

Viscoelastic Effects on Unsteady Two-Dimensional and Mass Transfer of a Viscoelastic Fluid in a Porous Channel with Radiative Heat Transfer

Utpal Jyoti Das

Department of Mathematics, Rajiv Gandhi University, Itanagar, India

Email: utpaljyotidas@yahoo.co.in

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ABSTRACT

An analysis of oscillatory flow of a viscoelastic fluid and mass transfer along a porous oscillating channel with radiative heat transfer in presence of first-order chemical reaction is considered. The problem is concerned with the flow through a channel in which the viscoelastic fluid is injected on one boundary of the channel with a constant velocity, while it is sucked off at the other boundary with the same velocity. The two boundaries are considered to be in close contact with the two plates placed parallel to each other. The effect of temperature oscillations at the plate (upper wall) where the suction takes place is taken into consideration. The plates are supposed to be oscillating with a given velocity in their own planes. Analytical expressions for velocity profile, the temperature, concentration profile, wall shear stress on the upper wall are obtained. The profiles of the velocity and skin friction have been presented graphically for different values of the viscoelastic parameters with the combination of the other flow parameters encountered in the problem under investigation. It is observed that velocity decrease with the increasing values of the viscoelastic parameter in comparison with Newtonian fluid. Also, the wall shear stress increase with the increasing values of the viscoelastic parameter.

Keywords: Viscoelastic Fluid; First-Order Chemical Reaction; Porous Wall; Oscillating Channel; Radiative Heat Transfer

1. Introduction

During past several years considerable interest has been evinced in the study of the problem of hydrodynamic flow in a porous channel with radiative heat transfer because of its various applications in physiology and in engineering devices such as blood flow in arteries, transpiration cooling of re-entry vehicles and rocket busters, cross-hatching on ablative surfaces. Pulsatile flow of a fluid in a porous channel has been investigated by Wang [1], as well as Bhuyan and Hazarika [2] by considering the periodic pressure gradient. Raptis [3] studied the unsteady free convective flow through a porous medium bounded by an infinite vertical limiting surface with constant suction and time dependent temperature. The effect of Hall current and wall temperature oscillation on convective flow in a rotating fluid through porous medium was studied by Ram [4]. On the other hand several other researchers (e.g. Makinde and Mhone [5], Prakash and Ogulu [6] as well as Mehmood and Ali [7]) investigated the effects of heat transfer in the flow of fluids. Adhikary and Misra [8] investigated the effects of porosity of the channel wall, magnetic field and radiative heat transfer on unsteady flow of an electrically conducting fluid

through a channel. Ghosh [9] investigated the hydrodynamic fluctuating flow of a viscoelastic fluid in a porous channel, where the channels oscillate with a given velocity in their own planes. The effect of mass transfer on the flow past an infinite vertical oscillating plate in the presence of constant heat flux has been studied by Soundalgekar *et al.* [10]. Kim and Lee [11] reported an analytical study on the MHD oscillatory flow of a micropolar fluid over a vertical porous plate. Chamkha [12] studied unsteady two dimensional convective heat and mass transfer boundary layer flow of a viscous incompressible electrically conducting temperature-dependent heat absorbing fluid along a semi infinite vertical permeable moving plate with thermal and concentration buoyancy effects.

The purpose of the present work is to investigate the effects of viscoelastic parameter on unsteady two dimensional hydrodynamic flow and heat transfer of a viscoelastic fluid in a porous channel. The problem is concerned with the flow through a channel in which the fluid is injected in one plate with a constant velocity and it is sucked off by the other with the same velocity. The plates are considered to be oscillating with a given veloc-

ity in their own planes. In the upper wall the oscillation of the temperature is considered. It is assumed that the chemical reaction is of first-order. One of the most popular models for non-Newtonian fluids is the model that is called the second-order fluid or fluid of second grade. It is reasonable to use the second-order fluid model to do numerical calculations. The effects of viscoelastic parameter with the combinations of the other flow parameters have been studied thoroughly and presented graphically.

2. Mathematical Formulation

Consider the channel flow between two oscillating porous plates $y=0$ and $y=h$, the fluid is being injected by one plate with constant velocity V and sucked off by the other plate with the same velocity. Then the continuity equation reduces to $\frac{\partial u^*}{\partial x^*} = 0$ so that u^* is the function of y^* and t^* only.

The constitutive equation for the incompressible second-order fluid is

$$S = -pI + \mu_1 A_1 + \mu_2 A_2 + \mu_3 (A_1)^2 \quad (1)$$

where S is the stress tensor, p is the hydrostatic pressure, $A_n, n=1,2$ are the kinematic Rivlin-Ericksen tensors, μ_1, μ_2, μ_3 are the material co-efficients describing the viscosity, visco-elasticity and cross-viscosity respectively, where μ_1 and μ_3 are positive and μ_2 is negative (Coleman and Markovitz [13]). The Equation (1) was derived by Coleman and Noll [14] from that of the simple fluids by assuming that the stress is more sensitive to the recent deformation than to the deformation that occurred in the distant past.

The momentum equations are given by

$$\begin{aligned} & \rho \left(\frac{\partial u^*}{\partial t^*} + V \frac{\partial u^*}{\partial y^*} \right) \\ &= -\frac{\partial p^*}{\partial x^*} + \mu_1 \frac{\partial^2 u^*}{\partial y^{*2}} + \mu_2 \left(\frac{\partial^3 u^*}{\partial y^{*2} \partial t^*} + V \frac{\partial^3 u^*}{\partial y^{*3}} \right) \quad (2) \\ & -\frac{\mu_1 u^*}{k^*} + g \rho \beta_T (T - T_0) + g \rho \beta_C (C - C_0) \\ & 0 = -\frac{\partial p^*}{\partial y^*} + (2\mu_2 + \mu_3) \frac{\partial}{\partial y^*} \left(\frac{\partial u^*}{\partial y^*} \right)^2 \end{aligned}$$

so that $0 = -\frac{\partial p^*}{\partial y^*}$, assuming $2\mu_2 + \mu_3 = 0$ as $\mu_2 < 0$

and

$$\mu_3 > 0. \quad (3)$$

The heat transfer equation may be put in the form

$$\frac{\partial T}{\partial t^*} + V \frac{\partial T}{\partial y^*} = \frac{k'}{\rho C_p} \frac{\partial^2 T}{\partial y^{*2}} - \frac{1}{\rho C_p} \frac{\partial q}{\partial y^*} \quad (4)$$

The concentration equation may be put in the form

$$\frac{\partial C}{\partial t^*} + V \frac{\partial C}{\partial y^*} = D \frac{\partial^2 C}{\partial y^{*2}} \quad (5)$$

where p^* is the pressure, ρ the density of the fluid, k permeability factor, q the radiative heat flux, β_T the coefficient of volume expansion due to temperature, β_C the coefficient of volume expansion due to concentration, g the gravitational acceleration, k' the coefficient of thermal conductivity, C_p the specific heat at constant pressure, C the concentration of species, D chemical molecular diffusivity.

The corresponding boundary conditions of the oscillatory motion are:

$$\begin{aligned} u^* &= U_0 e^{i\omega^* t^*}, T = T_w + e^{i\omega^* t^*} (T_w - T_0), \\ C &= C_w + e^{i\omega^* t^*} (C_w - C_0) \quad \text{at } y^* = h \\ u^* &= U_0 e^{i\omega^* t^*}, T = T_0, C = C_0 \quad \text{at } y^* = 0 \quad (6) \end{aligned}$$

In these equations, we have taken into account the temperature oscillation on the upper plate $y^* = h$, while the lower plate $y^* = 0$ is maintained at the fixed temperature T_0 .

The heat flux may be expressed (Cogley *et al.* [15]) as

$$\frac{\partial q}{\partial y^*} = 4\alpha_1^2 (T - T_0) \quad (7)$$

where $\alpha_1 \ll 1$ is the mean radiation absorption coefficient.

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$\begin{aligned} y &= \frac{y^*}{h}, x = \frac{x^*}{h}, u = \frac{u^*}{h}, \text{Re} = \frac{Vh}{k_1}, \\ k &= \frac{k^*}{h^2 \rho}, P = \frac{hp^*}{\rho \nu_1 V}, t = \frac{t^* V}{h}, \theta = \frac{T - T_0}{T_w - T_0}, \\ \text{Gr} &= \frac{g \beta_T (T_w - T_0) h^2}{\nu_1 V}, \text{Pr} = \frac{Vh \rho C_p}{k'}, \\ N^2 &= \frac{4\alpha_1^2 h^2}{k'}, \omega = \frac{\omega^* h}{V}, \phi = \frac{C - C_0}{C_w - C_0}, \\ \text{Gm} &= \frac{g \beta_C (C_w - C_0) h^2}{\nu_1 V}, \text{Sc} = \frac{Vh}{D}. \quad (8) \end{aligned}$$

where Re the Reynolds number, Gr the Grashof number for heat transfer, Pr the Prandtl number, N the radiation Parameter, ω the angular frequency, Gm the Grashof number for mass transfer, Sc the Schmidt number.

In view of Equations (7) and (8), Equations (2)-(4) and

(5) reduce to the following dimensionless form:

$$\text{Re} \left(\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - \frac{u}{k} + \text{Gr}\theta + \text{Gm}\phi + \alpha \left(\frac{\partial^3 u}{\partial y^2 \partial t} + \frac{\partial^3 u}{\partial y^3} \right) \quad (9)$$

$$0 = -\frac{\partial p}{\partial y} \quad (10)$$

$$\text{Pr} \left(\frac{\partial \theta}{\partial t} + \frac{\partial \theta}{\partial y} \right) = \frac{\partial^2 \theta}{\partial y^2} + N^2 \theta \quad (11)$$

$$\frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial y} = \frac{1}{\text{Sc}} \frac{\partial^2 \phi}{\partial y^2} \quad (12)$$

The corresponding dimensionless boundary conditions are:

$$u = U_0 e^{i\omega t}, \theta = 1 + e^{i\omega t}, \phi = 1 + e^{i\omega t} \quad \text{at } y = 1$$

$$u = U_0 e^{i\omega t}, \theta = 0, \phi = 0 \quad \text{at } y = 0 \quad (13)$$

where $\alpha = \frac{\mu_2 V}{\mu_1 h}$ is the viscoelastic parameter.

3. Method of Solution

From (9) and (10), it follows that $\frac{\partial p}{\partial x}$ is a function of t alone. For the present study, we consider

$$\frac{\partial p}{\partial x} = A + B e^{i\omega t}, \quad (14)$$

A and B being undetermined constants. To solve Equations (9), (11) and (12) subject to boundary conditions (13), we write the velocity, temperature, and concentration in the form

$$u(y, t) = u_s(y) + u_p(y, t) = u_s(y) + u_f(y) e^{i\omega t} \quad (15)$$

$$\theta(y, t) = \theta_s(y) + \theta_p(y, t) = \theta_s(y) + \theta_f(y) e^{i\omega t} \quad (16)$$

and

$$\phi(y, t) = \phi_s(y) + \phi_p(y, t) = \phi_s(y) + \phi_f(y) e^{i\omega t} \quad (17)$$

where $u_s(y)$, $u_p(y, t)$, $\theta_s(y)$, $\theta_p(y, t)$, $\phi_s(y)$, $\phi_p(y, t)$ respectively represent the steady and unsteady parts of the velocity, temperature and concentration.

Substituting the above expressions in (9), (11) and (12) and comparing the like terms, we have derived the equations that govern the corresponding steady and unsteady

flow and heat transfer of the problem under consideration. They are given below:

Steady Case:

$$\alpha \frac{d^3 u_s}{dy^3} + \frac{d^2 u_s}{dy^2} - \text{Re} \frac{du_s}{dy} - \frac{u_s}{k} = A - \text{Gr}\theta_s - \text{Gm}\phi_s \quad (18)$$

$$\frac{d^2 \theta_s}{dy^2} - \text{Pr} \frac{d\theta_s}{dy} + N^2 \theta_s = 0 \quad (19)$$

$$\frac{d^2 \phi_s}{dy^2} - \text{Sc} \frac{d\phi_s}{dy} = 0. \quad (20)$$

with the boundary conditions:

$$u_s = 0, \theta_s = 1, \phi_s = 1 \quad \text{at } y = 1$$

$$u_s = 0, \theta_s = 0, \phi_s = 0. \quad \text{at } y = 0 \quad (21)$$

Unsteady Case:

$$\alpha \frac{d^3 u_f}{dy^3} + (1 + i\alpha\omega) \frac{d^2 u_f}{dy^2} - \text{Re} \frac{du_f}{dy} - \left(i\omega \text{Re} + \frac{1}{k} \right) u_f = B - \text{Gr}\theta_f - \text{Gm}\phi_f \quad (22)$$

$$\frac{d^2 \theta_f}{dy^2} - \text{Pr} \frac{d\theta_f}{dy} + (N^2 - i\omega \text{Pr} \theta_s) \theta_f = 0 \quad (23)$$

$$\frac{d^2 \phi_f}{dy^2} - \text{Sc} \frac{d\phi_f}{dy} - i\omega \text{Sc} \phi_f = 0. \quad (24)$$

with the boundary conditions:

$$u_f = U_0, \theta_f = 1, \phi_f = 1 \quad \text{at } y = 1$$

$$u_f = U_0, \theta_f = 0, \phi_f = 0 \quad \text{at } y = 0 \quad (25)$$

On solving the Equations (19), (20), (23) and (24) along with the boundary conditions (20) and (24), are found as

$$\theta_s = \frac{1}{e^{m_1} - e^{m_2}} (e^{m_1 y} - e^{m_2 y}), \quad (26)$$

$$\phi_s = \frac{1}{e^{\text{Sc}} - e^{-\text{Sc}}} (e^{\text{Sc} y} - e^{-\text{Sc} y}), \quad (27)$$

$$\theta_f = \frac{e^{i\omega t}}{e^{m_3} - e^{m_4}} (e^{m_3 y} - e^{m_4 y}), \quad (28)$$

$$\phi_f = \frac{1}{e^{\eta} - e^{-\eta_2}} (e^{\eta y} - e^{-\eta_2 y}). \quad (29)$$

where

$$m_1 = \frac{\text{Pr} + \sqrt{\text{Pr}^2 - 4N^2}}{2}, m_2 = \frac{\text{Pr} - \sqrt{\text{Pr}^2 - 4N^2}}{2},$$

$$m_3 = \frac{\text{Pr} + \sqrt{\text{Pr}^2 - 4(N^2 - i\omega \text{Pr})}}{2},$$

$$m_4 = \frac{\text{Pr} - \sqrt{\text{Pr}^2 - 4(N^2 - i\omega \text{Pr})}}{2},$$

$$r_1 = \frac{\text{Sc} + \sqrt{\text{Sc}^2 + 4i\omega \text{Sc}}}{2}, r_2 = \frac{\text{Sc} - \sqrt{\text{Sc}^2 + 4i\omega \text{Sc}}}{2}.$$

We note that $|\alpha| < 1$ for small shear rate and so we can assume the following:

For steady case

$$u_s(y) = u_{s0}(y) + \alpha u_{s1}(y) + O(\alpha^2),$$

and for unsteady case

$$u_f(y) = u_{f0}(y) + \alpha u_{f1}(y) + O(\alpha^2) \quad (30)$$

Substituting (30) in (18) and (22) together with boundary conditions (21) and (25) up to first order of α and equating the co-efficient of like powers of α , we obtain the following sets of ordinary differential equations and corresponding boundary conditions:

$$\frac{d^2 u_{s0}}{dy^2} - \text{Re} \frac{du_{s0}}{dy} - \frac{u_{s0}}{k} = A - \text{Gr} \theta_s - \text{Gm} \phi_s \quad (31)$$

$$\frac{d^3 u_{s0}}{dy^3} + \frac{d^2 u_{s1}}{dy^2} - \text{Re} \frac{du_{s1}}{dy} - \frac{u_{s1}}{k} = 0. \quad (32)$$

with

$$u_{s0} = u_{s1} = 0, \quad \text{at } y = 1$$

$$u_{s0} = u_{s1} = 0, \quad \text{at } y = 0 \quad (33)$$

and

$$\frac{d^2 u_{f0}}{dy^2} - \text{Re} \frac{du_{f0}}{dy} - \left(i\omega \text{Re} + \frac{1}{k} \right) u_{f0} = B - \text{Gr} \theta_f - \text{Gm} \phi_f \quad (34)$$

$$\alpha \frac{d^3 u_{f0}}{dy^3} + \frac{d^2 u_{f1}}{dy^2} - \text{Re} \frac{du_{f1}}{dy} - \left(i\omega \text{Re} + \frac{1}{k} \right) u_{f1} = -i\omega \frac{d^2 u_{f0}}{dy^2}. \quad (35)$$

with

$$u_{f0} = U_0, u_{f1} = 0, \quad \text{at } y = 1$$

$$u_{f0} = U_0, u_{f1} = 0, \quad \text{at } y = 0 \quad (36)$$

The Equations (31), (32) and (34), (35) are solved under the boundary conditions (33) and (36) respectively. Substituting these solutions in (30), we get the expressions for u_s and u_f , and thus the expression for u but due brevity the solutions are not presented here.

Nusselt number: From the temperature field, the rate of heat transfer coefficient can be obtained, which in terms of Nusselt number Nu_p across the upper wall is given by

$$\text{Nu}_p = - \left[\frac{\partial \theta}{\partial y} \right]_{y=1} \quad (37)$$

Skin-friction: Knowing the velocity field, the expression for the non-dimensional wall shear stress at the upper plate is given by

$$\tau_w = \left[\mu_1 \left(\frac{\partial u}{\partial y} \right) + \mu_2 \left(\frac{\partial^2 u}{\partial y \partial t} + \nu \frac{\partial^2 u}{\partial y^2} \right) \right]_{y=1} \quad (38)$$

Sherwood number: From the concentration field, the rate of mass transfer coefficient in terms of Sherwood number Sh_p across the upper wall is given by

$$\text{Sh}_p = - \left[\frac{\partial \phi}{\partial y} \right]_{y=1} \quad (39)$$

4. Results and Discussion

The purpose of this study is to bring out the effects of viscoelastic parameter α on the governing flow. The effects of viscoelastic parameter on the fluctuation of axial velocity distribution, shear stress, heat transfer, concentration profiles are evaluated numerically. The predicted variation of fluctuation of the axial velocity with different values of α and for porous permeability parameter (k), mass Grashof number (Gm), radiation parameter (N) with fixed values of $\text{Re} = 1, \text{Pr} = 3,$

$t = \frac{\pi}{2}, \text{Sc} = 0.5$ are shown in **Figures 1-3**. From the **Fig-**

ures 1-3, it is observed that the fluctuation of axial velocity is parabolic in nature and the values of u_p decrease with the increasing values of the viscoelastic parameter $|\alpha|, (\alpha = 0, -0.05, -0.1)$ in comparison with Newtonian fluid. It is observed that the axial velocity u_p increase with the increase of permeability parameter (**Figure 1**), mass Grashof number (**Figure 2**) and radiation parameter (**Figure 3**) for both Newtonian and non-Newtonian cases.

The wall shear stress is calculated from the Equation (38). **Figure 4** shows that the wall shear stress τ increases as the values of the viscoelastic parameter $|\alpha| (\alpha = 0, -0.05, -0.1)$ increase in comparison to Newtonian fluid. Also, the wall shear stress at the upper plate increase with the increase of mass Grashof number for both Newtonian and non-Newtonian cases.

The temperature and concentration profiles are calculated from the Equations (37) and (39) respectively. It is observed that the temperature and concentration profiles are not significantly affected by the viscoelastic parameter.

k	Gm	N	α	case
0.5	1	1	0	I
0.5	1	1	-0.05	II
0.5	1	1	-0.1	III
1	1	1	0	IV
1	1	1	-0.05	V
1	1	1	-0.1	VI

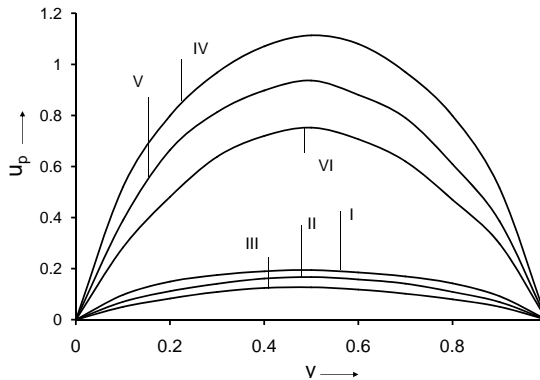


Figure 1. Variation of u against y .

k	Gm	N	α	case
0.5	1	1	0	I
0.5	1	1	-0.05	II
0.5	1	1	-0.1	III
1	1.5	1	0	IV
1	1.5	1	-0.05	V
1	1.5	1	-0.1	VI

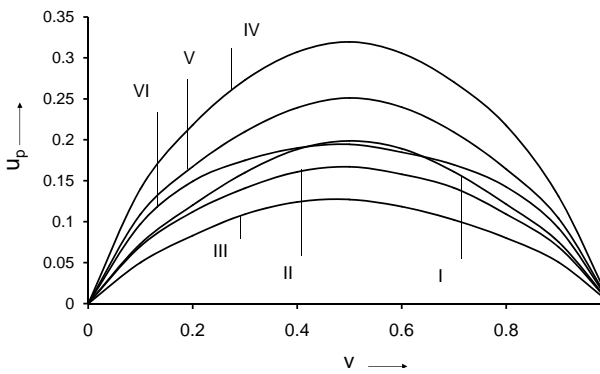


Figure 2. Variation of u against y .

5. Conclusions

The governing equations for unsteady heat and mass transfer flow of chemically reacting viscoelastic fluid through a channel with radiative heat transfer were formulated. The plates were supposed to be oscillating with a given velocity in their own planes. The effect of viscoelastic parameter of the fluid on the governing flow were analysed for various values of parameters under consideration. The conclusions of the study are as follows:

k	Gm	N	α	case
0.5	1	1	0	I
0.5	1	1	-0.05	II
0.5	1	1	-0.1	III
1	1	0.5	0	IV
1	1	0.5	-0.05	V
1	1	0.5	-0.1	VI

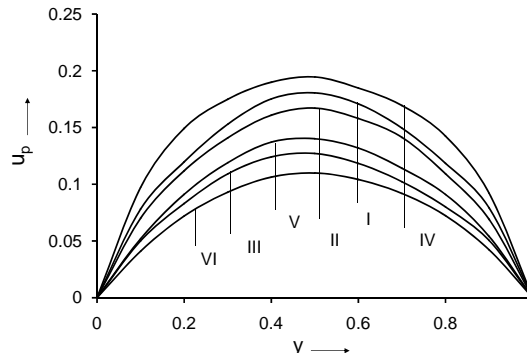


Figure 3. Variation of u against y .

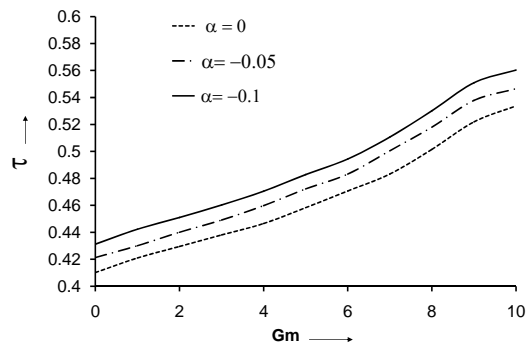


Figure 4. Wall shear stress versus N when $t = \pi$.

- 1) Velocity decrease with the increasing values of the viscoelastic parameter in comparison with Newtonian fluid;
- 2) Velocity increases significantly with increasing permeability parameter and mass Grashof number as compared to radiation parameter for both Newtonian and non-Newtonian cases;
- 3) Wall shear stress increase with the increasing values of the viscoelastic parameter;
- 4) Temperature and concentration profiles are not significantly affected by the viscoelastic parameter.

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