of the results it will be considered the three-phase transmission line as shown in the **Figure 1** to study the currents and voltages.

In the differential equations of the line, it's considered that the parameters are constants. For calculation of the voltages and currents in a three-phase line it will used the modal transformation method, in which the three-phase system is decomposed into three decoupled single-phase circuits, called alpha, beta and zero being equivalent to the original system [5]. The **Figures 2** to **4** show the different modes in transmission line to the uncoupled mode *n* generic.

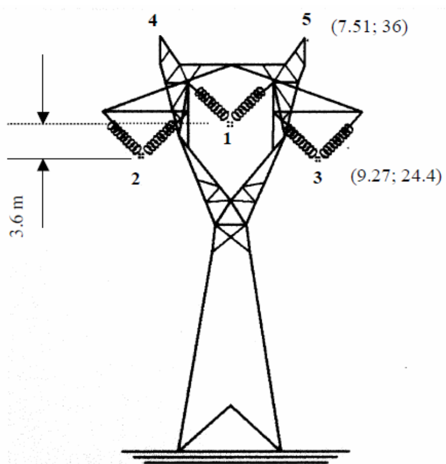

Figure 1. Three-phase transmission line used in the simulations.

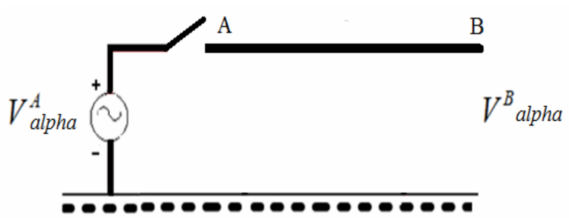

Figure 2, The mode alpha in the transmission line.

Figure 3. The mode beta in the transmission line.

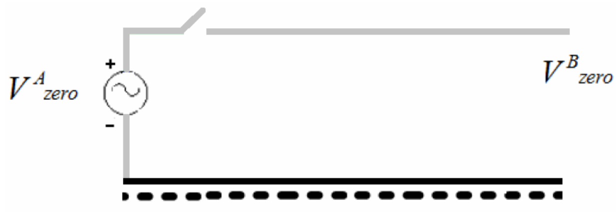

Figure 4. The mode zero in the transmission line.

The line showed in **Figure 1** has the impendence and the admittance as (10) and (11):

$$Z_n(\omega) = R_n + j\omega L_n \tag{10}$$

$$Y_n(\omega) = G_n + j\omega C_n \tag{11}$$

The R_n and L_n are the longitudinal parameters and G_n e C_n are the transverse parameters of line per unit length, considering the mode *n* of propagation. In the **Figure 1** $I_A^n(\omega)$ and $I_B^n(\omega)$ are the currents at ends A and B line, while the $V_A^n(\omega)$ and $V_B^n(\omega)$ are the voltages and these ends in the mode *n*. The equations for the currents in the frequency domain are given by (12) and (13):

$$I_A^n(\omega) = Y^n{}_{AA}(\omega) V_A^n(\omega) + Y^n{}_{AB}(\omega) V_B^n(\omega) \tag{12}$$

$$I_B^n(\omega) = Y^n{}_{BA}(\omega) V_A^n(\omega) + Y^n{}_{BB}(\omega) V_B^n(\omega) \tag{13}$$

The terms $Y^n{}_{AA}(\omega)$ to $Y^n{}_{BB}(\omega)$ are evaluated as (14) to (17):

$$Y^n{}_{AA}(\omega) = \frac{1}{Z^n{}_C}\coth(\gamma^n(\omega)d) \tag{14}$$

$$Y^n{}_{AB}(\omega) = -\frac{1}{Z^n{}_C}\operatorname{csc}h(\gamma^n(\omega)d) \tag{15}$$

$$Y^n{}_{BA}(\omega) = \frac{1}{Z^n{}_C}\coth(\gamma^n(\omega)\, d) \tag{16}$$

$$Y^n{}_{BB}(\omega) = -\frac{1}{Z^n{}_C}\operatorname{csc}h(\gamma^n(\omega)d) \tag{17}$$

Equations (14) by (17), the terms $Z_c^n(\omega)$ and $\gamma^n(\omega)$ are the characteristic impendence and propagation constant in the mode *n* and can be written as (18) and (19):

$$Z^n{}_C(\omega) = \sqrt{\frac{Z^n(\omega)}{Y^n(\omega)}} \tag{18}$$

$$\gamma^n(\omega) = \sqrt{Z^n(\omega) Y^n(\omega)} \tag{19}$$

The $Z_c^n(\omega)$ and $\gamma^n(\omega)$ are complex numbers and $\gamma^n(\omega)$ can be written as (20).

$$\gamma^n(\omega) = a^n + jb^n \tag{20}$$

The real part of $\gamma^n(\omega)$ is the attenuation constant, which corresponds to the amplitude of the wave as it travels in the conductor. The imaginary part is called the phase constant. Thus each mode of propagation *n* have the characteristic impedance, attenuation constant, phase constant and propagation velocity different. As illustration the simulation of a three-phase transmission line in to the process of energization. The physical configuration of the three-phase circuit is shown in **Figure 5**. The three-phase line has the length of 100 km and frequency of 60 Hz.

5. Transient Responses due to Energization Procedure

In the **Figure 5** shows a three-phase transmission line with the receiving open end B that will be used for the study of the model, while the phase1 is energized by a DC voltage source and the phases 2 and 3 are in short circuit in the sending end A.

The transmission line in the **Figure 5** will be energized by a DC voltage source of 20 kV. The **Figures 6** to **8** show the behavior of the voltages for each propagation modes alpha, beta and zero.

The **Figures 6** to **8** show the voltages in the alpha, beta, and zero modes for receiving open end of the three-phase line. It can be seen that each propagation mode behaves as a single-phase line energized by a constant voltage

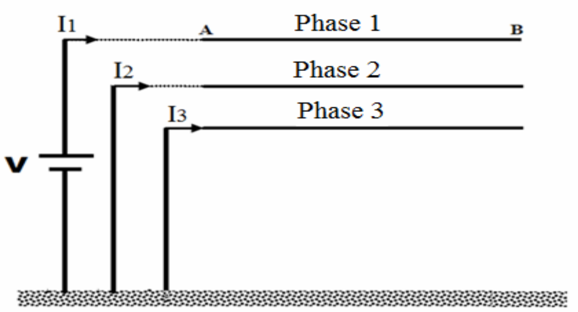

Figure 5. Three-phase transmission line energized with DC source.

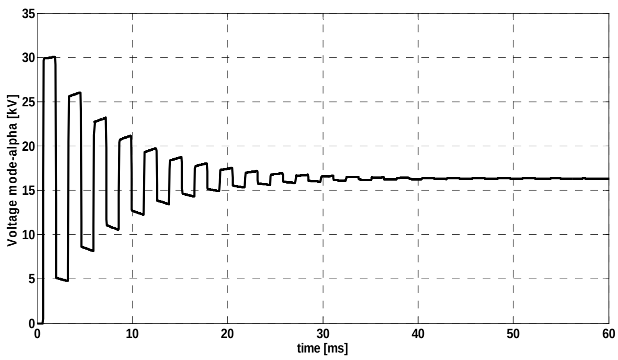

Figure 6. Voltage at the receiving end of the mode alpha.

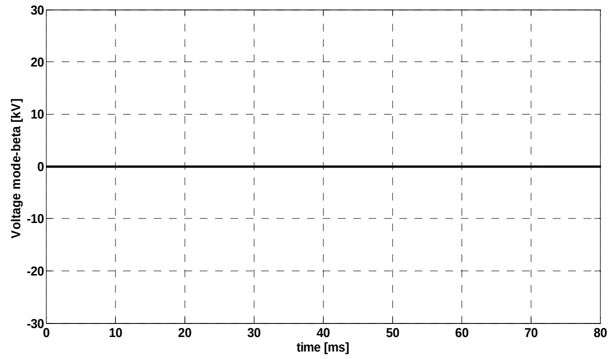

Figure 7. Voltage at the receiving end of in the mode beta.

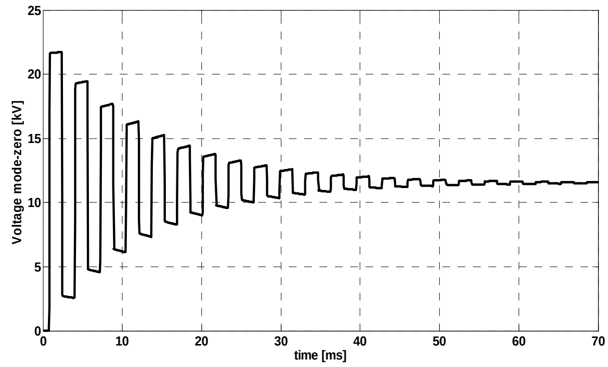

Figure 8. Voltage at the receiving end of in the mode zero.

source and each mode has a different propagation velocity and different steady-state values. The values of the voltages of phases 1, 2 and 3 are obtained by linear combination of the voltages modes shown in **Figures 6** to 8 as (21):

$$[V_{1,2,3}]=\left\{[T_{clarke}]^T\right\}^{-1}[V_{qm}] \tag{21}$$

The vector $[V_{1,2,3}]$ represents the voltages in phases 1, 2 and 3 and $[V_{qm}]$ represents the three-phase line voltage alpha, beta and zero modes of **Figures 6** to **8**. The figure 9 shows the behavior of voltage in the receiving open end B of a three-phase line, using (21).

In **Figure 9** the behavior of voltage in the phase 2 and 3 are influenced by the behavior of voltage in phase 1 due to the mutual inductances of the line. The **Figure 10** and **11** shows the behavior of currents in the phases 1, 2 and 3 at the receiving open end.

In the **Figure 10**, when the voltage in phase 1 is positive, the voltage in phases 2 and 3 become negative, obeying the Faraday-Neumann's law. When voltage in the phase 1 remains constant, the flows induced in the others phases remains constant and there is no induced voltages in the phases 2 and 3. When the phase voltage decreases, the induced voltages in the phases 2 and 3 are positive. The **Figures 10** and **11** show the same behavior obtained in the **Figure 9**. Considering the three-phase transmission line will be energized by symmetrical three- phase voltage source, balanced and dephased in 120°, as shown in the **Figure 12**. Where V_m is the peak voltage, ω is the angular frequency and f is the frequency. The transmission line of **Figure 12** will be energized by an alternating source of 255 V (line-neutral voltage) corresponding to a line-line voltage of 440 kV and a frequency of 60 Hz..

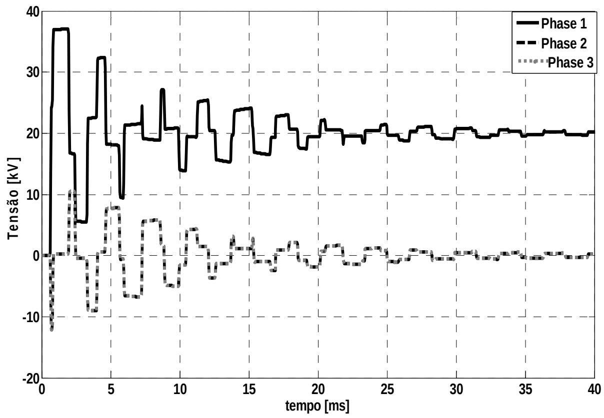

Figure 9. Voltages in the receiving end B of a three- phase transmission line.

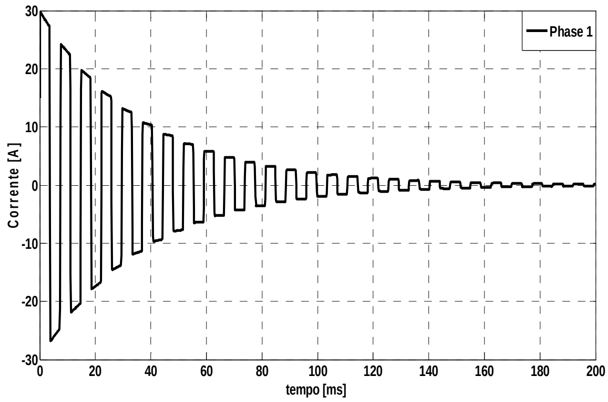

Figure 10. The current in the sending end A of a three-phase transmission line.

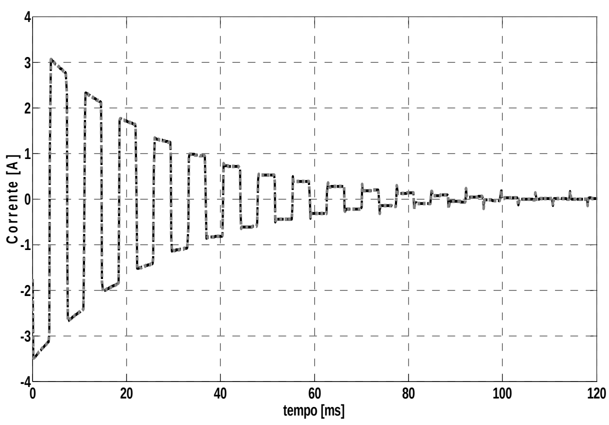

Figure 11. The current in the sending end A of a three-phase transmission line.

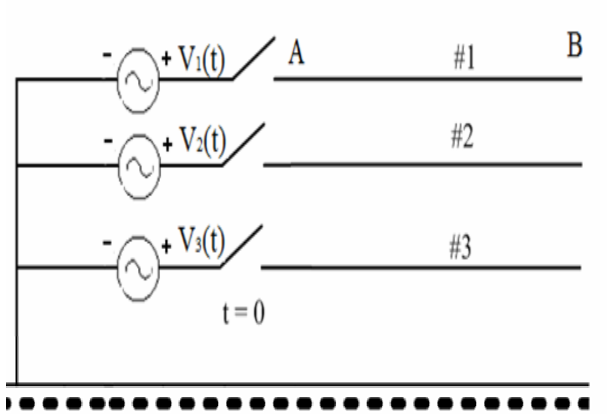

Figure 12. Three-phase line transmission energized with sinusoidal source.

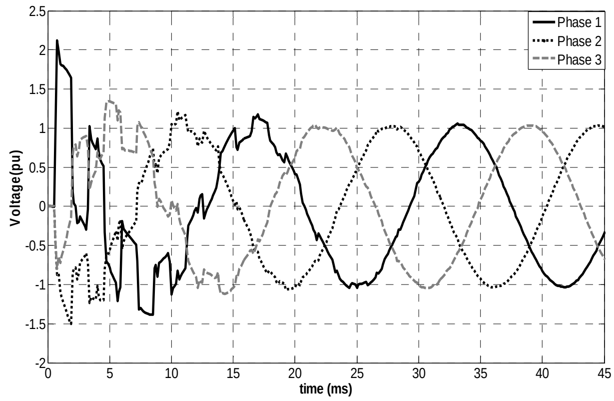

Figure 13. Voltages in the open end B of three-phase transmission line of 440 kV.

The voltages at the recede opening end B as shown **Figure 13**.

The **Figure 13** shows that the peak voltage for a phase 1 reaches approximately 1.7 pu in the transient period. As the same behavior to figure 9, when the voltage in the phase 1 is a positive, in the phases 2 and 3 become negatives and when the voltage in phase 1 is negative, the induced voltages in the phases 2 and 3 are positives and the voltage in the system will reach the 1 pu value in steady-estate.

6. Conclusions

In this work with the method of modal transformation is possible to obtain currents and voltages in a three-phase transmission line. Due to the difficulty of solving the second order differential equations that model the polyphase transmission line, the system has n coupled phases can be represented by n decoupled single-phase systems that are equivalent to the original system, as represented in the **Figures 2** to **4**.

In **Figure 5** the line is energized with a voltage source

DC in the phase 1 and the others phases are in short circuit. It's obtained the alpha, beta, and zero propagation modes, and in which each mode the velocity, attenuation and steady-state values are different, as shown in the **Figures 6** to **9**. Using (21) were obtained the voltages at receiving open end B. In the **Figure 9**, the peak value in the phase 1 is approximately 2 times its steady state value. At steady-state value, the voltage in the phase 1 will reach a value of DC source of 20kV and the in others phases the value steady-state will be zero because there is no variation in mutual flow. The **Figure 10** and **11** shows the behavior of current in the phases 1, 2 and 3 of the line. When the current in phase 1 is positive, the currents in the phases 2 and 3 are negative, inducing negative voltages at the end B of the line. When the current in phase 1 is negative, the current in the phases 2 and 3 are positive, inducing positive voltages in the end B of the line. In the **Figure 13** the voltage in the phase 1, reaches a value of 2,0 per unit and the induced voltages in phases 2 and 3 present value of 1 pu, approximately. Thus the model of modal transformation can be used to study the transient electromagnetic transmission line phase subjected to the energization process.

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