Studying Effect of Magnetizing Curve Nonlinearity Index on the Occurring Chaotic Ferroresonance Oscillation in Autotransformers

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Abstract

This paper investigates the effect of iron core saturation characteristic degree on the onset of chaotic ferroresonance and duration of transient chaos in an autotransformer. The transformer chosen for study has a rating of 50 MVA, 635.1 kV, the magnetization characteristic of the autotransformer is modeled by a single-value two-term polynomial with q = 5, 7, 11. The core loss is modeled by a linear resistance and is considered a fix value for it. Simulation results are derived by using MATLAB software and nonlinear dynamics tool such as bifurcation and phase plan diagrams. It is shown settling down to the chaotic region is increased when degree of core nonlinearity is gone from 5 to 11.

Keywords: Linear Core Losses, Chaotic Ferroresonance, Bifurcation Diagrams, Autotransformers

1. Introduction

Ferroresonance is initiated by improper switching operation, routine switching, or load shedding involving a high voltage transmission line. It can result in Unpredictable over voltages and high currents. The prerequisite for ferroresonance is a circuit containing iron core inductance and a capacitance. Such a circuit is characterized by simultaneous existence of several steady-state solutions for a given set of circuit parameters. The abrupt transition or jump from one steady state to another is triggered by a disturbance, switching action or a gradual change in values of a parameter. Typical cases of ferroresonance are reported in [1-4]. Theory of nonlinear dynamics has been found to provide deeper insight into the phenomenon. In [5,6] is among the early investigations in applying theory of bifurcation and chaos to ferroresonance. The susceptibility of a ferroresonance circuit to a quasi-periodic and frequency locked oscillations has been presented in [7]. In this case, investigation of ferroresonance has been done upon the new branch of chaos theory that is quasiperiodic oscillation in the power system and finally ferroresonance appears by this route. Effect of initial conditions on occurring ferroresonance oscillation is investigated in [8,9]. Effect of circuit breaker grading capacitance on ferroresonance in voltage transformer is investigated in [10]. Analysis of ferroresonance modes in power transformers using preisach-type hysteretic magnetizing inductance is described in [11]. Effect of a connected MOV arrester in parallel to the power transformer is illustrated in [12]. Analysis of ferroresonance phenomena in power transformers including neutral resistance effect is investigated in [13]. Effect of circuit breaker shunt resistance on chaotic ferroresonance in voltage transformer has shown in [14]. In this work ferroresonance is controlled by considering C.B resistance effect. Controlling ferroresonance in voltage transformer by considering circuit breaker shunt resistance including transformer nonlinear core losses effect is done in [15]. Decreasing Ferroresonance Oscillation in Potential Transformers Including Nonlinear Core Losses by Connecting Metal Oxide Surge Arrester in Parallel to the Transformer is studied in [16]. This paper studies the effect of linear core losses and iron core saturation characteristics degree on the global behavior of a ferroresonance circuit. The circuit under study represents a case of ferroresonance that occurred on 1100 kV system of Bonneville Power Administration as described in [17].



2. Circuit Descriptions and Modeling

Three-phase diagram for the circuit is shown in **Figure 1**. The 1100 kV transmission line was energized through a bank of three single-phase as reported in autotransformers. Ferroresonance occurred in phase A when this phase was switched off on the low-voltage side of the autotransformer; phase C was not yet connected to the transformer at that time [17]. The autotransformer is modeled by a T-equivalent circuit with all impedances referred to the high voltage side. The magnetization branch is modeled by a nonlinear inductance in parallel with a nonlinear resistance and these represent the nonlinear saturation characteristic, and nonlinear hysteresis curve [17]. Iron core saturation characteristic is given by:

$$i_{Im} = a\lambda + b\lambda^q \tag{1}$$

The exponent q depends on the degree of saturation. It was shown that for proper representation of the saturation characteristics of a power transformer the exponent q may take the values 5, 7, and 11. In this paper, the core losses model is described by a linear resistance. The polynomial of order seven and the coefficient *b* of Equation (1) are chosen for the best fit of the saturation region. It was shown in this paper that for lower order polynomials, chaos occurred for larger value of input voltage, also for polynomials of order 5 and 7, chaos did not appear for low losses and only fundamental and subharmonic resonances are obvious.

Figure 2 shows the hysteresis curve of the iron core of the auto transformer that is simulated and obtained for the transformer that is studied in this paper. It is thus important to have as accurate an approximation to the magnetization curve as possible. Because of the nonlinear nature of the transformer magnetizing characteristics, the behavior of the system is extremely sensitive to change in system parameter and initial conditions. A small change in the value of system voltage, capacitance or losses may lead to dramatic change in the behavior of it. A more suitable mathematical language for studying ferroresonance and other nonlinear systems is provided by nonlinear dynamic methods. Mathematical tools that are used in this analysis are phase plan diagram, time domain simulation and bifurcation diagram.

The circuit in **Figure 1** can be reduced to a simple form by replacing the proposed power system with the thevenin equivalent circuit as shown in **Figure 3**. **Figure 4** shows the equivalent circuit of the power system including auto transformer that is derived by applying thevenin theorem to the initial single line diagram of the power system. By using the steady-state solution of MATLAB SIMULINK with the data of the 1100 Kv



Figure 1. System modeling of the power system.



Figure 2. Hystersis curve of the iron core of the autotransformer.



Figure 3.Thevenin circuit of the power system that is shown in Figure 1.



Figure 4. Initial circuit of ferroresonance circuit.

transmission line [17], E_{th} and z_{th} were found to be:

$$E_{\rm th} = 130.1 \,\rm kV; \ z_{\rm th} = -j1.01243E + 0.5 \,\Omega$$

2.1. Nonlinear Dynamics and Equation

The resulting circuit to be investigated is shown in **Fig-ure 4** where Zth represents the Thevenin impedance. The behavior of this power system can be described by the following system of nonlinear differential equations, by applying KVL and KCL laws to the circuit in the case of considering linear core losses effect can be driven as below:

$$-e_{\rm th} + v_c + R_2 \cdot i + v_m + v_{L2} = 0 \tag{2}$$

$$v_{L_2} = L_2 \frac{\mathrm{d}i}{\mathrm{d}t} \tag{3}$$

$$v_c = \frac{1}{C} \int i \mathrm{d}t \tag{4}$$

$$-e_{\rm th} + v_c + R_2 \cdot i + v_m + v_{L_2} = 0 \tag{5}$$

$$\frac{\mathrm{d}v_c}{\mathrm{d}t} = \frac{1}{C} \left(a\lambda + b\lambda^q + h_0 + \frac{1}{R_m} \frac{\mathrm{d}\lambda}{\mathrm{d}t} \right) \tag{6}$$

$$i = (i_{Lm} + i_{Rm}) = \left(a\lambda + b\lambda^{q} + \frac{1}{R_{m}}\frac{d\lambda}{dt}\right)$$
(7)

$$v_{L_2} = L_2 \frac{\mathrm{d}i}{\mathrm{d}t} = L_2 \left(a \frac{\mathrm{d}\lambda}{\mathrm{d}t} q b \lambda^{q-1} \frac{\mathrm{d}\lambda}{\mathrm{d}t} + \frac{1}{R_m} \frac{\mathrm{d}^2 \lambda}{\mathrm{d}t^2} \right)$$
(8)

$$e_{\rm th} = v_c + R_2 \cdot \left(\frac{1}{R_m} \frac{d\lambda}{dt} + a\lambda + b\lambda^q\right) + \frac{d\lambda}{dt} + L_2 \left(\frac{1}{R_m} \frac{d^2\lambda}{dt^2} + a\frac{d\lambda}{dt} + qb\lambda^{q-1}d\right)$$
(9)

The base values of the power system parameters are listed in **Table 1**.

Also, the per unit values of the nonlinear index coefficients are listed in **Table 2**.

2.2. Simulation Results and Discussion

Ferroresonance in three phase systems can involve large power transformers, distribution transformers, or instrument transformers. The general requirements for ferroresonance are an applied or induced source voltage, a saturable magnetizing inductance of a transformer, a capacitance, and little damping. The capacitance can be in the form of capacitance of underground cables or long transmission lines, capacitor banks, coupling capacitances between double circuit lines or in a temporarily-ungrounded system, and voltage grading capacitors in HV circuit breakers. Other possibilities are generator surge

Table 1. Basic parameters value of the power system.

$V_{\rm base}$	Ibase	$C_{p.u}$	$R_{\rm p.u}$	R _{core}	R _{base}	L _{sp.u}
635.1 kV	78.72 A	0.079	0.0014	556.68	8067 Ω	0.0188

Table 2.	Different	value	of q	with	its	coefficient.
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q/coefficient	А	В
5	0.0071	0.0034e(-19)
7	0.000375	7.3824e(-24)
11	0.000375	1.5648e(-37)

capacitors and SVC's in long transmission lines. Due to nonlinearities, increased capacitance does not necessarily mean an increased likelihood of ferroresonance. Operating guidelines based on linear extrapolations of capacitance may not be valid. Also, as mentioned previously, the smaller the load on the transformer's secondary, the less the system damping is and the more likely ferroresonance will be. Therefore, a highly capacitive line and little or no load on the transformer are prerequisites for ferroresonance. So, in the case of simulation, time domain simulations were performed using fourth order Runge-Kutta method and validated against MATLAB SIMULINK, and effect of changing in the value of system capacitances doesn't investigate in this paper. The initial conditions as calculated from steady-state solution of MATLAB are:

$$\lambda = 0,0; v_m = 1.67 \text{ pu}; v_C = 1.55 \text{ pu}$$

The circuit in **Figure 4** is analyzed by first modeling the core loss as a constant linear resistance. Simulation has been done in three categories, first: system simulation with linear core losses effect with considering degree of core nonlinearity q = 5, second system simulation with linear core losses effect with considering degree of core nonlinearity q = 7 and finally investigation of chaotic ferroresonance by considering q = 11.

Figure 5 Shows the phase plan diagram of voltage and flux linkage of the transformer with q = 5, according to this plot, trajectory of the system has a chaotic behavior and over voltages of transformer reaches to 6 p.u. Corresponding time domain simulation in **Figure 6** clearly shows the chaotic behavior of the voltage on the transformer, value of time is considered as a per unit value and it is shows amplitude of this oscillation is reached to 5 p.u.

Figure 7 Shows the phase plan diagram of over voltage on transformer with q = 7, it is shown when the degree of q increases from 5 to 7, amplitude of over voltage goes up and reaches to 6 p.u. also chaotic behavior has more nonlinear oscillation when compared it with the previous case of simulation.



Figure 5. Phase plan diagram for q = 5 considering linear core losses.







Phase Plan Diagram of voltage on transformer with q = 7 including linear core losses

Figure 7. Phase plan diagram for q = 7 considering linear core loss.

Figure 8 shows time domain simulation when input voltage of the system is 4 p.u. This plot shows the chaotic signal with much subharmonic resonance in it.

The circuit in **Figure 4** is analyzed by first modeling the core loss as a constant linear resistance. **Figures 9** and **10** show the phase plan diagram and time domain simulating with q = 11. It is shown that when q is increased, nonlinear phenomena in the transformer are begun in the low value of the input voltage. It was shown that the chaotic behavior begins at a value of $E_p = 2$ p.u for q = 5 and $E_p = 1.5$ p.u for q = 7, 11 where represents the amplitude of eth(t). Another tool that can shows manner of the system in vast variation of parameters is bifurcation diagram. In **Figure 11**, voltage of the system is increased to 8 p.u, and ferroresonance is begun in 2.5 p.u, it is shown that if input voltage goes up due to the abnormal operation or switching action, ferroresonance phenomena is occurred. In **Figure 11** input voltage is increased up to 8 p.u and over voltage on transformer has been analyzed according to input voltage variation. In point (1) period 3 appears, in point (2) period 5 appears and jumping in the trajectory of the system has been take placed, in point (3) chaos has been begun and in point (4) system behavior comes out from chaotic region but there are many subharmonic resonance in the system. By repeating simulation, between point (5), (6) and (7) again system has a nonlinear behavior. After the point (7) system has a periodic manner with the period 9 behavior.

Bifurcation diagrams in this paper are traced period doubling route to chaos and it is begun from period1, period 3, period 5 and by this manner goes to chaotic region. In the big value of input voltage, system behavior is completely nonlinear and chaotic over voltages reach to 3 p.u; this amplitude is very dangerous for transformer







Figure 9. Phase plane diagram for q = 11 considering linear core losses.



Figure 10. Corresponding time domain signal that includes chaotic motion.



Figure 11. Bifurcation diagram for q = 5 considering linear core losses.

and can cause core failure. **Figure 12**. shows system over voltage with q = 7. In this plot, one jump is occurred in point (2), and chaos is appeared in point (3). By increasing degree of q, it is shown chaos appear in 2 p.u. by comparing this plot with the previous case, it is clearly shows ferroresonance oscillation is begun 0.5 p.u earlier than the case of considering q = 5.

Bifurcation diagram in **Figure 13** shows more nonlinearity with q = 11, it is shown the chaotic overvoltage after point (1) and period doubling oscillation is occurred in E = 1 p.u. Amplitude of this case is reached to 2.5 p.u. **Figure 13** shows system over voltage with q = 11, according to this plot, in point (1), period 3 appears and in this point ferroresonance is begun. By comparing the bifurcation diagram in **Figures 12** and **13** by **Figure 11**, it is concluded that degree of nonlinear core can cause occurring the ferroresonance over voltages in the low value of the input voltage. In the real system, when the input voltage due to the abnormal switching or other unwanted phenomena reaches to 4 p.u, transformer core is heated and can cause transformer core failure.

3. Conclusions

The dynamic behavior of an autotransformer is shown by the dynamical tool such as bifurcation and phase plan diagrams. In this paper, effect of changing the value of degree q on occurring ferroresonance over voltages is studied, and is shown the nonlinear iron core and its degree "q" has a great effect of occurring nonlinear oscillation in the autotransformer. When q is 11, chaotic over voltages are more nonlinear and amplitude of this case is higher than the other cases of study. Power transformer is modeled with q = 11, so settling down to ferroresonance



Figure 12. Bifurcation diagram for q = 7 considering linear core loss.



Figure 13. Bifurcation diagram for q = 11 considering linear core loss.

in these transformers is more dangerous and should consider protecting tool on this transformers such as damping resistor loads, MOV surge arrester and neutral earth resistance.

4. References

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