Engineering, 2016, 8, 646-656
http://www.scirp.org/journal/eng
ISSN Online: 1947-394X
ISSN Print: 1947-3931

Application of Fractal Theory in Brick-Concrete Structural Health Monitoring

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How to cite this paper: Yang, C.M. Zhao, X., Yao, Y.F. and Zhang, Z.Q. (2016) Application of Fractal Theory in Brick-Concrete Structural Health Monitoring. *Engineering*, **8**, 646-656.
http://dx.doi.org/10.4236/eng.2016.89058

Received: August 4, 2016
Accepted: September 26, 2016
Published: September 29, 2016

Abstract

In order to monitor and forecast the deformation of the brick-concrete building, by taking a brick-concrete building as research object, fiber grating sensors were used to collect the monitoring data and double logarithmic curve of limit value characteristic and monitoring data were obtained based on the fractal theory. Constant dimension fractal method cannot be used to analyze the data directly. With the method of variable dimension fractal, we accumulate data, and the double logarithmic curve is smooth. Piecewise fractal dimensions are close. The outer interpolation method is used to calculate the fractal dimension of the next point and then back calculate the vertical displacement. The relative errors are calculated by comparing the forecast values and monitoring values, and the maximum relative error is 5.76%. The result shows that the fractal theory is suitable to use in the forecast of the deformation and the accuracy is good.

Keywords

Brick-Concrete Building, Real-Time Monitoring, Fiber Grating Sensors, Constant Dimension Fractal, Variable Dimension Fractal, Log-Log Line, Prediction

1. Introduction

In China, the structure is generally divided into brick-concrete building, post and panel structure, and reinforced concrete structure, and these buildings during service are bound to produce cumulative damage affected by corrosion, fatigue, aging and other factors, so it is particularly important to monitor the buildings which are on active service.

Health monitoring is an important means to understand the situation of the buildings in use and to layout reasonable monitoring position; analyzing and processing monitoring data will be a key step to master the state of buildings in use; reasonable data processing and effective monitoring models can help staffs to discover abnormities and they can take appropriate measures to ensure the security of the persons and property within the building [1]-[3]. With the progress and mutual integration of modern testing, analysis technics, computer technology, mathematical theory and wireless communication technology, and the traditional monitoring methods are changing to an online, dynamic, real-time direction [4] [5]. Fiber grating technology applying on-line monitoring is a major breakthrough in the field of civil engineering structures or buildings, it has aroused widespread attention [6]-[9].

Fractal geometry developed by the Mandelbrot [10] in the 1970s is a new branch of mathematics, it focuses on the similarity between the parts and the whole, starting directly from the complex non-linear system, and it recognizes the inherent regularity [11] [12] by the no simplified and abstract study itself. The fractal theory applied to fault diagnosis and monitoring of structural damage is only appeared recently; fault diagnosis and damage identification of fractal theory has been widely developed in the field of machinery, aerospace, ships, vehicles [13]-[16], but in the field of civil engineering [17] seems less. Yongqiang Jin [18] makes a more reliable prediction to dam uplift pressure with the application of fractal theory. Displacement-time curve variability of buildings under various loads also satisfies the self-similarity of fractal theory in the process of service, therefore, the fractal theory applied to health monitoring and early warning is theoretically feasible. The traditional health monitoring methods are time series, neural network, etc., the theory is applied to analyze the structure by the observational data, and analyze the characteristics of a specific observed quantity in particular period of time and then predict its future variation by extrapolation methods, but it requires a large amount of observation data. When the observed quantities and the range of time change, the model also need to do a great change, and we cannot find any common between observed quantities of building systems. Fractal theory is a very important part in the modern non-linear science research, it can describe some kinds of scientific complexity of nature and various non-linear systems encountered, and it requires only small amount of data (15 - 30 values) to predict accurately, so the calculation is faster than traditional forecasting methods; it is a new method of processing prototype observation data of buildings.

Fractal theory means the parts in some way are similar to the whole, under normal circumstances it can be regarded as the state of gathering of fragments. It generally has the following characteristics: 1) fractal sets have a ratio of the details of any small scales, or have a fine structure; 2) fractal sets have some self-similar forms, they may be approximate self-similarity or statistical self-similarity; 3) the fractal dimension of fractal sets is strictly greater than its corresponding topological dimension.

The remote monitoring and warning system used in the experiment includes: fiber sensing systems, signal acquisition and transmission systems, data processing and monitoring and warning system. Fiber grating sensor system includes: types of fiber grating sensors, modulation system and installation of fiber grating sensors. The system of signal transmission and collection includes a correction of fiber grating sensors, application of module, storage structure and methods of vast amounts of real-time data. Data processing, monitoring and warning system is a key part of this experiment, including visualization system of data analyzing and structure running status, and the function of disaster early warning. This study is based on the fractal theory, using the methods of constant dimension fractal and variable dimension fractal to deal with the data collected by fiber grating sensors, and we can predict the vertical displacement of the next point in time, and then to achieve the goal of monitoring and early warning.

2. Constant Dimension Fractal and Variable Dimension Fractal

At present, the application of constant dimension fractal is described by Equation (1)

$$N = \frac{C}{r^D} \tag{1}$$

Here: r is the characteristic linearity; N is the function associated with r; C is undetermined constant; D is the dimension. Because D is a constant, Equation (1) in log-log line is a straight line, so any two data points (r_i, N_i), (r_j, N_j) Can determine D and C of the Equation (1) as follows:

$$D = \frac{\ln(N_i/N_j)}{\ln(r_j/r_i)} \tag{2}$$

$$C = N_i r_i^D = N_j r_j^D \tag{3}$$

If there is a negative number in the logarithm operation, all the values of this sequence plus a constant to eliminate the impact of negative number. However, in the curve of log-log line, if there is a nonlinear function, this constant dimension fractal cannot be dealt with. Variable dimension fractal can be introduced to solve the problem, specifically as follows:

Because the fractal dimension D is a function of the characteristic linearity r:

$$D = F(r) \tag{4}$$

Then a Function relationship $N = f(r)$ between N and r can be described by available variable dimension fractal form, $f(r) = \frac{C}{r^D}$, we can obtain that:

$$D = \frac{\ln C - \ln f(r)}{\ln r} \tag{5}$$

That is a form of variable dimension fractal.

We can know from the Equation (5), any functions the same as $N = f(r)$ can be translated into constant dimension fractal forms, however, in practical engineering, the function is unknown, only discrete data points. In response to this problem, literature [18] presented that data can be accumulated according to a series of conversions, transformed data always can be dealt with in the method of available constant dimension fractal. First, we should plot the raw data points $(k = 1, 2, \cdots, n)$ in a curve of log-log line, and substantially arranged N_k in a sequence, that is:

$$\{N_k\} = \{N_1, N_2, N_3, \cdots\} \quad (k = 1, 2, \cdots, n) \tag{6}$$

Then use the basic sequence to construct other cumulative sequence. If you construct a first order accumulation sequence $\{Z1\}$, here, $Z1_1 = N_1$, $Z1_2 = N_1 + N_2$, $Z1_3 = N_1 + N_2 + N_3$, $Z1_4 = N_1 + N_2 + N_3 + N_4$, $\cdots$, similarly we can construct an S-order cumulative sequence. Such as:

$$\begin{aligned}
&\{Z1_k\} = \{N_1, N_1 + N_2, N_1 + N_2 + N_3, \cdots\} \\
&\{Z2_k\} = \{Z1_1, Z1_1 + Z1_2, Z1_1 + Z1_2 + Z1_3, \cdots\} \\
&\{Z3_k\} = \{Z2_1, Z2_1 + Z2_2, Z2_1 + Z2_2 + Z2_3, \cdots\} \\
&\{Z4_k\} = \{Z3_1, Z3_1 + Z3_2, Z3_1 + Z3_2 + Z3_3, \cdots\} \\
&\qquad \vdots \\
&\{ZS_k\} = \{Z(S-1)_1, Z(S-1)_1 + Z(S-1)_2, Z(S-1)_1 + Z(S-1)_2 + Z(S-1)_3, \cdots\}
\end{aligned} \tag{7}$$

and $k = 1, 2, 3, \cdots, n$. Then we build variable dimension fractal model of cumulative sequence of each order, in terms of fist order accumulation, in the log-log line coordinate, the individual data points $(Z1_k, r_k)$ are connected in turn, so we can obtain piecewise variable dimension fractal model. Finally, choose the best conversion in the piecewise variable dimension fractal model and calculate the fractal dimensions of the subsections, then select a best broken respective line, then we use the interpolation method to calculate fractal parameters need to be predicted of each subsection, finally we anti-derived the data need to be predicted by the Equation (2).

3. Monitoring General Situation about Brick-Concrete Structure

3.1. Monitoring Purpose and the Arrangement of Measuring Points

The test takes the telecommunication building of Hebei University as research object. The telecommunication building belongs to brick-concrete building, 6 layers, 5 layers of main building, built in 1973.12-1976.12, parts of beams, columns exists aging, corrosion and other phenomena. Damage of the aging, corrosion place can easily occur in the future, the main building appears minute vibrations, deformation under the environmental loads, then the vibrations and deformation may lead to crack development, even worse, the whole building may collapse. To make sure the safety of the whole building, teachers and students, we analyze the status of the telecommunication building, then make sure the physical types and the appropriate sensor type and the sensor location.

In the outer wall surface of each floor of typical aging parts, we put FBG surface crack meters to monitor the development of main cracks. In the southeast, northeast, southwest and northwest corners of the main building roof, we install a FBG fiber level to monitor the vertical deformation of main building under outside interference. This is because the top of the building is the most sensitive to external factors, and it seems inconspicuous to artificial disturbance. In the typical beams, columns of the building, we arrange FBG strain gauge to monitor the strain of the beams and columns under outside interference, monitoring point arrangement is as shown in **Figure 1**, each monitoring instrument parameters are as shown in **Table 1** (instrument parameters), the monitor-

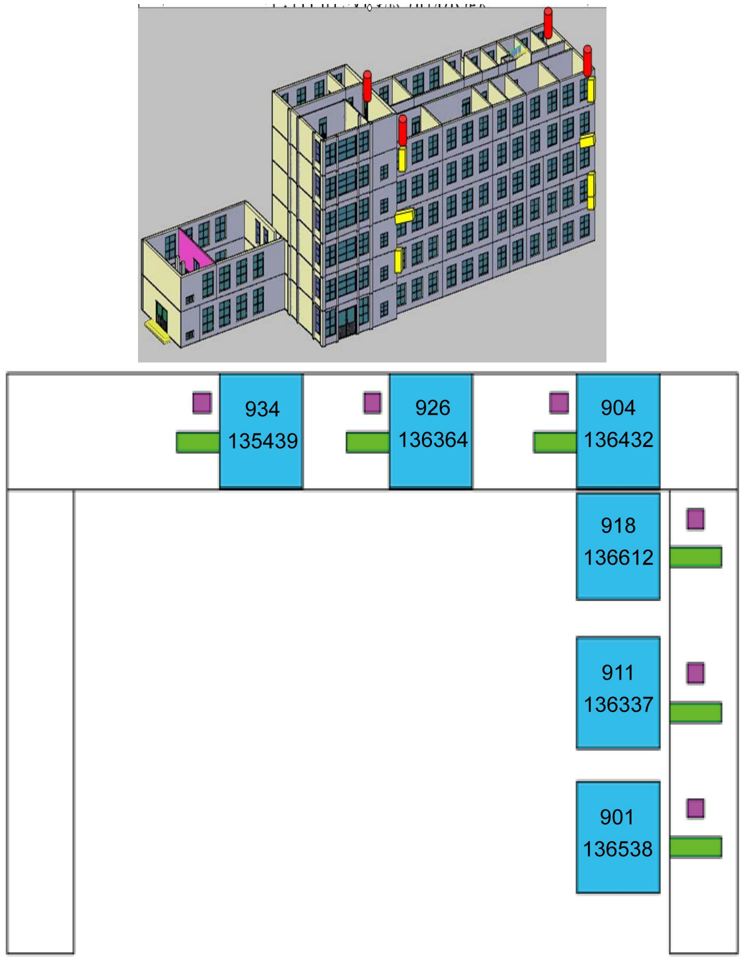

Figure 1. Structure model and arrangement of measuring points.

Table 1. Instrument parameters.

Equipment name	Model	Standard Range	Accuracy /%FS	Sensitivity /%FS	Temperature compensation	frequency /Hz
FBG meter Crack	BSIL-GS600	200 mm	0.3	0.1	Internal	100
FBG Level	BGK-FBG-4675T	100 mm	≤0.1	0.1	Internal	100
FBG strain gauge	BSIL-GS220T	±1500 με	0.3	0.1	Internal	100

ing devices are as shown in Figure 2 (Measuring device).

3.2. Data Analysis

Due to the huge amount of test data, the test is in order to deal with the level monitoring data, we cut out 30 s data of vertical displacement from the northeast corner of the telecommunication building roof, we cut out a data point every second, a total of 30 data points, we take these points as the research object, then we numbers them in chronological order, we use the former 20 points to build prediction model, then use the latter

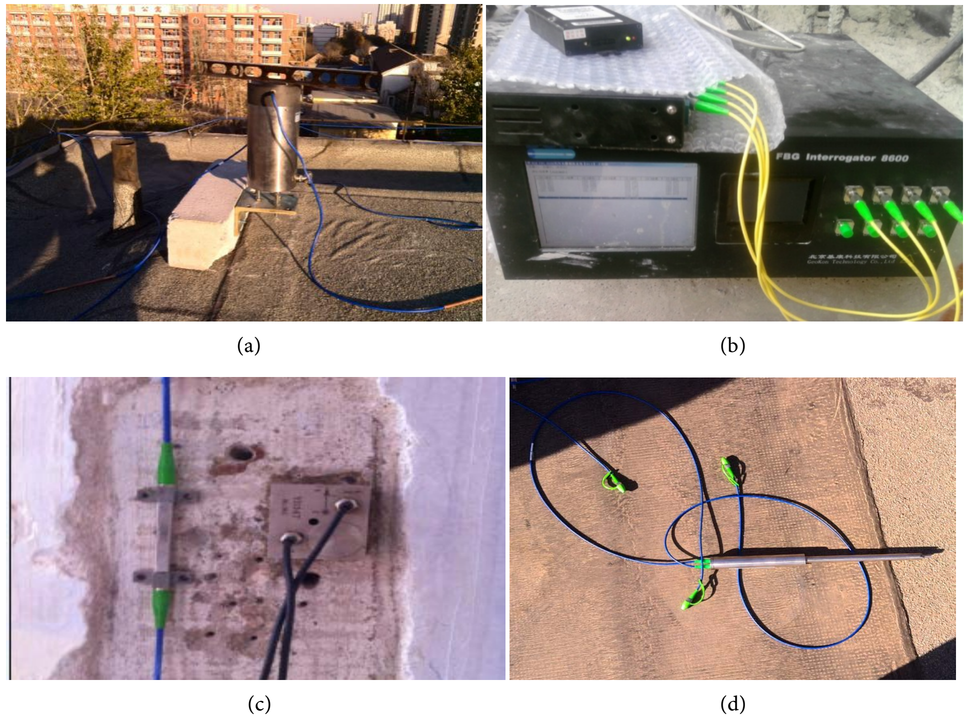

Figure 2. Measuring device. (a) roof level; (b) speed fiber grating demodulator; (c) strain gage; (d) crack meter.

10 points to check the correctness of the prediction model.

Table 2 shows that: this part of the vertical displacement includes positive number and negative number, however, the negative number will be unable to operate in the logarithmic coordinates, then we plus 0.2 for them all to eliminate the impact, the processed data is shown in Table 3 (after processing).

The data after processing can obtain piecewise fractal dimension of monitoring data according to Equation (2), Table 4 (Piecewise fractal dimension of monitoring data):

We build log-log coordinate according to the data in Table 4 (Piecewise fractal dimension of monitoring data), then draw the $(N_i, r_i)(i = 1, 2, 3, 4, \cdots, 20)$ in the log-log coordinate, we can see from the Figure 3 (The log-log curve of the original monitoring data): log-log curve has a big fluctuation, piecewise fractal dimension (D) includes negative number and positive number, so it is difficult to predict.

So we deal with the original monitoring data by first-order accumulation, depending on Equation (7), then we obtain the piecewise fractal dimension (Table 5 the monitoring data of first-order accumulative piecewise fractal dimension). Draw $(r_k, Z1_k)$ in the log-log coordinates (Figure 4, the monitoring data of first-order accumulative double logarithmic curve),we can see in the Table 4 (Piecewise fractal dimension of monitoring data): the original complex curve change into a relatively smooth curve, each of the piecewise fractal dimension seems similar, so we can predict unknown piecewise fractal dimension according to the known piecewise fractal dimension, then we can anti-derived displacement, this will achieve the purpose of early warning.

Table 2. The original data of northeast.

Sequence (r)	Monitoring values (N) mm	Sequence (r)	Monitoring values (N) mm
1	−0.064	11	0.120
2	−0.032	12	0.096
3	0.008	13	−0.048
4	−0.120	14	−0.056
5	−0.088	15	0.008
6	0.096	16	−0.144
7	0.040	17	−0.008
8	0.024	18	0.040
9	−0.032	19	0.016
10	−0.040	20	−0.032

Table 3. After processing.

Sequence (r)	Monitoring values (N) mm	Sequence (r)	Monitoring values (N) mm
1	0.136	11	0.320
2	0.168	12	0.296
3	0.208	13	0.152
4	0.080	14	0.144
5	0.112	15	0.208
6	0.296	16	0.056
7	0.240	17	0.192
8	0.224	18	0.240
9	0.168	19	0.216
10	0.160	20	0.168

3.3. The Prediction on of Monitoring Data

By comparing Figure 3 (The log-log curve of the original monitoring data) and 4 (the monitoring data of first-order accumulative double logarithmic curve), a cumulative fist-order log-log curve (fractal dimension curve) can act as a predictive model to predict the data of the telecommunication building, and using equations to predict the 10 values in the next time. Specific methods are as follows:

1. Figure out the total increment $X = D_{19,20} - D_{1,2}$ by Table 4 (Piecewise fractal dimension of monitoring data), so the average of the neighboring piecewise fractal dimension is. $\bar{X} = X/19$

2. Figure out the piecewise fractal dimension after the19th subsection.

$$D_{k,k+1} = D_{19,20} + (k-19)\bar{X} \quad (k > 19) \tag{8}$$

Table 4. Piecewise fractal dimension of monitoring data.

Sequence(r)	Measurements (N) mm	ln r	ln N	D
1	0.136	0.000000	−1.995100	----
2	0.168	0.693147	−1.783791	−0.304855
3	0.208	1.098612	−1.570217	−0.526738
4	0.080	1.386294	−2.525729	3.321417
5	0.112	1.609438	−2.189256	−1.507874
6	0.296	1.791759	−1.217396	−5.330489
7	0.240	1.945910	−1.427116	1.360484
8	0.224	2.079442	−1.496109	0.516678
9	0.168	2.197225	−1.783791	2.442475
10	0.160	2.302585	−1.832582	0.463089
11	0.320	2.397895	−1.139434	−7.272563
12	0.296	2.484907	−1.217396	0.895302
13	0.152	2.564949	−1.883875	8.326616
14	0.144	2.639057	−1.937942	0.729570
15	0.208	2.708050	−1.570217	−5.329889
16	0.056	2.772589	−2.882404	20.331690
17	0.192	2.833213	−1.650260	−20.324360
18	0.240	2.890372	−1.427116	−3.903917
19	0.216	2.944439	−1.532477	1.948712
20	0.168	2.995732	−1.783791	4.899577

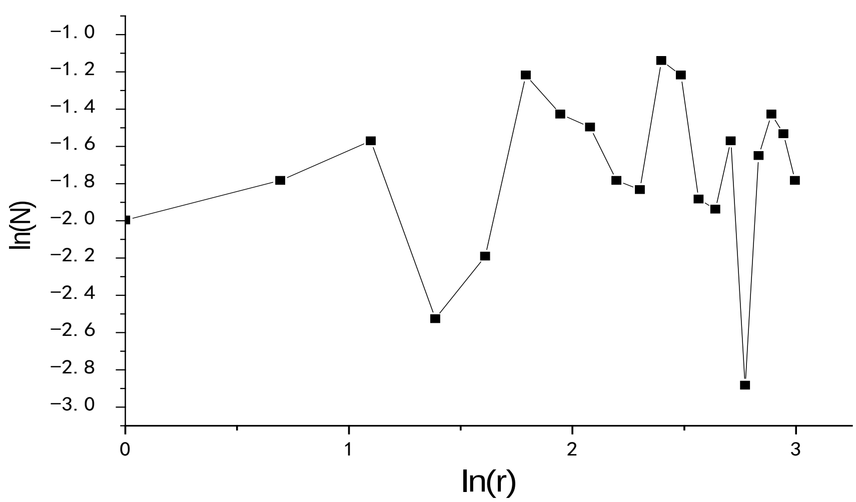

Figure 3. The log-log curve of the original monitoring data.

Table 5. The monitoring data of first-order accumulative piecewise fractal dimension.

sequence (r)	Measurements (N) mm	$\ln r$	$\ln N$	D'
1	0.136	0.000000	−1.995100	----
2	0.168	0.693147	−1.190728	−1.160464
3	0.208	1.098612	−0.669431	−1.285677
4	0.080	1.386294	−0.524249	−0.504661
5	0.112	1.609438	−0.350977	−0.776503
6	0.296	1.791759	0.000000	−1.925050
7	0.240	1.945910	0.215111	−1.395456
8	0.224	2.079442	0.381172	−1.243605
9	0.168	2.197225	0.489806	−0.922323
10	0.160	2.302585	0.583332	−0.887680
11	0.320	2.397895	0.747635	−1.723880
12	0.296	2.484907	0.878797	−1.507401
13	0.152	2.564949	0.940007	−0.764724
14	0.144	2.639057	0.994732	−0.738449
15	0.208	2.708050	1.068840	−1.074138
16	0.056	2.772589	1.087888	−0.295144
17	0.192	2.833213	1.150572	−1.033975
18	0.240	2.890372	1.223775	−1.280691
19	0.216	2.944439	1.285368	−1.139198
20	0.168	2.995732	1.330782	−0.885384

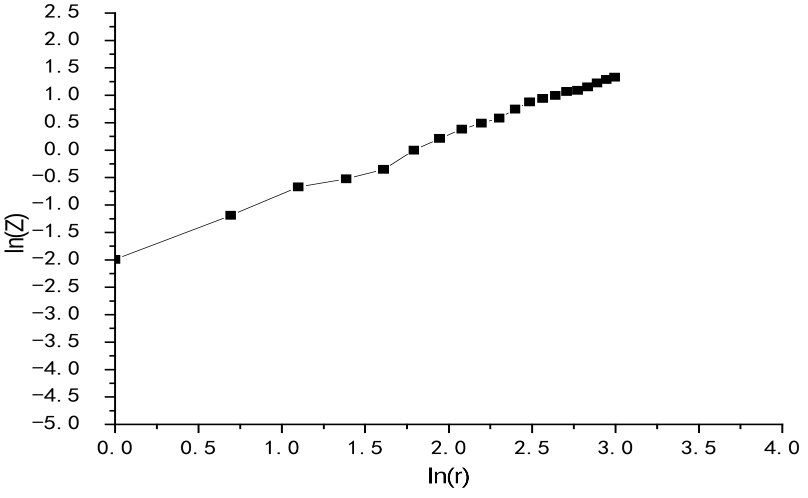

Figure 4. The monitoring data of first-order accumulative double logarithmic curve.

3. depending on the equation we can obtain:

$$N'_{k+1} = N_k \left(r_k / r_{k+1} \right)^{D_{k,k+1}} \quad \left(20 \le k \le 40 \right) \tag{9}$$

We can predict the displacement of the sequence (21 ~ 30), substituting D of Equation (8) into Equation (9). According to the result, we can discover that the relative error is between −5.74% and +5.76% (Table 6, the prediction results of variable dimension fractal). So the method of variable dimension fractal can predict the structural deformation.

4. Conclusion

The result of the test shows that FBG sensing technology can achieve the goal of the remote real-time dynamic prediction to the deformation and displacement of the brick-concrete buildings. Through processing the displacement-time graph, we can visually monitor the state of buildings. Health monitoring data usually presents self-similarity and satisfies the conditions of application of fractal theory. Fractal theory can make a reasonable assessment quickly for the health status of the buildings which are in use after analyzing the dimension changes of displacement curve. When the range of the measurements and time changes, there is no need to change the prediction model, and the similarity of systematic measurements of the brick-concrete structures can be reflected. Displacement data which FBG sensors collect meet the fractal characteristics, but D, piecewise fractal dimension, has a big fluctuation, then variable dimension fractal can be a predictive model to monitor brick-concrete buildings, and the relative error between predictive value and true value ranges from −5.74% to +5.76%, so accuracy of the prediction is better than others. By setting the alarm value of the building, fractal theory provides a new type of monitoring, early warning methods for the practical engineering. Related conclusions have yet to be studied further.

Table 6. The prediction results of variable dimension fractal.

Sequence (r)	Measurements (N) mm	First-order cumulative fractal dimension D'	Predictive value (N') mm	Relative error %
21	0.176	−0.870906	0.175	−0.57
22	0.188	−0.856428	0.183	−2.66
23	0.185	−0.841950	0.195	+5.41
24	0.196	−0.827472	0.192	−2.04
25	0.191	−0.812994	0.202	+5.76
26	0.203	−0.798516	0.197	−2.96
27	0.195	−0.784038	0.206	+5.64
28	0.190	−0.769560	0.199	+4.73
29	0.209	−0.755082	0.197	−5.74
30	0.221	−0.740604	0.213	−3.62

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