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An Optimal Control Theory Based Analysis of Brushless DC Motor Drive

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Abstract

The application of BLDC motor drives in industries is becoming more popular nowadays. An error will occur in the drive that is originated by some disturbances which are the major problems to reduce the stability of the system. To obtain the minimum performance index, the optimal control signal is formulated, which is the main objective of this paper. Based on quadratic performance index, the optimal control system of BLDC motor drive is a design which spotlights in this paper. The complexity of the mathematical expressions has been reduced by using state space approach to the BLDC system. The burden to the control engineers has reduced based on tedious computation by using thus optimal design. To provide the desired operating performance, this optimal design helps to realize the BLDC system with practical components.

Keywords

1. Introduction

Presently, consumers demand for lower energy costs, higher efficiency, better performance, reduced acoustic noise and more convenient features. The traditional technologies like DC motor and AC motor cannot meet these demands. Hence, motor manufactures are gravitating towards Brushless Direct Current (BLDC) motor. The use of the BLDC motor in these applications is becoming very common due to the features of high efficiency, high flux density per unit volume and high power density due to the absence of field winding. In addition, the absence of brushes leads to high reliability, low maintenance requirements and low electromagnetic interference problems [1] [2]. This drive can be used for variable speed applications like Electrical Vehicles, Robotics, etc.

Due to change in load variations, BLDC motor exhibits oscillations in speed response and hence the stability of the system gets affected. The stability issues can be recovered by two ways, namely reducing the gain of speed controller and regulating output method [3]-[5]. But reducing the gain increases the oscillations and decreases the system performance. The regions of instability occur due to changes in the reference input command. Because of oscillations in the output of BLDC motor, it cannot achieve precise speed control. In the next method of regulating the system output modification cannot achieve the time optimal operation [6]-[8]. In the classical design method of control systems, the designer is given a set of specifications in time domain or in frequency domain with the system configuration. In practice, it may not be possible to satisfy all the desired specifications [9]-[11]. The optimal control design is aimed at obtaining a best possible system of a particular type with respect to a certain performance index or design criterion. The optimal control problem of systems has always been the key focus of control fields in the past several decades. The optimal control approaches are mostly based on linearization methods or numerical computation methods [12] [13].

The optimal control design is aimed to obtain a best possible system of a particular type with respect to a certain performance index. In the optimal control design, the performance index replaces the conventional design criteria. A transfer function for the BLDC drive is derived in this paper. In this paper, for digitally PWM controlled BLDC motor drive, the optimal control system is designed. To achieve this optimization, the state and control variables are formulated in this paper. Based on optimal control theory, the BLDC performance characteristics close to the optimal control system are synthesized.

2. Digital PWM Control of BLDC Motors

By changing the applied voltage across the motor phases, the speed of BDC motor can be controlled. This is achieved by utilizing pulse amplitude modulation, PWM or hysteresis control. Another method of speed control involves sensorless techniques. **Figure 1** shows the block diagram for digital PWM control for a BLDC motor drive system. The derivation of this model is based on the assumption that the induced currents in the rotor due to stator harmonic fields are neglected and the iron and stray losses are also neglected [4].

The torque equation is given by,

$$T_{em} = J\frac{\mathrm{d}\omega}{\mathrm{d}t} + B\omega + T_L \tag{1}$$

where T_{em}, $\omega(t)$, B, J and T_L denote electromagnetic torque, rotor angular velocity, viscous friction constant, rotor moment of inertia and load torque respectively.
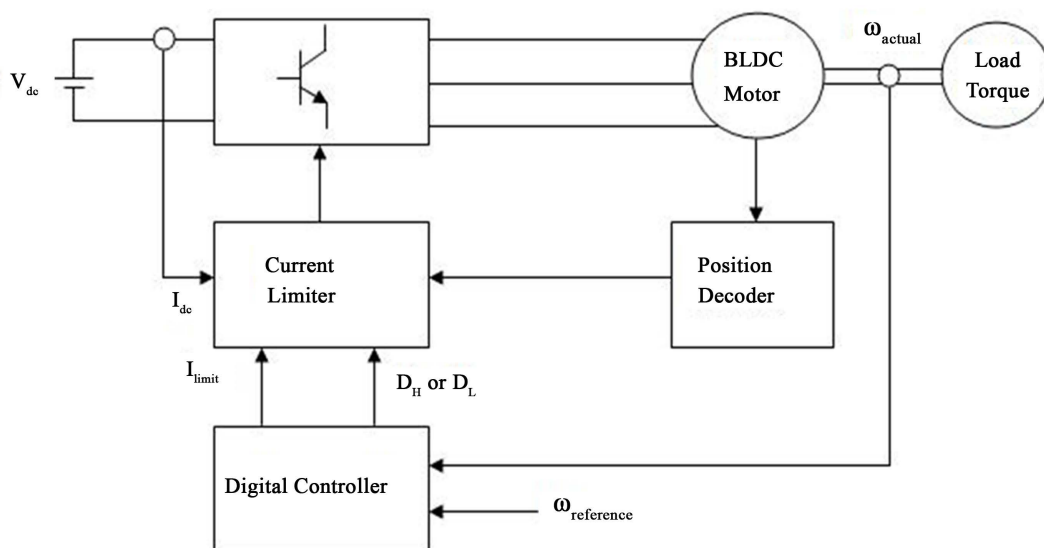
$$T_{em}\alpha I \tag{2}$$

Figure 1. Block diagram for digital PWM control for a BLDC motor drive system.

$$T_{em} = k_t I \tag{3}$$

$$k_t I = J \frac{d\omega}{dt} + B\omega + T_L \tag{4}$$

where K_t = torque constant and I = average current. For the purpose of analysis, the digital controller was considered equivalent to a proportional controller with high gain and saturation.

From the above equations it is possible to derive the transfer function

$$\frac{\omega(s)}{V(s)} = \frac{\dfrac{K_t}{JL_a}}{s^2 + \left(\dfrac{JL_a + BL_a}{JL_a}\right)s + \left(\dfrac{BR_a + K_t K_\infty}{JL_a}\right)} \tag{5}$$

The state variable equation for this BLDC drive is given by

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{BR_a + k_t k_e}{JL_a} x_1 - \frac{JR_a + BL_a}{JL_a} x_2 + \frac{k_t}{JL_a} u
\end{aligned} \tag{6}$$

Arranging in matrix form, the following equations are obtained

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \dfrac{-(BR_a + k_t k_e)}{JL_a} & \dfrac{-(JR_a + BL_a)}{JL_a} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{k_t}{JL_a} \end{bmatrix} u \tag{7}$$

The output equation is given by,

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{8}$$

3. State Variable Feedback

The design of a state feedback BLDC motor control system is based on a suitable selection of a feedback system structure. If the state variables are known, then they can be utilized to design a feedback controller so that the input becomes $u = kx$. It is necessary to measure and utilize the state variables of the system in order to control the speed of the BLDC motor. This design approach of state variable feedback control gives sufficient information about the stability of the BLDC drive system. Feedback control system will be chosen so that,

$$u(t) = -k_1 x_1 - k_2 x_2 \tag{9}$$

Then Equations (6) & (7) become,

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{BR_a + k_t k_e + k_1 k_t}{JL_a} x_1 - \frac{JR_a + BL_a + k_2 k_t}{JL_a} x_2
\end{aligned} \tag{10}$$

Arranging in matrix form the following state equation is obtained

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \dfrac{-(BR_a + k_t k_e + k_1 k_t)}{JL_a} & \dfrac{-(JR_a + BL_a + k_2 k_t)}{JL_a} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{11}$$

This is in the form $\dot{x} = Hx$
where

$$H = \begin{bmatrix} 0 & 1 \\ \dfrac{-(BR_a + k_t k_e + k_1 k_t)}{JL_a} & \dfrac{-(JR_a + BL_a + k_2 k_t)}{JL_a} \end{bmatrix} \tag{12}$$

Let $k_1=1$ and determine a suitable value for k_2 so that the performance index is minimized.

$$J = \int_0^\infty X^{\mathrm{T}} X \mathrm{d}t = X^{\mathrm{T}}(0) PX(0) \tag{13}$$

$$H^{\mathrm{T}} P + PH = -I \tag{14}$$

$$\begin{bmatrix} 0 & \dfrac{-(BR_a + k_t k_e + k_1 k_t)}{JL_a} \\ 1 & \dfrac{-(JR_a + BL_a + k_2 k_t)}{JL_a} \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}$$

$$+ \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ \dfrac{(BR_a + k_t k_e + k_1 k_t)}{JL_a} & \dfrac{-(JR_a + BL_a + k_2 k_t)}{JL_a} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \tag{15}$$

Completing matrix multiplication, addition and solving the following are obtained

$$P_{11} = \frac{(JR_a + BR_a + k_t k_e + k_t)}{2(JR_a + BL_a + k_2 k_t)} + \frac{(JR_a + BL_a + k_2 k_t)}{2(BR_a + k_t k_e + k_t)} \tag{16}$$

$$P_{12} = \frac{JR_a}{2(JR_a + k_t k_e + k_t)} \tag{17}$$

$$P_{22} = \frac{JL_a(JL_a + BR_a + k_t k_e + k_t)}{2(JR_a + BL_a + k_2 k_e + k_t)(BR_a + k_t k_e + k_t)} \tag{18}$$

$$J = P_{11} + 2P_{12} + P_{22} \tag{19}$$

To minimize as a function of k_2

$$\frac{\partial J}{\partial k_2} = 0 \tag{20}$$

Therefore,

$$k_2 = \frac{-JR_a BL_a}{K_t} \pm \frac{\sqrt{(JR_a BL_a)^2 - (JL_a + BR_a + k_t k_e + \mathrm{k}_t) + J_2 Ra_2 + 2JR_a BL_a + B^2 La^2}}{k_t} \tag{21}$$

$$k_2 = 1.01499 \tag{22}$$

$$J_{\min} = 1.47 \tag{23}$$

The system matrix H obtained for the compensated system is,

$$H = \begin{bmatrix} 0 & 1 \\ -2629.476 & -2833.7 \end{bmatrix} \tag{24}$$

The feedback control signal is obtained as,

$$u = -x_1 - 1.015 x_2 \tag{25}$$

This optimal controlled BLDC drive system results in a minimum value for the performance index. Also, the control law given by Equation (25) yields optimal result for any initial state under the given performance index. This design based on the quadratic performance index yields a stable control system for the BLDC drive system.

4. Optimal System with Control Energy Considerations

This section considers the expenditure of control signal energy for the BLDC drive system. To account for the

expenditure of the optimal energy of the performance index $J = \int_0^\infty X^T X \mathrm{d}t = X^T(0) PX(0)$ is chosen where λ is

the scalar weighting factor and I = identity matrix. **Table 1** gives parameters of BLDC drive systems. We will choose a feedback control system so that,

$$u = kx = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = kIx \tag{26}$$

Therefore the matrix Q is

$$Q = \left(I + \lambda k^T k \right) = \left(1 + \lambda k^2 \right) I \tag{27}$$

Let,

$$X^T(0) = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{28}$$

$$H^T P + PH = -Q = -\left(1 + \lambda k^2 \right) I \tag{29}$$

$$J = P_{11} \tag{30}$$

To simplify the algebra without any loss, let $k_1 = k_2 = k$

$$H = \begin{bmatrix} 0 & 1 \\ \dfrac{\left(BR_a + k_t k_e + kk_t \right)}{JL_a} & \dfrac{-\left(JR_a + BL_a + kk_t \right)}{JL_a} \end{bmatrix} \tag{31}$$

$$\begin{bmatrix} 0 & \dfrac{-\left(BR_a + k_t k_e + kk_t \right)}{JL_a} \\ 1 & \dfrac{-\left(JR_a + BL_a + kk_t \right)}{JL_a} \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}$$

$$+ \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ \dfrac{\left(BR_a + k_t k_e + kk_t \right)}{JL_a} & \dfrac{-\left(JR_a + BL_a + kk_t \right)}{JL_a} \end{bmatrix} = \begin{bmatrix} -\left(1 + \lambda k^2 \right) & 0 \\ 0 & -\left(1 + \lambda k^2 \right) \end{bmatrix} \tag{32}$$

$$P_{11} = \frac{\left(JR_a + BR_a + kk_t \right)\left(1 + \lambda k^2 \right)}{2\left(BR_a + k_t k_e + kk_t \right)} + \frac{\left(JL_a + BR_a + k_t k_e + kk_t \right)\left(1 + \lambda k^2 \right)}{2\left(JR_a + BL_a + kk_t \right)} \tag{33}$$

$$P_{12} = \frac{JL_a \left(1 + \lambda k^2 \right)}{2\left(BR_a + k_t k_e + kk_t \right)} \tag{34}$$

$$P_{22} = \frac{JL_a \left(JL_a + BR_a + k_t k_e + kk_t \right)\left(1 + \lambda k^2 \right)}{2\left(JR_a + BL_a + kk_t \right)\left(BR_a + k_t k_e + kk_t \right)} \tag{35}$$

Table 1. BLDC drive parameters.

S. No	Parameter	Symbol	Unit	Value
1	Stator Winding Resistance	Ra	Ω	1.4
2	Stator Winding Inductance	L_a	H	0.0066
3	Rotor Inertia	J	Kg-m^2	0.00176
4	Motor Viscous Friction Coefficient	B	Nm/rad/sec	0.00038818
5	Torque Constant	k_t	Nm/Amp	0.03
6	Velocity Constant	k_e	Volts/rad	0.0000181

To obtain minimum performance index, set $\dfrac{\partial J}{\partial k} = 0$, following equations are obtained

$$ak^5 + bk^4 + ck^3 + dk^2 + ek + f = 0 \tag{36}$$

where

$$a = 8k_t^4 \lambda \tag{37}$$

$$b = 8JR_a BL_a K_t^3 + 8BR_a k_t^3 \lambda + 8k_t^4 k_e \lambda + 8k_t^3 JR_a \lambda + 8K_t^3 BL_a \lambda + 8BR_a k_t^4 k_e \lambda + 2k_t^3 JL_a \lambda - 2k_t^4 \tag{38}$$

$$
\begin{aligned}
c = {} & 4JR_a BL_a k_t \lambda \left(3R_a Bk_t + 4k_t^2 k_e + JR_a k_t + BL_a k_t - \frac{k_t^2}{\lambda} + 2k_t + 4BJR_a \lambda k_t \left(2Ra^2 kt + 3Rak_t^2 k_e \right) \right. \\
& \left. + 4B^2 L_a R_a \lambda k_t^2 \left(2 + 3k_t k_e \right) + 4BR_a k_t \lambda \left(BR_a k_t + 2k_t^2 k_e + BR_a k_t^2 k_e + k_t^3 k_e^2 \right) - 2k_t^4 \right)
\end{aligned}
\tag{39}
$$

$$
\begin{aligned}
d = {} & \left(J^2 R_a^2 + B^2 L_a^2 + 2JR_a BL_a \right)\left(6BR_a K_t \lambda + 6k_t^2 k_e \lambda + 4Jk_t R_a \lambda + 4k_t BL_a \lambda \right. \\
& \left. - 2k_t^2 - 2JR_a \lambda k_t - 2BL_a \lambda k_t \right) + 8J^2 R_a^3 B^2 L_a k_t \lambda + 8JR_a^2 B^3 L_a^2 k_t \lambda - 2JR_a k_t^3 \\
& + 8BR_a^2 k_t^2 k_e J^2 L_a \lambda + 8BR_a^2 k_t^2 k_e L_a^2 - 2BL_a k_t^3 - 2JL_a k_t^3 - 2BR_a k_t^3 - 2k_t^4 k_e
\end{aligned}
\tag{40}
$$

$$
\begin{aligned}
e = {} & \left(J^2 R_a^2 + B^2 L_a^2 + 2JR_a BL_a \right)\left(4BJR_a^2 \lambda + 4B^2 L_a R_a \lambda + 4k_t k_e JR_a \lambda + 4Bk_t k_e JL_a \lambda \right) \\
& - 4J^2 BR_a^2 L_a k_t^2 - 4JB^2 R_a L_a^2 k_t^2 - 4BR_a k_t^3 k_e JL_a - 4B^2 R_a^2 k_t^3 k_e - 2BR_a k_t 4k_e^2 \\
& + \left(B^2 R_a^2 + k_t^2 k_e^2 + 2BR_a k_t k_e \right)\left(4J^2 R_a L_a \lambda + 4JR_a^2 B\lambda + 4BJL_a^2 \lambda + 4B^2 L_a R_a \lambda + 4BL_a \lambda k_t k_e - 2k_t^2 \right)
\end{aligned}
\tag{41}
$$

When $\lambda = 0$, Solving the above equations we get,

$$k^4 + k^3 + 0.871k^2 + 3.29 \times 10^{-4} k + 5.61 \times 10^{-4} = 0 \tag{42}$$

Applying Lin's Method the above equation becomes

$$\left(k^2 + 0.998k + 0.868 \right)\left(k^2 - 3.63 \times 10^{-4} k + 6.46 \times 10^{-4} \right) = 0 \tag{43}$$

5. Quadratic Optimal Control

This section discusses about the design of a stable control system for BLDC drive based on quadratic performance indexes. The system design will be stable by using the quadratic optimal control scheme has the main advantage except in the case where the system is not controllable. The matrix "P" is determined from the solution of the matrix Riccatti equation. This optimal control is called the Linear Quadratic Regulator (LQR) [10] [11]. The optimal feedback gain matrix k can be obtained by solving the following Riccatti equation for a positive-definite matrix "P".

$$A^{\mathrm{T}} P + PA - PBR^{-1}B^{\mathrm{T}} P + Q = 0 \tag{44}$$

Let,

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & \mu \end{bmatrix}; \left(\mu \geq 0 \right) \tag{45}$$

$$
\begin{bmatrix} 0 & -\dfrac{\left(BR_a + k_t k_e \right)}{JL_a} \\ 1 & -\dfrac{\left(JR_a + BL_a \right)}{JL_a} \end{bmatrix}
\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}
+
\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}
\begin{bmatrix} 0 & 1 \\ -\dfrac{\left(BR_a + k_t k_e \right)}{JL_a} & -\dfrac{\left(JR_a + BL_a \right)}{JL_a} \end{bmatrix}
$$

$$
-\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}
\begin{bmatrix} 0 \\ \dfrac{k_t}{JL_a} \end{bmatrix}
[1]
\begin{bmatrix} 0 & \dfrac{k_t}{JL_a} \end{bmatrix}
\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}
+
\begin{bmatrix} 1 & 0 \\ 0 & \mu \end{bmatrix} = 0
\tag{46}
$$

Solving we obtain the following three equations

$$\frac{P_{12}^2 k_t^2}{J^2 L_a^2} + \frac{2(BR_a + k_t k_e)}{JR_a} - 1 = 0 \tag{47}$$

$$P_{11} - \frac{P_{12}(JR_a + BL_a)}{JL_a} - \frac{P_{22}(BR_a + k_t k_e)}{JL_a} - \frac{P_{12}P_{22}k_t^2}{J^2 L_a^2} = 0 \tag{48}$$

$$2P_{11} - \frac{2P_{22}(JR_a + BL_a)}{JL_a} - \frac{P_{22}^2 k_t^2}{J^2 L_a^2} + \mu = 0 \tag{49}$$

$$k = R^{-1}B^\mathrm{T}P \tag{50}$$

Solving these three equations we get,

$$P = \begin{bmatrix} -1.09 \times 10^{-3} + \sqrt{1 + 13298\mu} & 3.8 \times 10^{-4} \\ 3.8 \times 10^{-4} & -3.18 \times 10^{-5} + 3.35 \times 10^{-5}\sqrt{1 + 13298\mu} \end{bmatrix} \tag{51}$$

The optimal feedback gain matrix

$$k = R^{-1}B^\mathrm{T}P \tag{52}$$

$$k = \begin{bmatrix} 0.981 & -0.08 + 0.0865\sqrt{1 + 132.98\mu} \end{bmatrix} \tag{53}$$

$$u = -kx = -0.981x_1 - \left(-0.08 + 0.0865\sqrt{1 + 132.98\mu}\right)x_2 \tag{54}$$

This control signal yields an optimal result for any initial state under the given performance index. **Figure 2** shows the block diagram for optimal control of the BLDC drive system.

6. Conclusion

To achieve the desired system response, the BLDC drive system had designed with state variable feedback. Also the performance index value is minimum for BLDC drive which was designed by optimal control. In accordance with the expenditure of energy and resources, the control signal is often included in the performance integral. From the foregoing analysis, the value of K_2 is obtained as 1.0149 so that the performance index is minimized. The minimum value of performance index is obtained as 1.47. This optimal controlled BLDC drive system results in a minimum value for the performance index. Also, the control law given by Equation (54) yields optimal
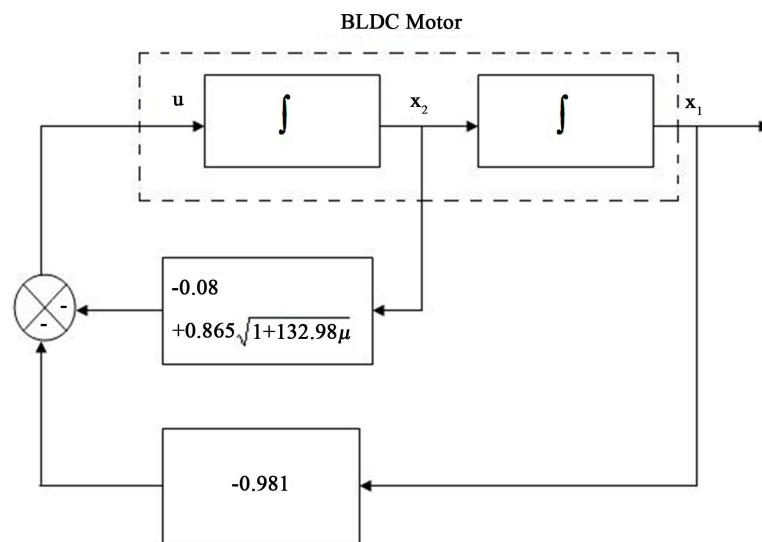

Figure 2. Optimal control of the BLDC drive system.

results for any initial state under the given performance index. This design based on the quadratic performance index yields a stable control system for the BLDC drive system.

References

[1] Pillay, P. and Krishnan, R. (1989) Modelling, Simulation, and Analysis of Permanent Magnet Drives, Part I. The Permanent Magnet Synchronous Motor Drive. *IEEE Transaction on Industrial Applications*, **25**, 265-273. http://dx.doi.org/10.1109/28.25541

[2] Pillay, P. and Krishnan, R. (1989) Modelling, Simulation, and Analysis of Permanent Magnet Drives, Part II. The Permanent Magnet Synchronous Motor Drive, *IEEE Transaction on Industrial Applications*, **25**, 274-279. http://dx.doi.org/10.1109/28.25542

[3] Krishnan, R. (2009) Electric Motor Drives, Modelling, Analysis and Control. PHI Learning Private Limited, New Delhi.

[4] Sathyan, A., Milivojevic, N., Lee, Y.J. and Krishnamoorthy, M. (2009) An FPGA-Based Novel Digital PWM Control Scheme for BLDC Motor Drives. *IEEE Transaction Industrial Electronics*, **35**, 3040-3049. http://dx.doi.org/10.1109/TIE.2009.2022067

[5] Kuo, C.-F.J. and Hsu, C.-H. (2005) Precise Speed Control of a Permanent Magnet Synchronous Motor. *International Journal of Advanced Manufacturing Technology*, **28**, 942-949. http://dx.doi.org/10.1007/s00170-004-2447-2

[6] Venkitaraman, R. and Ramaswami, B. (1981) Thyristor Converter-Fed Synchronous Motor Drive. *Electric Machines and Electro Mechanics*, **6**, 433-449.

[7] Pan, C.-T. and Fang, E. (2008) A Phase-Locked-Assisted Internal Model Adjustable-Speed Controller for BLDC Motors. *IEEE Transaction Industrial Electronics*, **55**, 3415-3425. http://dx.doi.org/10.1109/TIE.2008.922600

[8] Rodigeuz, F., Desai, P. and Emadi, A. (2004) A Novel Digital Control Technique for Trapezoidal Brushless DC Motor Drives. *Proceedings on Electronics and Technology Conference*, Chicago, November 2014.

[9] Rodigeuz, F. and Emadi, A. (2005) A Novel Digital Control Technique for Brushless DC Motor Drives: Conduction-Angle Control. *Proceedings of IEEE International Conference on Electric Machines and Drives*, San Antonio, 15 May 2005, 308-314.

[10] Ogata, K. (1998) Modern Control Engineering. International Edition, Prentice-Hall of India, New Delhi.

[11] Dorf, R.C. (1998) Modern Control Systems. Addison Wesley Longman Inc., Boston.

[12] Murugan, M., Jeyabharath, R. and Sarankumar, V. (2015) An Approach of PFC in BLDC Motor Drives Using BLSEPIC Converter. *TELKOMNIKA Indonesian Journal of Electrical Engineering*, **14**, 215-221.

[13] Niasar, A.H., Moghbeli, H. and Vahedi, A. (2006) Implementation of Four-Switch Brushless DC Motor Drive Based on TMS320LF2407 DSP. *IEEE International Conference on Signal Processing and Communications*, Dubai 24-27 November 2007, 332-335.