

# Economic Dispatch with Multiple Fuel Options Using CCF

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## Abstract

This paper presents an efficient analytical approach using Composite Cost Function (CCF) for solving the Economic Dispatch problem with Multiple Fuel Options (EDMFO). The solution methodology comprises two stages. Firstly, the CCF of the plant is developed and the most economical fuel of each set can be easily identified for any load demand. In the next stage, for the selected fuels, CCF is evaluated and the optimal scheduling is obtained. The Proposed Method (PM) has been tested on the standard ten-generation set system; each set consists of two or three fuel options. The total fuel cost obtained by the PM is compared with earlier reports in order to validate its effectiveness. The comparison clears that this approach is a promising alternative for solving EDMFO problems in practical power system.

**Keywords:** Economic Load Dispatch, Composite Cost Function, Multiple Fuel Options, Piecewise Quadratic Function, Mathematical Model

## 1. Introduction

The economic dispatch problem in a power system is to determine the optimal combination of power outputs for all generating units which will minimize the total cost while satisfying the load and operational constraints [1]. The economic dispatch problem is very complex to solve because of its colossal dimension, a non-linear objective function, and a large number of constraints. Conventional techniques lambda iteration method and quadratic programming offer good results, but when the search space is nonlinear and has discontinuities, they become very complicated with a slow convergence ratio and do not always seek the optimal solution. New numerical methods are needed to cope with these difficulties, especially those with high speed search for the optimal and not being trapped in local minima [2].

The stochastic search algorithms such as Simulated Annealing (SA) and Genetic Algorithm (GA) have been applied to determine the optimal generation schedule for economic dispatch problem in a power system [3]. SA is applied in many power system problems, but setting the control parameters of the SA algorithm is a difficult task, and the convergence speed is slow when applied to a real system. The GA methods have been employed to successfully solve complex optimization problem, recent research has identified some deficiencies in GA per-

formance. Particle swarm optimization method (PSO) has been applied for solving economic dispatch problems with various operating constraints [4].

A novel optimization approach, Artificial Immune System (AIS) has been applied to solve constrained economic load dispatch problem [5]. This approach utilizes the clonal selection principle and evolutionary approach wherein cloning of antibodies is performed followed by hyper mutation. A novel coding scheme for practical economic dispatch by modified particle swarm optimization approach has also been proposed to solve economic dispatch problem [6]. The heuristic search technique, Differential Evolution has been suggested for solving economic dispatch problems [7]. Bacterial Foraging-Nelder Mead method has been applied for the solution of economic dispatch problems [8].

In certain fossil fire systems, the generation cost function is represented as a segmented piecewise quadratic function. The generating unit, supplied with multi-fuel sources like coal, natural gas or oil suffers with the problem of determining the most economic fuel to burn. Such a problem has been solved using the Hierarchical Method (HM) of Lagrangian multipliers method to find the incremental fuel cost for subsystems comprising sets of units [9]. The solution searches for the optimal for various choices of fuel and generation range of the units iteratively. A Hopfield neural network approaches to

economic dispatch problems has been proposed [10]. An improved adaptive Hopfield Neural Network (HNN) approach has been proposed for finding the solution for economic dispatch with multiple fuel options [11]. The HNN suffers with slow convergence rate and normally takes a large number of iterations. A hybrid real coded GA method has been presented for solving the economic dispatch problem with multiple fuel options [12].

An enhanced Lagrangian neural network has been applied to solve the economic load dispatch problems with piecewise quadratic cost functions [13]. In this method the convergence speeds are enhanced by employing by momentum technique and providing criteria for choosing the learning rate. Economic dispatch solutions with piecewise quadratic cost functions has been solved by using improved genetic algorithm [14]. In order to improve the effectiveness of genetic algorithm multi-stage algorithm and directional crossover methods are proposed and projection method is introduced to satisfy a linear equality constraint from power balance. The heuristic search techniques such as PSO [15], Taguchi method (TM) [16], Evolutionary Programming (EP) [17] and its improved version are also been applied to solve the economic dispatch problems with multiple fuel options [18,19].

The CCF is a non-iterative direct method, gives the most economic dispatches of the online units with less computation time. In this paper, the CCF is used to solve the economic dispatch problem with multiple fuel options.

## 2. Problem Formulation

### 2.1. Nomenclature

$a_i, b_i, c_i$	Fuel cost coefficients of the units
$a_{sj}, b_{sj}, c_{sj}$	Fuel cost coefficients of the set $S$ with $k$ fuel options
$A, B, C$	Composite cost coefficients
$A_S, B_S, C_S$	Composite cost coefficients of the set $S$
$A_P, B_P, C_P$	Composite cost coefficients of the plant
$FC_{Sj}$	Fuel cost function of the set $S$ with $k$ fuel options, in \$/h
$FC_S$	Fuel cost function of the set $S$ , in \$/h
$FC_P$	Fuel cost function of the plant, in \$/h
$k$	Number of fuel options in a plant
$N$	Number of generation units
$P_S^{\min}$	Minimum power generation of set $S$ , in MW
$P_S^{\max}$	Maximum power generation of set $S$ , in MW
$P_S$	Economic dispatch of the set $S$ , in MW
$P_G$	Power generation of the plant, in MW

$P_D$	Power demand, in MW
$S$	Set of generating units in a plant
$\lambda$	Incremental production cost, in \$/MWh

### 2.2. Economic Dispatch Problem with Multiple Fuels

The main objective of economic dispatch is to find the optimal combination of power generation that minimizes the total generation cost while satisfying equality and inequality constraints. A piecewise quadratic function is used to represent the input-output curve of a generator with multiple fuel options. For a generator with  $k$  fuel options, the cost curve is divided into  $k$  discrete regions between lower and upper bounds. The economic dispatch problem with piecewise quadratic function is defined as

$$\text{Minimize } \sum_{i=1}^N F_i(P_i)$$

$$F_i(P_i) = \begin{cases} a_{i1}P_i^2 + b_{i1}P_i + c_{i1}, & \text{fuel 1, } P_i^{\min} \leq P_i \leq P_{i1} \\ a_{i2}P_i^2 + b_{i2}P_i + c_{i2}, & \text{fuel 2, } P_{i1} \leq P_i \leq P_{i2} \\ \vdots \\ a_{ik}P_i^2 + b_{ik}P_i + c_{ik}, & \text{fuel } k, P_{i(k-1)} \leq P_i \leq P_i^{\max} \end{cases} \quad (1)$$

where  $F_i(P_i)$  is the fuel cost function of the  $i^{\text{th}}$  unit,  $P_i$  is the power output of the  $i^{\text{th}}$  unit,  $N$  is the number of generating units in the system and  $a_{ik}$ ,  $b_{ik}$  and  $c_{ik}$  are cost coefficients of the  $i^{\text{th}}$  unit using fuel type  $k$ .

Minimization of the generation cost is subjected to the following constraints:

1) *The power balance constraints*

$$\sum_{i=1}^N P_i = P_D \quad (2)$$

where  $P_D$  is the total system demand in MW.

2) *Generating capacity constraints*

$$P_i^{\min} \leq P_i \leq P_i^{\max} \quad (3)$$

where,  $P_i^{\min}$  and  $P_i^{\max}$  are the minimum and maximum power outputs of the  $i^{\text{th}}$  unit.

### 3. Composite Cost Function (CCF)

The composite cost coefficients were reported in the literature [9].

$$A = 1/(1/a_1 + 1/a_2 + \dots + 1/a_N) \quad (4)$$

$$B = (b_1/a_1 + b_2/a_2 + \dots + b_N/a_N)A \quad (5)$$

$$2AP_G + B = \lambda; \text{ where } P_G = P_D$$

$$P_{Gi} = \frac{\lambda - b_i}{2a_i} \text{ where } i = 1, 2, \dots, N \quad (6)$$

$$C = (c_1 + c_2 + \dots + c_N) - (b_1^2/4a_1 + b_2^2/4a_2 + \dots + b_N^2/4a_N) + B^2/4A \quad (7)$$

$$FC_P = A_P P_G^2 + B_P P_G + C_P \quad (8)$$

The  $A$ ,  $B$  and  $C$  are the composite cost coefficients and can be easily calculated by using Equations (4), (5) and (7) respectively. For a particular load demand, the optimal generation of units is directly computed using Equation (6).

#### 4. Proposed Approach for Economic Dispatch with MFO

The proposed methodology consist two stages of the most economic fuel identification and economic scheduling. In the first stage, the composite cost function of the plant is developed as detailed in the previous section. The incremental cost of the plant for a particular load demand is calculated and the generation dispatch for each unit is determined. The dispatch of each set directly indicates the most economical fuel and feasible operating region. In the second stage, the dispatch of the generating units is refined within the feasible operating region. The composite cost function is developed with the selected fuels and is solved to obtain the most economic dispatch of generating units.

##### 4.1. Identification of the Most Economic Fuel

Consider a plant consists of ‘ $S$ ’ set of generation, each set consists of ‘ $k$ ’ fuel options.

The fuel cost function of the set ‘ $S$ ’ with ‘ $k$ ’ fuel options is,

$$FC_{S_j} = a_{S_j} P_S^2 + b_{S_j} P_S + c_{S_j},$$

$$P_S^{\min} \leq P_S \leq P_S^{\max}; \quad (9)$$

$$S = 1, 2, \dots, N; j = 1, 2, \dots, k$$

The composite cost function of set ‘ $S$ ’ is calculated by using Equations (4), (5) and (7).

$$FC_S = A_S P_S^2 + B_S P_S + C_S \quad (10)$$

$$A_S = 1/(1/a_1 + 1/a_2 + \dots + 1/a_k) \quad (11)$$

$$B_S = (b_1/a_1 + b_2/a_2 + \dots + b_k/a_k) A_S \quad (12)$$

$$C_S = (c_1 + c_2 + \dots + c_k) - (b_1^2/4a_1 + b_2^2/4a_2 + \dots + b_k^2/4a_k) + B_S^2/4A_S \quad (13)$$

In this manner, the CCF for the plant is calculated and is given as,

$$FC_P = A_P P_G^2 + B_P P_G + C_P \quad (14)$$

The composite coefficients are constant for any load demands.

The incremental production cost of the plant for the demand is calculated by

$$\lambda = 2A_P P_G + B_P; \text{ where } P_G = P_D \quad (15)$$

The economic dispatch of the set ‘ $S$ ’ is calculated as,

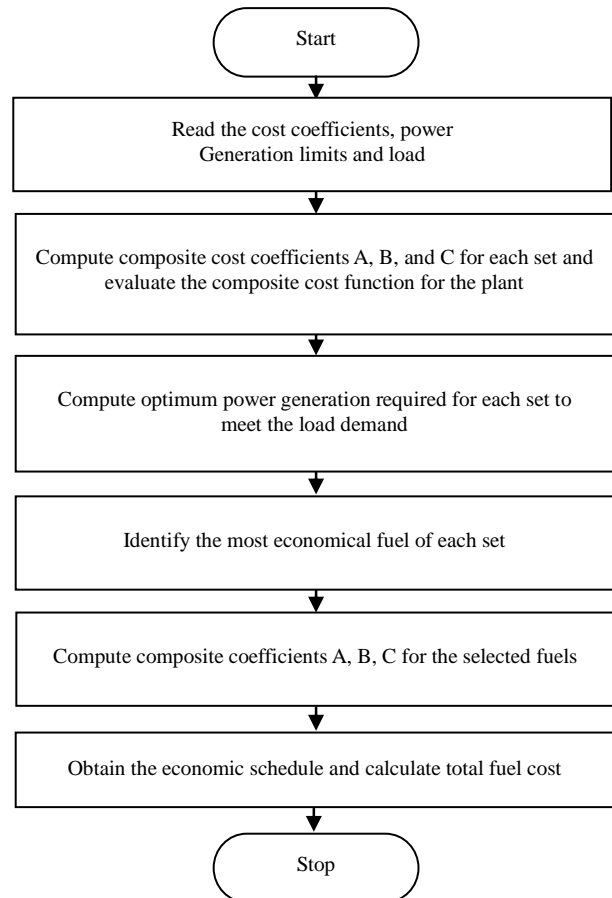
$$P_S = \frac{\lambda - B_S}{2A_S} \quad (16)$$

The above equation provides the dispatch of each set to meet the load demand. Based on this dispatch, the most economical fuel and the feasible operating region of each set can be easily identified.

##### 4.2. Economic Dispatch of the Selected Fuels

The composite cost function is developed using the most economic fuels. The generation dispatch is refined within the feasible limits as detailed in Section 3.

The computational flow of the proposed methodology for solving economic dispatch problem with multiple fuel options is presented as a flow chart in **Figure 1**.



**Figure 1.** Flow chart of the proposed method.

## 5. Numerical Simulation Results and Discussion

The proposed technique has been implemented in MATLAB on a 2.10 GHz core2Duo processor PC. The simulation studies have been carried out on ten-generating unit system with multiple fuel options [9]. This problem includes one objective function with ten variable parameters ( $P_1, P_2, \dots, P_{10}$ ), one equality and twenty inequality constraints, *i.e.* power balance constraint and maximum and minimum limits of each generating unit.

By the proposed strategy, the economic dispatch solution for the given system can be obtained in two stages: 1) evaluate the composite fuel cost function for the plant and is solved to identify the most economic fuel and feasible operating region of each set, and 2) the optimal dispatches are calculated by solving composite fuel cost function with selected fuels.

The implementation of the proposed strategy for the given system is detailed as follows. The equivalent cost function of set 'S' is calculated using the Equations (11), (12) and (13). In the selected sample system the number of sets in the plant is ten and the number of units is twenty nine.

For example, the equivalent cost function of the set 5 is,

$$FC_5 = 0.000044807P_5^2 - 0.0493P_5 + 91.5841 \quad (17)$$

Similarly, for set 8, the equivalent cost function is,

$$FC_8 = 0.000067369P_8^2 - 0.04404P_8 + 126.6098 \quad (18)$$

In this manner, the equivalent cost function of each set is determined. By combining these cost coefficients, the equivalent cost function of the plant is calculated.

The equivalent cost function of the plant is,

$$FC_p = 7.3287e^{-06}P_G^2 + 0.3933P_G - 1518 \quad (19)$$

The incremental cost of the plant is obtained by using Equation (15).

For a load demand of 2400 MW, the  $\lambda$  is 0.4285 and the dispatch of each set is determined by using Equation (16). Then, the dispatches of the set 5 and 8 are 278 MW and 265 MW respectively. In this manner, the dispatch of each set is identified and it indicates the most economical fuel and the feasible operating region. For the set 5, the dispatch is 278 MW, then the most economical fuel is 1 and the feasible operating region is 190 MW to 338 MW. For the set 8, the dispatch is 265 MW, then the most economical fuel is 3 and the feasible operating region is 200 MW to 265 MW.

Similarly, the economic dispatch is performed to determine the optimal dispatches within the feasible oper-

ating region by using the composite cost function of the selected fuels. During the calculation of dispatch, if the generation of any unit violates the effective limits, its generation are fixed at the violated limit. Then that unit is eliminated from the dispatch procedure. The generation of all the units except the violated unit is recalculated using the above procedure with total generation equal to the load demand minus the generation of the limit violated unit.

The simulation is performed for various load demands of 2400 MW, 2500 MW, 2600 MW and 2700 MW. The optimal dispatches obtained by the proposed methodology and HM [9] for the above mentioned load demands are compared and the comparison is presented in **Table 1**. These results signify that the proposed CCF always provides better solution than HM [9]. Though the HM method is an iterative mathematical approach, suffers with the assumption of initial lambda values. In addition, two operating points having same incremental cost also exist and it requires valid assumption to choose the optimal fuel for a particular demand, improper selection will lead infeasible solution. Additionally, the proposed CCF is a direct or non iterative method, it does not demand any initial guess values for economic dispatch of units for the given load demand.

The comparison of total fuel cost obtained by proposed methodology, HM [9], HNN [10], AHNN [11], HGA [12], Modified PSO (MPSO) [15], TM [16], Improved Fast EP (IFEP) [17], Fast EP (FEP) [17], Classical EP (CEP) [17], PSO [18], and Improved EP (IEP) [19] is presented in **Table 2**. As seen the comparison, the generation costs obtained by CCF are lowest among the results. Moreover, the optimal fuel cost obtained through the CCF method is exactly same as the HGA [12], except the load demand of 2600 MW. However, the proposed method directly provides the optimal schedule and it utilizes the CCF for fuel selection and economic scheduling. These comparison results confirm that the CCF provides better solution quality.

The salient features of proposed approach over existing methods are:

- The approach is conceptually simple.
- It is a non iterative method.
- The simplified generalized expression directly gives the most economical fuel and the feasible operating region for a particular load demand.
- It also provides the most economic schedule of generation with less computational effort.
- It requires negligible computational time, hence it is suitable for on line applications.
- The performance of the proposed is independent of system size; hence it is suitable for system of any size.

**Table 1. Comparison of simulation results between proposed method and HM.**

UNIT	LOAD DEMAND = 2400 MW				LOAD DEMAND = 2500 MW			
	PM		HM [9]		PM		HM [9]	
	FT	GEN (MW)	FT	GEN (MW)	FT	GEN (MW)	FT	GEN (MW)
1	1	189.7405	1	193.2	2	206.5190	2	206.6
2	1	202.3427	1	204.1	1	206.4573	1	206.5
3	1	253.8953	1	259.1	1	265.7391	1	265.9
4	3	233.0456	3	234.3	3	235.9531	3	236.0
5	1	241.8297	1	249.0	1	258.0177	1	258.2
6	3	233.0456	1	195.5	3	235.9531	3	236.0
7	1	253.2750	1	260.1	1	268.8635	1	269.0
8	3	233.0456	3	234.3	3	235.9531	3	236.0
9	1	320.3832	1	325.3	1	331.4877	1	331.6
10	1	239.3969	1	246.3	1	255.0562	1	255.2
UNIT	LOAD DEMAND = 2600 MW				LOAD DEMAND = 2700 MW			
	PM		HM [9]		PM		HM [9]	
	FT	GEN (MW)	FT	GEN (MW)	FT	GEN (MW)	FT	GEN (MW)
1	2	216.5442	2	216.4	2	218.2499	2	218.4
2	1	210.6058	1	210.9	1	211.6626	1	211.8
3	1	278.1441	1	278.5	1	280.7228	1	281.0
4	3	239.0967	3	239.1	3	239.6315	3	239.7
5	1	275.5154	1	275.4	1	278.4973	1	279.0
6	3	239.0967	3	239.1	3	239.6315	3	239.7
7	1	285.7585	1	285.6	1	288.5845	1	289.0
8	3	239.0967	3	239.1	3	239.6315	3	239.7
9	1	343.8134	1	343.3	3	428.5216	3	429.2
10	1	271.5861	1	271.9	1	274.9667	1	275.2

**Table 2. Comparison of the total cost with different techniques.**

TECHNIQUES	TOTAL FUEL COST (\$/h)			
	2400 MW	2500 MW	2600 MW	2700 MW
HM [9]	488.500	526.700	574.030	625.180
HNN [10]	487.870	526.130	574.260	626.120
AHNN [11]	481.700	526.230	574.370	626.240
HGA [12]	481.7226	526.2388	574.3808	623.8092
MPSO [15]	481.723	526.239	574.381	623.809
TM [16]	481.6	-----	-----	623.7
IFEP [17]	-----	526.25	-----	-----
FEP [17]	-----	526.26	-----	-----
CEP [17]	-----	526.25	-----	-----
PSO [18]	-----	-----	-----	623.88
IEP [19]	481.779	526.304	574.473	623.851
PROPOSED METHOD	481.7226	526.2338	574.0105	623.8092

## 6. Conclusions

The economic dispatch problem with multiple fuel options is a complex optimization problem whose importance may increase as competition in power generation intensifies. This paper presents economic dispatch problem with multiple fuel options using composite cost function. The proposed CCF based solution for economic dispatch with MFO offers a best contribution in the area of economic dispatch. In contrast to the HM [9], this approach fully explores the cost coefficients and gives a promising value of power for providing improved economic dispatch. The HM method requires the valid assumptions such as initial value of lambda and it iteratively solves the problem. The proposed methodology is a non-iterative method that directly gives the optimal generation schedule of the generating set and it does not require any assumptions. The numerical results demonstrate that the proposed approach offers a better convergence rate, minimum cost to be achieved and better solution than the existing methods. As power systems are usually large scale systems, the proposed method may be suggested for the solution of economic load dispatch problems and it is also suitable for online applications.

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