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Stability Analysis of Multi-Dimensional Linear Time Invariant Discrete Systems within the Unity Shifted Unit Circle

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Abstract

This technical brief proposes a new approach to multi-dimensional linear time invariant discrete systems within the unity shifted unit circle which is denoted in the form of characteristic equation. The characteristic equation of multi–dimensional linear system is modified into an equivalent one-dimensional characteristic equation. Further formation of stability in the left of the z-plane, the roots of the characteristic equation $f(z) = 0$ should lie within the shifted unit circle. Using the coefficients of the unity shifted one dimensional equivalent characteristic equation by applying minimal shifting of coefficients either left or right and elimination of coefficient method to two triangular matrixes are formed. A single square matrix is formed by adding the two triangular matrices. This matrix is used for testing the sufficient condition by proposed Jury's inner determinant concept. Further one more indispensable condition is suggested to show the applicability of the proposed scheme. The proposed method of construction of square matrix consumes less arithmetic operation like shifting and eliminating of coefficients when compare to the construction of square matrix by Jury's and Hurwitz matrix method.

Keywords

Stability, Multi-Dimensional, Unity-Shifting, Characteristics Equations, Inner Determinants

1. Introduction

The stability problem of multidimensional discrete polynomials is receiving more attention due to the emerging widespread applications. In recent years, multivariable functions have been increasing the applications in analyses

and synthesizes problem of discrete continues system. Various algebraic stability test algorithms have been proposed for multi dimension system. However, they required huge amount of computation time for all. The stability investigation associated with multidimensional digital filters used in the areas like seismology needs to be considered. Other applications arise in obtaining reliability properties of impedances of networks and transmission lines which represent multidimensional continuous systems in the form of multidimensional continuous filters. Foremost among them is the stability of two and multi dimension system which find the application in the process of bio medical, sonar and radar data. The study of multi dimension discrete shift invariant system has received considerable attention among the researches. The stability problem is an important issue in the design and analyses of multi dimension linear discrete system. A huge amount of research was dedicated to developing technique for multi dimension system. Jury proposed the stability test for multi dimension system. It should be motivated by practical applications. It is done mainly by engineers in the mathematical literate. The stability analysis of multidimensional digital system is much more complex than single dimensional system. Result for the general case of any multidimensional realization is still warranted which will the topic of future investigation. The Jury's inner wise determinant method of stability test is useful for numerical testing of stability and for the design of linear time invariant discrete systems. At the result of this paper multi-dimensional discrete system is converted into single dimensional discrete system. The necessary condition is that the roots of the characteristic equation should lie within the left half of the s-plane is verified. Then the characteristic equation forms the $(n + 1) * (n + 1)$ matrix and the inner determinant is calculated with the help of inner Jury's concept and the sufficient condition is the inner determinant value is positive should be verified. This shows that the system is stable.

2. Literature Survey

To have a knowledge about the various stability problem in designing and analysing the multi-dimensional linear time invariant discrete system. Many practical tests have been developed to solve the stability problem. The new criterion stability of the multi-dimensional system is obtained by using the functional schur co-efficient which been discussed bySerban et al. in [1] [2] and it is illustrated by means of various examples. Benidir had demonstrated in [3] about the stability conditions and its properties. The number of final constraints should be represented as $\frac{1}{2}\left[n(n+1)\right]$ in these constraints n will be reduced to obtain the stability criterion which was proposed by Jury et al. in [4]. Bose et al. presented in [5] a simplified form to test the stability of multi-dimensional system by as curtaining the robust stability of linear discrete system. Sivanandam et al. in [6] revealed the stability test for multi-dimensional system. In this multi-dimensional digital filter is a total of 2^n different cases are considered and by extending the Routh test. The necessary and sufficient conditions are verified as in one-dimensional. A new transform, which is applied to the denominator polynomial of unstable multi-dimensional to yield a stable polynomial and the stability of one-dimensional and two-dimensional are obtained using discrete Hilbert transform was addressed by Dameravenkata et al. in [7]. Anderson et al. in [8] gave a simplified form for checking the stability by converting the multivariable polynomial into the single degree polynomial. The reduction method which reduces a non-linear n-dimensional equation to one-dimensional non-linear equation then comparing the results using the proposed matrix norm approach was proposed by Jury et al. in [9] [10]. In most of the cases this method provides less restrictive and it can be used for any order-dimensional if it is a direct realization. Bose had proposedin [11] a repeated application of an extended schurcohn formulation and also tested the positive definiteness of an arbitrary binary quadratic form. serban et al. in [12] revealed a new multidimensional BIBO stability algorithm. Hu Focused in [13] on the condition for multi-dimensional variables to test the stability. Justice et al. in [14] Proved stability theorem for multidimensional system. Strintzis proposed a simple approach in [15] [16] to task the numerical testing of stability condition and establishing the stabilization. Mastorakis et al. in [17] made a approach to the stability problem where it reduced it into an appropriate constrained optimization using the Neural Network because it provides computational speed and by performing many experiments. Bose et al. in [18] explained the robust stability of discrete system which depends upon the bilinear transformation. Kurosawa et al. had addressed in [19] the various algebraic stability testing algorithm multi-dimensional system. To determine whether or not a polynomial in several real variables is positive procedure was proposed by Bose et al. in [20] [21] gave a simplified form a reduction of multivariable polynomial into several variable polynomials by the finite number of rational operations. Huang and Jury introduced the concept in [22]

[23] of inner wise matrix. The stability test of 2-D and multi-dimensional system reduces to several applications of the stability test of one-dimensional system. Many results on stability tests of such system had recently been presented by Hertz *et al.* in [24] and it reveals that simplifications and computational complexity which grows steeply with dimensionality of the system and which cause the test to become almost impractical for large n. Ezra zeheb *et al.* in [25] had proposed a new theorem which can be used to derive new simplified procedure for a multi-dimensional stability test, presented for the continuous analog case as well as for the discrete digital case.

3. Proposed Method

A Multidimensional (M-D) linear discrete system [26] described by the transfer function

$$G(z_1,\cdots,z_m) = \frac{A(z_1,\cdots,z_m)}{B(z_1,\cdots,z_m)} \tag{1}$$

The Multidimensional system is stable if and only if

$$
\begin{aligned}
&B(0,\cdots,0,z_m) \neq 0 \ \text{ for } |z_m| \leq 1 \\
&B(0,\cdots,0,z_{m-1},z_m) \neq 0 \ \text{ for } |z_{m-1}| \leq 1, |z_m| \\
&B(0,z_2,\cdots,z_{m-1},z_m) \neq 0 \ \text{ for } |z_2| \leq 1, |z_3| = \cdots = |z_m| = 1 \\
&B(z_1,z_2,\cdots,z_m) \neq 0 \ \text{ for } |z_1| \leq 1, |z_2| = \cdots = |z_m| = 1
\end{aligned}
\tag{2}
$$

Let $B(Z_1,Z_2,Z_3) = Z_3 + Z_1 + Z_1^2 Z_2 - Z_2 + 4 = 0$

1. with $Z_2 = Z_3 = 0$,

$$B(Z_1) = Z_1 + 4 = 0, |Z_1| \leq 1, \text{ the reciprocal of } Z_1 \text{ and with } \left[\frac{1}{Z_1}\right] = X$$

$$T_1(x) = 4x + 1 = 0, |x| < 1.$$

Thus $|x| = \frac{1}{4} < 1$ (Satiesfied)

2. with $Z_3 = 0$

$$B(Z_1,Z_2) = Z_1 + Z_1^2 Z_2 - Z_2 + 4 = 0, \text{ the reciprocal of } Z_2 \text{ with } \left[\frac{1}{z_2}\right] = X$$

$$T_2(x) = 4x^3 + 1 = 0, |x| < 1$$

Thus $x^3 = -\frac{1}{4}$, indicating $|x| < 1$.

Using the Equations (1) and (2) stability checking involves more complexity and computation cost also increased. Assuming that $G(z_1,\cdots,z_m)$ has no nonessential singularity of the second kind. The above theorem is known as the theorem of Anderson and Jury [8]. t is well known that, for the purposes of stability testing, we need more practical tests than the above theorem. For two dimensional systems, a great variety of practical tests have been developed in the last three decades and some of these are Jury's two dimensional test [9], Schur-Chon test [1], Inner's test [22], Zeheb-Walach test [5] [6]. There are also a variety of special results and other considerations [25].

However for multi-dimensional systems (M-D) systems $(m > 3)$, we have a complete lack of such tests, though we must refer to the contributions of [2] [3] [6] [7]. Hence, it is difficult to check as to whether a given multi-dimensional polynomial $B(z_1,\cdots,z_m)$ corresponds to the characteristic polynomial [27] of the stable multi-dimensional system, when $m > 2$. In this paper a simple method is proposed to find the stability of the given higher order system, we mean that it corresponds to the characteristic polynomial of a stable (Or) unstable multi-dimensional system.

3.1. Proposed Test Procedure

The previous necessary condition as mentioned in [8] and one sufficient condition referred in [21] [23] and (2) may be rewritten using reciprocal, since all the roots $\left(\dfrac{1}{z_n}\right)$ are assumed to be lying within a unit circle surface

$$B\left(\frac{1}{z_1},\frac{1}{z_2},\cdots,\frac{1}{z_n}\right) = T_0\left(\frac{1}{z_1},\frac{1}{z_2},\frac{1}{z_3},\cdots\right)\left(\frac{1}{z_n}\right)^n + T_1\left(\frac{1}{z_1},\frac{1}{z_2},\frac{1}{z_3},\cdots\right)\left(\frac{1}{z_n}\right)^{n-1} + \cdots + T_n\left(\frac{1}{Z_1}\right) = 0 \tag{3}$$

After simplification the above equation may be written as Equation (4)

$$M\left(Z_1, Z_2, Z_3\right)\Big|_{(Z_1=Z_2=Z_3=Z)} = F(Z) = 0 \tag{4}$$

$F(Z) = 0$ Is one dimensional equation and for stability $|Z| < 1$.

Let

$$F(Z) = a_n Z^n + a_{n-1} Z^{n-1} + \cdots + a_0 = 0 \tag{5}$$

where a_i —are the coefficients and n is the degree of $f(z) = 0$.

The Equation (5) is written as the following Equation (6)

$$F(Z) = Z^n + \left(\frac{a_{n-1}}{a_n}\right) Z^{n-1} + \cdots + \left(\frac{a_0}{a_n}\right) = 0 \tag{6}$$

For unity shifted [8] unit circle state that in $F(Z)$, z is replaced as $Z = x + 1$.

The Equation (6) can be written as

$$F(x+1) = (x+1)^n + \left(\frac{a_{n-1}}{a_n}\right)(x+1)^{n-1} + \cdots + \left(\frac{a_0}{a_n}\right) = f(x) \tag{7}$$

Then the unity shifted $f(x)$ can be analyzed by algebraic method using following necessary and sufficient condition for stability. The Proof for the above equations is discussed by Jury in [26] [27].

3.2. Proposed Necessary Conditions

The Unity shifted equivalent one dimensional characteristic equation tested using following necessary conditions [26] [28].

(i) $f(0) > 0$ $\tag{8}$

(ii) $f(-2) > 0$ for $\left[\text{Even } f(x)\right]$

(iii) $f(-2) < 0$ for $\left[\text{Odd } f(x)\right]$ $\tag{9}$

3.3. Proposed Sufficient Conditions

Using the coefficients of unity shifted $f(x)$ along with two triangular matrices [26] [28] referred in Equations (10) and (11) as given below.

$$[X] = \begin{pmatrix} a_n & a_{n-1} & a_{n-2} & \cdot & \cdot & \cdot & a_0 \\ 0 & a_n & a_{n-1} & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & a_n & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & a_n & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_n \end{pmatrix} \tag{10}$$

$$[Y] = \begin{pmatrix} a_n & a_{n-1} & a_{n-2} & \cdot & \cdot & \cdot & a_0 \\ a_{n-1} & a_{n-2} & a_{n-3} & \cdot & \cdot & \cdot & 0 \\ a_{n-2} & a_{n-3} & 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ a_0 & 0 & 0 & \cdot & \cdot & \cdot & 0 \end{pmatrix}$$ (11)

Adding $[X]$ and $[Y]$ to the above equation, a square matrix is formed as in Equation (12).

$$[H] = [X] + [Y]$$ (12)

This square matrix $[S]$ is said to be positive inner wise when all the determinants with the center element $[S]$ and to be scrambled outwards and must be positive [23]. This procedure is used for testing the sufficient condition that $|x| < 1$. In Jury method [22] two matrices $H_1 = X + Y$ and $H_2 = X - Y$ were formed to determine inner determinants starting with the centre elements and proceeding outwards up to the entire matrix are positive. The proposed scheme accounts all the coefficient of the characteristic equation in order to form the matrix followed by applying left shifting and right shifting principle to form X and Y matrix respectively. The single square matrix $H = X + Y$ has been constructed from X and Y matrix with respect to Jury's proposal in [26], hence this proposed method reduces the number of computations compared to jury method [22].

4. Illustrations

Example 1: [3]

$$f(Z) = 1 + Z_1 + Z_1 Z_2 + Z_1 Z_3 + 5 Z_1 Z_2 Z_3$$

Convert the two dimensional equation into one dimensional equation

$$f(Z) = 1 + \frac{1}{Z_1} + \frac{1}{Z_1 Z_2} + \frac{1}{Z_1 Z_3} + \frac{5}{Z_1 Z_2 Z_3}$$

Put $Z_1 = Z_2 = Z_3 = Z$

$$f(Z) = 1 + \frac{1}{Z} + \frac{1}{Z^2} + \frac{1}{Z^2} + \frac{5}{Z^3}$$

$$f(Z) = Z^3 + Z^2 + 2Z + 5$$

For unity shifting the unit circle to left half of Z-plane $(X = Z - 1)$ put $Z = X + 1$.

$$f(X+1) = (X+1)^3 + (X+1)^2 + 2(X+1) + 5$$

$$f(X+1) = f(X) = X^3 + 4X^2 + 7X + 10 = 0$$

Necessary Conditions:

1) $f(0) = 10 > 0$ (satisfied)

2) $f(-2) = 4 > 0$ (satisfied)

Sufficient conditions:

To check the requirements of sufficiency, we construct X and Y

$$X = \begin{pmatrix} 1 & 4 & 7 & 10 \\ 0 & 1 & 4 & 7 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}, Y = \begin{pmatrix} 1 & 4 & 7 & 10 \\ 4 & 7 & 10 & 0 \\ 7 & 10 & 0 & 0 \\ 10 & 0 & 0 & 0 \end{pmatrix}$$

Add matrix X and Y

$$H = X + Y = \begin{pmatrix} 2 & 8 & 14 & 20 \\ 4 & 8 & 14 & 7 \\ 7 & 10 & 1 & 4 \\ 10 & 0 & 0 & 1 \end{pmatrix}$$

The determinants are

$$\nabla_2 = \begin{vmatrix} 8 & 14 \\ 10 & 1 \end{vmatrix} = -0.132 < 0 \, (\text{Not satisfied})$$

$$\nabla_4 = \begin{vmatrix} 2 & 8 & 14 & 20 \\ 4 & 8 & 14 & 7 \\ 7 & 10 & 1 & 4 \\ 10 & 0 & 0 & 1 \end{vmatrix} = 17424 > 0 \, (\text{satified})$$

The given system satisfied the all necessary condition but not satisfied the one sufficient condition therefore the system is unstable.

Example 2: [25]

$$f(Z) = Z_3 Z_4 + Z_1 Z_4 + Z_4 + z_2 + 5$$

Convert the multi-dimensional equation into one-dimensional equation

$$f(Z) = \frac{1}{Z_3 Z_4} + \frac{1}{z_1 Z_4} + \frac{1}{Z_4} + \frac{1}{Z_2} + 5$$

Put $Z_1 = Z_2 = Z_3 = Z_4 = Z$

$$f(Z) = 5 + \frac{1}{Z} + \frac{1}{Z} + \frac{1}{Z^2} + \frac{1}{Z^2}$$

$$f(Z) = 5Z^2 + 2Z + 2$$

For unity shifting the unit circle put $Z = X + 1$.

$$f(X+1) = 5(X+1)^2 + 2(X+1) + 2$$

$$f(X+1) = f(X) = 5 + 12X + 9 = 0$$

Necessary conditions:

1) $f(0) = 9 > 0 \, (\text{satisfied})$

2) $f(-2) = 5 > 0 \, (\text{satisfied})$

Sufficient conditions:

To check the requirements of sufficiency, we construct X and Y and Add matrix X and Y

$$X = \begin{pmatrix} 5 & 12 & 9 \\ 0 & 5 & 12 \\ 0 & 0 & 5 \end{pmatrix}, Y = \begin{pmatrix} 5 & 12 & 9 \\ 12 & 9 & 0 \\ 9 & 0 & 0 \end{pmatrix}, H = X + Y = \begin{pmatrix} 10 & 24 & 18 \\ 12 & 14 & 12 \\ 9 & 0 & 5 \end{pmatrix}$$

The determinants are

$$\nabla_1 = 14 > 0 \, (\text{satisfied}), \nabla_3 = \begin{vmatrix} 10 & 24 & 18 \\ 12 & 14 & 12 \\ 9 & 0 & 5 \end{vmatrix} = -416 < 0 \, (\text{Not satified})$$

The given system satisfied the all necessary conditions but not satisfied the one sufficient condition therefore the system is unstable.

Example 3: [13]

Let
$$f(Z) = 1 - 2Z_1 + 0.3Z_2 + 3Z_3Z_2 + 4Z_1Z_2 - 0.04Z_1^2 + 4Z_1Z_2^2 - 5Z_2Z_3^2$$
$$+ 0.7322Z_1Z_2Z_3 - 2Z_1Z_2Z_3^2 + 3.024Z_1^2Z_3^2 - Z_2^2Z_3^2$$

Convert the multi-dimensional equation into one-dimensional equation

$$f(Z) = 1 - \frac{2}{Z_1} + \frac{0.3}{Z_2} + \frac{3}{Z_3Z_2} + \frac{4}{Z_1Z_2} - \frac{0.04}{Z_1^2} + \frac{4}{Z_1Z_2^2} - \frac{5}{Z_2Z_3^2} + \frac{0.7322}{Z_1Z_2Z_3} - \frac{2}{Z_1Z_2Z_3^2} + \frac{3.024}{Z_1^2Z_3^2} - \frac{1}{Z_2^2Z_3^2}$$

Put $Z_1 = Z_2 = Z_3 = Z$

$$f(Z) = 1 - \frac{2}{Z} + \frac{0.3}{Z} + \frac{3}{Z^2} + \frac{4}{Z^2} - \frac{0.04}{Z^2} + \frac{4}{Z^3} - \frac{5}{Z^3} + \frac{0.7322}{Z^3} - \frac{2}{Z^4} + \frac{3.024}{Z^4} - \frac{1}{Z^4}$$

$$f(Z) = Z^4 - 1.7Z^3 + 1.04Z^2 - 0.268Z + 0.024$$

For unity shifting the unit circle put $Z = X + 1$.

$$f(X+1) = (X+1)^4 - 1.7(X+1)^3 + 1.04(X+1)^2 - 0.268(X+1) + 0.024$$

$$f(X+1) = f(X) = X^4 + 2.3X^3 + 1.94X^2 + 0.712X + 0.096 = 0$$

Necessary conditions:

1) $f(0) = 0.096 > 0$ (satisfied)

2) $f(-2) = 4.032 =$ (satisfied)

Sufficient conditions:

To check the requirements of sufficiency, we construct X and Y

$$X = \begin{pmatrix} 1 & 2.3 & 1.94 & 0.712 & 0.096 \\ 0 & 1 & 2.3 & 1.94 & 0.712 \\ 0 & 0 & 1 & 2.3 & 1.94 \\ 0 & 0 & 0 & 1 & 2.3 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, Y = \begin{pmatrix} 1 & 2.3 & 1.94 & 0.712 & 0.096 \\ 2.3 & 1.94 & 0.712 & 0.096 & 0 \\ 1.94 & 0.712 & 0.096 & 0 & 0 \\ 0.712 & 0.096 & 0 & 0 & 0 \\ 0.096 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Add matrix X and Y

$$H = X + Y = \begin{pmatrix} 2 & 4.6 & 3.88 & 1.424 & 0.192 \\ 2.3 & 2.94 & 3.012 & 2.036 & 0.712 \\ 1.94 & 0.712 & 1.096 & 2.3 & 1.94 \\ 0.712 & 0.096 & 0 & 1 & 2.3 \\ 0.096 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The determinants are

$$\nabla_1 = 1.096 > 0 \text{ (satisfied)}$$

$$\nabla_3 = \begin{vmatrix} 2.94 & 3.012 & 2.036 \\ 0.712 & 1.096 & 2.3 \\ 0.096 & 0 & 1 \end{vmatrix} = 1.5284 > 0 \text{ (satified)}$$

$$\nabla_5 = \begin{vmatrix} 2 & 4.6 & 3.88 & 1.424 & 0.192 \\ 2.3 & 2.94 & 3.012 & 2.036 & 0.712 \\ 1.94 & 0.712 & 1.096 & 2.3 & 1.94 \\ 0.712 & 0.096 & 0 & 1 & 2.3 \\ 0.096 & 0 & 0 & 0 & 1 \end{vmatrix} = 0.02621 > 0 \text{ (satisfied)}$$

The given system satisfies the all necessary and sufficient conditions therefore the system is stable.

5. Conclusion

In this paper, an overview of the most important results for the stability of m-D discrete systems was made. Various approaches to this important problem for the analysis and design of m-D discrete systems have been studied. A simple and direct method for stability testing of m-dimensional linear discrete system has been proposed. The implementation of the method has been discussed and compared to jury [22] method which requires minimum arithmetic operation and the results illustrated by three examples.

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