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Fast Algorithm for Prediction of Airfoil Anti-icing Heat Load*

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ABSTRACT

Many flight and icing conditions should be considered in order to design an efficient ice protection system to prevent ice accretion on the aircraft surface. The anti-icing heat load is the basic knowledge for the design of a thermal anti-icing system. In order to help the design of the thermal anti-icing system and save the design time, a fast and efficiency method for prediction the anti-icing heat load is investigated. The computation fluid dynamics (CFD) solver and the Messinger model are applied to obtain the snapshots. Examples for the calculation of the anti-icing heat load using the proper orthogonal decomposition (POD) method are presented and compared with the CFD simulation results. It is shown that the heat loads predicted by POD method are in agreement with the CFD computation results. Moreover, it is obviously to see that the POD method is time-saving and can meet the requirement of real-time prediction.

Keywords: Anti-icing; Heat Load; Proper Orthogonal Decomposition

1. Introduction

Aircraft icing is a thermodynamic phenomenon that leads to the formation of ice on aircraft in flight. Severe in-flight icing is a serious hazard that destroys the clean aerodynamic shape of the airfoil, resulting in increased drag, decreased lift, reduced aircraft stability, performance and controllability. Thermal anti-icing system, used as the most popular means of ice prevention in icing conditions, operates to keep the surface temperature and the water collected by the aircraft above the freezing point.

During the process of the design of the anti-icing system, many flight and icing conditions should be considered for the calculation of the water droplet impingement and the anti-icing heat load to have knowledge of how about the most severe condition and how much heat flux should be supplied for anti-icing. Generally, the CFD method is used for the air flow simulation around the object and then the thermal analysis are applied to calculate the heat load [1] considering the whole flight and icing conditions under FAR 25 Appendix C. This would lead to very large computation work and expend much computation time. In order to help the design and save the design time, a fast and efficiency method for prediction the anti-icing heat load is investigated in this paper.

The proper orthogonal decomposition (POD) [2] technique is a powerful reduced-order model (ROM), which can effectively reduce the degree of freedom for physical problems with high precision, thus reducing the calculation time and saving the data storage significantly. POD has been used extensively in the field of flow and heat transfer computation [3,4] to predict these problems accurately in very short time.

In this paper the CFD method is used to obtain the snapshots (samples) for the POD analysis. The POD method is studied and used for the fast prediction of the heat loads. The effects of the variables for the POD prediction of the heat load are investigated and the results are compared with the CFD simulation results.

2. POD Methodology

POD method can rapidly reconstruct the flow information applying a data set called snapshots or samples of the physical properties. The essence of the POD method is the extraction of a set of eigenfunctions which can best describes the dominant features of the flow or heat transfer. Once these eigenfunctions are calculated, the approximate states of the flow field and heat transfer parameters can be obtained by superposition of them.

POD method requires a set of snapshots which can be obtained either from numerical calculations or tests. In this paper, the snapshots are obtained from the CFD analysis. The snapshot $\boldsymbol{U}^i$, a vector which describes a certain physical field, can be any type of the flow vari-

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able, such as velocity, pressure, ice shape [5,6], or anti-icing heat load, as in this paper. If each $\boldsymbol{U^i}$ has N sample values, a set of M snapshots forms a $N \times M$ matrix $\boldsymbol{U}$, which can be expressed by:

$$\boldsymbol{U} = \left\{\boldsymbol{U}^1, \ldots, \boldsymbol{U}^M\right\}. \tag{1}$$

As mentioned above, POD method presents the main characteristics of the flow by using a set of eigenfunctions. This means that a set of proper orthogonal decomposition modes $\boldsymbol{\psi}^j$, should be found, and then $\boldsymbol{U^i}$ can be expressed by:

$$\boldsymbol{U^i} = \sum_{j=1}^{M} \alpha_j^i \cdot \boldsymbol{\psi}^j, \tag{2}$$

where α_j^i is a set of sample coefficients for the proper orthogonal decomposition modes $\boldsymbol{\psi}^j$, can be calculated by:

$$\alpha_j^i = (\boldsymbol{\psi}^j, \boldsymbol{U}^i), \tag{3}$$

where j denotes the j^{th} proper orthogonal decomposition mode, and i denotes the i^{th} snapshot.

All the $\boldsymbol{\psi}^j$ form the proper orthogonal decomposition modes matrix $\boldsymbol{\psi}$:

$$\boldsymbol{\psi} = \left\{\boldsymbol{\psi}^1, \ldots, \boldsymbol{\psi}^M\right\}, \tag{4}$$

where M is the number of snapshots. The matrix $\boldsymbol{\psi}$, of the same dimensionality as $\boldsymbol{U}$, satisfies the constraint $\boldsymbol{\psi^T \cdot \psi = E}$. The elements of the modes are defined as:

$$\boldsymbol{\psi = U \cdot V}, \tag{5}$$

where $\boldsymbol{V}$ is the eigenvector matrix of $\boldsymbol{C}$:

$$\boldsymbol{C \cdot V} = \Lambda \cdot \boldsymbol{V}, \tag{6}$$

Λ is a diagonal matrix storing the eigenvalues λ_i of the positive definite covariance matrix $\boldsymbol{C}$. The entries of $\boldsymbol{C}$ are defined as:

$$C_{ij} = \sum_{k=1}^{N} U_k^i U_k^j \Big/ M. \tag{7}$$

Thus, the problem of solving the proper orthogonal decomposition modes changes into that of solving the eigenvalues and eigenvectors of the matrix $\boldsymbol{C}$. Obviously, $\boldsymbol{C}$ is a $M \times M$ matrix, so the matrix to be solved becomes much smaller, reducing from a size N (the number of grid points) to a size M (number of snapshots). This is one of the advantages of using the POD technique.

After the proper orthogonal decomposition modes $\boldsymbol{\psi}^j$ and sample coefficients α_j^i are obtained from the matrix $\boldsymbol{C}$ which can be calculated from the snapshot matrix $\boldsymbol{U}$, the approximate states of the flow field can be obtained by linear superposition of the proper orthogonal decomposition modes, shown as follows:

$$\boldsymbol{U^{pod}} = \sum_{j=1}^{M} \alpha_j^{pod} \cdot \boldsymbol{\psi}^j. \tag{8}$$

To predict a field which is not part of the snapshots, an interpolation method must be employed to obtain the POD coefficients α_j^{POD}. The linear interpolation method is employed to compute the coefficients in this paper.

Moreover, the magnitude of each eigenvalue λ_i determines the energy contained in the corresponding mode. The ratio of the energy contained in a certain mode can be measured by:

$$\text{Energy} = \frac{\lambda_i}{\sum_{i=1}^{M} \lambda_i}. \tag{9}$$

Therefore, if necessary, the modes that contain less energy can be negligible, resulting in the reduction of proper orthogonal decomposition modes. The approximation by POD calculation of the linear combination of eigenvectors can be written as:

$$\boldsymbol{U^{pod}} = \sum_{j=1}^{l} \alpha_j^{pod} \cdot \boldsymbol{\psi}^j, \tag{10}$$

where l is the proper orthogonal decomposition modes used for the construction of the approximation by POD, so $M \geq l$. If some proper orthogonal decomposition modes are ignored, the flow field reconstructed by POD will be different from that reconstructed by all modes. The inaccuracy can be controlled by the total energy of all the used proper orthogonal decomposition modes.

The most time cost during the calculation by POD is that of solving the eigenvalues and eigenvectors of the $M \times M$ matrix $\boldsymbol{C}$, and the total calculation process only takes a few seconds.

In this paper, the effectiveness of the POD method in prediction the anti-icing heat load on an airfoil is investigated and the effects of the ambient temperature, flight velocity and liquid water content (LWC) on the POD-reconstructed anti-icing heat load are studied.

3. Anti-icing Heat Load Prediction for Snapshots

The prediction of the anti-icing heat load generally has four steps: 1) air flow field calculation, 2) water droplet impingement calculation, 3) external convective heat transfer coefficient prediction, 4) anti-icing heat load analysis.

In this paper, for the snapshots, the Eulerian method is used for the calculation of the water droplet impingement. The boundary layer integrated method is used for the calculation of the external heat transfer coefficient. The anti-icing heat loads simulation is based on the Messinger model. The Computational Fluid Dynamic (CFD) tool Fluent and its User Defined Function (UDF) are applied to implement the above computations for the snapshots. Details about the method and analysis can be

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found in Reference [1,7,8] written by authors.

4. POD Validation and Results Analysis

In this paper, the POD method, at first, is applied to the prediction of anti-icing heat loads for 2D viscous flow under a one-variable problem, here being ambient temperature. The calculation process of the POD method is introduced in detail. Then, the anti-icing heat loads predictions by POD under two-variable and three-variable problems have been investigated.

An airfoil which has 344 control volumes is taken into consideration for the study. The anti-icing heat load samples of each control volume on the airfoil under certain flight and icing conditions are obtained by the CFD method. Besides three parameters which are the ambient temperature, the velocity and the LWC, the other parameters in all cases are the same, which are: 20μm as the mean volume diameter, 2° as the angle of attack, 90KPa as the ambient pressure, 25℃ as the surface temperature of the airfoil.

4.1. One Parameter POD Application for Heat Load

It is well known that the ambient temperature has a great influence on anti-icing heat loads. Therefore, firstly, the effect of the ambient temperature on the POD-predicted anti-icing heat loads on an airfoil is investigated and the effectiveness of the POD method is studied. In this section, the LWC and the flight velocity are 0. 6 g/m^3 and 90 m/s individually. The ambient temperature is from -30 to - ℃. The anti-icing heat loads, which are obtained by CFD method, at the ambient temperature of -30, -25, -20, -15, -10, -5℃ are taken as samples. So a 344 × 6 matrix forms and $M = 6$. The anti-icing heat loads under various ambient temperatures, predicted by the CFD method, are shown in **Figure 1**.

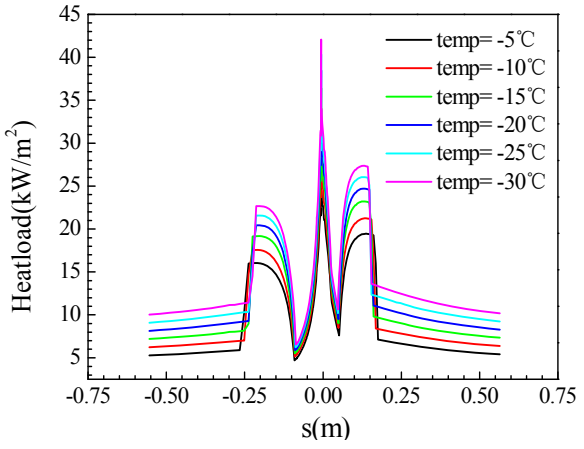

Figure 1. Anti-icing heat loads at different ambient temperatures (CFD method).

According to the sample matrix $\boldsymbol{U}$, the corresponding matrix $\boldsymbol{C}$ can be calculated using Equation (7), and $\boldsymbol{C}$ is a 6 × 6 matrix because there are 6 snapshots. The results of $\boldsymbol{C}$ matrix are showed in **Table 1**. Then the ordered eigenvectors and eigenvalues of matrix $\boldsymbol{C}$ are calculated from Equation (6). The eigenvalues are showed in **Table 2**.

It can be found that the first eigenvalue is much larger than others from **Table 2**, which means that the energy contained in the corresponding mode is very large. It can be concluded that the first mode is vital to the prediction and it determines the broad contour of the anti-icing heat loads distribution.

Next, the ultimately used proper orthogonal decomposition modes can be obtained by orthogonalizing the matrix $\boldsymbol{\psi}$, which can be calculated from Equation (5). Here all the proper orthogonal decomposition modes are used for calculation. The sample coefficient α^i for the corresponding proper orthogonal decomposition mode can be obtained using Equation (3). The results are shown in **Table 3**. It is obvious that each sample coefficient for the first proper orthogonal decomposition mode is much larger than other modes, indicating that the first mode makes a significant contribution on the POD calculation.

Table 1. C matrix values.

23720.93	22564.56	21285.61	19996.39	18478.02	16893.21
22564.56	21468.51	20254.55	19030.61	17585.86	16077.71
21285.61	20254.55	19118.81	17970.34	16609.72	15187.40
19996.39	19030.61	17970.34	16898.27	15622.51	14285.26
18478.02	17585.86	16609.72	15622.51	14465.47	13236.85

Table 2. Eigenvalues.

case	Λ
1	107679.734
2	101.829
3	18.207
4	4.686
5	1.147

Table 3. Sample coefficients.

	a1	a2	a3	a4	a5	a6
1	269.250	294.367	318.333	338.661	358.786	377.063
2	16.606	10.960	2.040	-2.865	-8.775	-11.213
3	-5.387	1.847	6.790	2.931	-1.625	-4.414
4	-2.170	4.166	-1.467	-1.593	-0.191	1.148
5	0.081	-0.290	0.805	-0.263	-1.917	1.550
6	0.085	-0.159	0.861	-1.384	0.593	0.016

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After the above calculations, the POD model has been basically established. The anti-icing heat loads at the ambient temperature of -23, -12℃ are predicted using the POD approach. The predicted results are compared with the results from CFD method. The POD coefficients α_j^{POD} are interpolated by the linear method, and the predicted anti-icing heat loads are calculated using Equation (10). **Figure 2** shows the heat loads comparisons between the POD solutions and the CFD method, which indicates that the results nearly the same between the POD and CFD method under the same conditions. Therefore, the POD method can be applied very well to predict the heat load when one parameter changes.

4.2. Two Parameters POD Application for Heat Load

In this section, the POD method is used to predict the anti-icing heat loads under two-variable problem, here being the ambient temperature and flight velocity. The LWC is 0.6 g/m^3. The samples are obtained at the ambient temperature of -30, -25, -20, -15, -10, -5 ℃ and the flight velocity of 90, 100, 110, 120, 130, 140 m/s. Thus the number of the snapshots is 36.

All the 36 proper orthogonal decomposition modes are applied for the prediction of the heat load. **Figure 3** compares the heat load results obtained using POD solutions to that acquired through the CFD method. Case 1 indicates that the ambient temperature is -23℃ and the flight velocity is 135 m/s. Case 2 indicates that the ambient temperature is -12℃ and the velocity is 125 m/s. It is obvious that the heat loads predicted by the POD method fit well with the CFD simulated results except a little deviation at the location where a large drop occurs. That might because that the runback water evaporates completely at that place and the heat load decreases remarkably, which causing non-derivability, leading to the deviation of heat load between POD and CFD method. The solution time taken by the POD method is less than two seconds also.

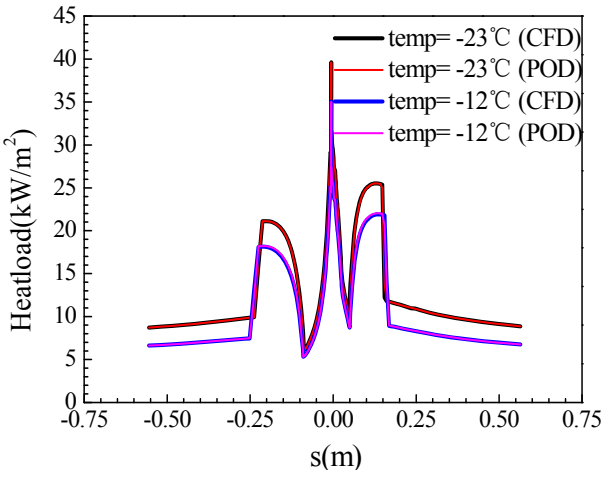

Figure 2. Anti-icing heat loads comparison of single parameter POD and CFD method.

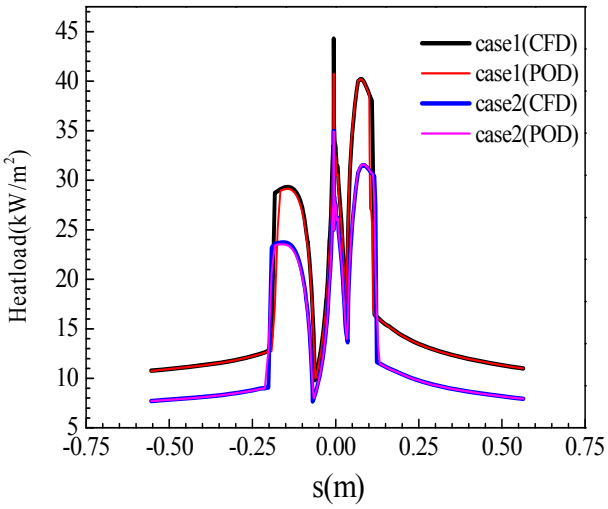

Figure 3. Anti-icing heat loads comparison of two parameter POD and CFD method.

4.3. Three Parameters POD Application for Heat Load

At last, the POD method is used to predict the anti-icing heat loads under three-variable problem, here being the ambient temperature, flight velocity and LWC. To generate the various samples, the ambient temperature, flight velocity and LWC are selected as follows. Therefore, in total, 216 snapshots are computed.

Temperature = -30, -25, -20, -15, -10 and -5 (℃)
Velocity = 90, 100, 110, 120, 130 and 140(m/s)
LWC = 0.3, 0.4, 0.5, 0.6, 0.7 and 0.8(g/m^3)

From calculating the eigenvectors and eigenvalues of matrix $\boldsymbol{C}$, 216 proper orthogonal decomposition modes is obtained. Because the number of proper orthogonal decomposition modes is very large, the first seventeen modes which contain more than 99.9% energy of all the modes are chosen for the prediction of the anti-icing heat loads. It is considered to be able to accurately describe the anti-icing heat loads.

The comparisons of the results between the three parameters POD solution and the CFD method are shown in **Figure 4**. Case 1 indicates that the ambient temperature is -23℃, the flight velocity is 135 m/s and the LWC is 0.35 g/m^3. Case 2 indicates that the ambient temperature is -12 ℃, the velocity is 125 m/s and the LWC is 0.56 g/m^3. More deviations can be seen near the locations where are non-derivability. But the deviations are in the acceptable region. Likewise, the solution time taken by the POD method is almost equal to the one-variable problem above, means time-saving.

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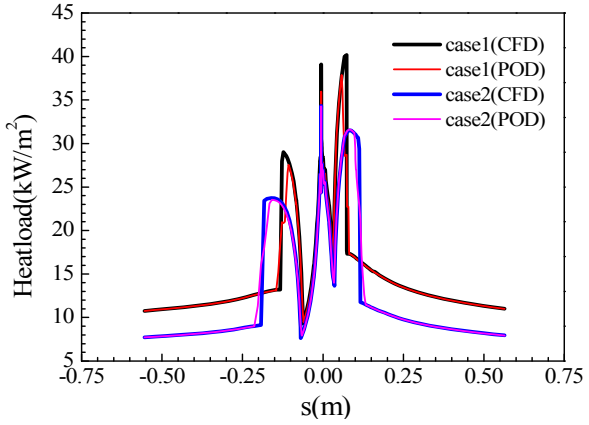

Figure 4. Anti-icing heat loads comparison of three parameter POD and CFD method.

5. Conclusions

The POD approach for reduced-order modeling is applied to the prediction of the anti-icing heat loads for a two-dimensional airfoil. Considering the influence of the ambient temperature, flight velocity and LWC, the anti-icing heat loads predictions by the POD method under one-variable, two-variable and three-variable problems have been investigated. The snapshots for the POD prediction are obtained by the CFD method. To validate the POD method for the heat load prediction, the heat load results from the POD solutions are compared with those from the CFD method. It can be concluded that the POD approach can predict the anti-icing heat loads remarkably well besides some slight disagreements at the locations where water evaporates totally. Comparing the time it takes via POD method and CFD solver, it can be obviously found that the POD method is much time- saving. It should be studied further on how to select the samples to reduce the number of the snapshots.

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