

# Diversity–Multiplexing Tradeoff and Outage Performance for 2×2 Dual-Polarized Uncorrelated Rice MIMO Channels\*

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## ABSTRACT

In this paper, diversity-multiplexing tradeoff (DMT) curve for 2×2 Dual-Polarized uncorrelated Rice MIMO channels is studied. Exact expressions for statistic information of mutual information exponent are derived. Impacts of channel parameters such as signal to noise ratio (SNR), k-factor and cross polarization discrimination (XPD) on mutual information exponent are analyzed. Compared to conventional single-polarized (SP) Rice MIMO systems, a qualitatively different behavior is observed for DP Rice systems. The work in this paper, allows to identify quantitatively for which channels (k-factor) and SNR levels the use of dual polarization becomes beneficial. Gamma or lognormal distribution are used to describe mutual information component, and a theoretical formulation for finite-SNR DMT curve in 2×2 DP uncorrelated Rice channels is presented for the first time, which is valid in low and medium SNRs when multiplexing gain is larger than 0.75.

**Keywords:** DMT curve; Dual-Polarized; Uncorrelated Rice Channel; Mutual Information Exponent; k-factor; Outage Probability Approximation

## 1. Introduction

Due to the space-cost of the conventional Single-polarized (SP) multiple-input multiple-output (MIMO) systems, dual-polarized (DP) MIMO has been receiving much attention as an attractive alternative for realizing MIMO architectures in compact devices [1-7]. Compared with SP MIMO, DP MIMO exhibits many different characteristics. For instance, in [1] it has been clearly illustrated that in Rice fading, after some k-factor (defined as the ratio of the power in the fixed exponent to the power in the variable exponent), error probability of zero-forcing detection method for polarization multiplexing starts to decrease with increasing k-factor, while SP systems perform the opposite. Moreover, various literatures, such as in [1-4], an idea has been well developed that polarization diversity works well only in correlated Rayleigh fading or Rice fading channels with LOS components. It is necessary to note that measurements have been done to get real parameters of DP channels, which helps in getting more accurate polarized channel model [6]. To go further, channel correlation and capac-

ity are discussed in these literatures, proving that such dual polarization has de-correlation effect on correlated channels from a practical aspect. Nevertheless, this result does not extend to diversity systems, such as Alamouti coded MIMO, where polarization confronts performance loss [2]. In conventional MIMO systems, it is known that there exists a fundamental tradeoff between achievable diversity and multiplexing gains of any transmission over  $n_t \times n_r$  MIMO channel, i.e., diversity-multiplexing tradeoff (DMT), as has been clearly illustrated in [8] for signal to noise ratio (SNR) approaching infinity. Moreover, it is also pointed out that DMT curve at finite SNR is quite different [9-13]. Under realistic propagation conditions, since SNR cannot reach infinity, it would be meaningful to study DMT behavior at finite SNRs that are practical in operating regimes. Up to now there are no literatures that investigate finite-SNR DMT for dual-polarized systems.

In previous literatures [9-13], DMT curve is discussed based on the assumption that, elements of  $HH^H$  follow Wishart distribution. However, for polarized MIMO, because of the asymmetric properties of the generalized channel matrix, random matrix theory results for Wishart matrices cannot be leveraged. Inspired by the idea proposed in [14], which used gamma, lognormal or weibull

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distribution to approximate outage capacity for dual-polarized MIMO in high SNR regime, by approximating mutual information exponent, we get theoretical DMT curve for DP in low and medium SNR regimes in  $2 \times 2$  uncorrelated Rice channels.

The rest of this paper is organized as follow. Section 2 describes the channel model developed for a  $2 \times 2$  Dual-Polarized uncorrelated Rice fading channels. Section 3 discusses statistic characteristic of mutual information exponent, outage probability and DMT curve and their approximations. Section 4 shows the simulation results. Finally, section 5 is the conclusion.

In this paper,  $E(x)$  and  $D(x)$  represents the expectation and variation of random variable  $x$ , respectively,  $*$  stands for the element-wise conjugation,  $H$  for conjugate transpose,  $\det(A)$  is the determinant of matrix  $A$ .

## 2. System Model and Definitions

Consider a system with one dual-polarized transmit and one dual-polarized receive antenna. The channel is assumed frequency-flat over the band of interest. The channel matrix is given by

$$H = \begin{pmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \end{pmatrix} \quad (1)$$

Assume that both transmitter and receiver employ the same polarization scheme, i.e. both of them employ horizontal/vertical or slanted polarization. Decomposing the channel matrix into the sum of a fixed exponent and a variable exponent as

$$H = \sqrt{\frac{k}{1+k}} \bar{H} + \sqrt{\frac{1}{1+k}} \tilde{H} \quad (2)$$

The elements of the matrix  $\bar{H}$  do not vary and satisfy  $|\bar{h}_{11}|^2 = |\bar{h}_{22}|^2 = 1$ ,  $|\bar{h}_{12}|^2 = |\bar{h}_{21}|^2 = \alpha_f$ . The elements  $\tilde{h}_{ij}$  of the matrix  $\tilde{H}$  are complex random variables, which satisfy

$$E[\tilde{h}_{11}]^2 = E[\tilde{h}_{22}]^2 = 1, E[\tilde{h}_{12}]^2 = E[\tilde{h}_{21}]^2 = \alpha \quad (3)$$

$$E(\tilde{h}_{11}\tilde{h}_{12}) = E(\tilde{h}_{12}\tilde{h}_{22}) = 0; E(\tilde{h}_{12}\tilde{h}_{21}) = E(\tilde{h}_{11}\tilde{h}_{22}) = 0 \quad (4)$$

where  $0 \leq \alpha_f \leq 1$ ,  $0 \leq \alpha \leq 1$  are related to the XPD for the fixed and variable exponent of the channel, respectively. Good discrimination of orthogonal polarizations amounts to small values of  $\alpha$  and  $\alpha_f$ , and vice versa. Clearly, when  $\alpha_f = 1, \alpha = 1$  the model becomes the conventional SP (single polarization) channel. For  $2 \times 2$  Dual-Polarized uncorrelated Rice MIMO channels  $h_{ij}$  are complex Gaussian random variables whose parameters are:

$$h_{ij} \sim \begin{cases} \text{CN}\left(\sqrt{\frac{k}{k+1}}, \frac{1}{k+1}\right), & i = j; i, j = 1, 2 \\ \text{CN}\left(\sqrt{\frac{k}{k+1}}\alpha_f, \frac{1}{k+1}\alpha\right), & i \neq j; i, j = 1, 2 \end{cases} \quad (5)$$

In [8], conventional asymptotic definitions of multiplexing and diversity gains for a MIMO channel are given by:

$$r^* = \lim_{\rho \rightarrow \infty} \frac{\log(R(\rho))}{\log \rho} \quad (6)$$

$$d^* = \lim_{\rho \rightarrow \infty} \frac{\log P_{out}}{\log \rho} \quad (7)$$

where  $r^*$  and  $d^*$  represent the asymptotic multiplexing and diversity gain respectively,  $\rho$  is the average SNR per receive antenna,  $R$  is the system data rate and  $P_{out}$  is the outage probability. Assuming that no CSI is available at the transmitter,  $P_{out}$  is defined by

$$P_{out} = P(I < R) = P(W < 2^R) \quad (8)$$

where  $I$  is the mutual information between received and transmitted signals over the MIMO channels, and  $W$  is the mutual information exponent satisfy  $I = \log W$ .

The asymptotic DMT is given by the piece-wise linear function connecting the points  $(i, d^*(i))$ , where  $d^*(i)$  is given by [8]:

$$d^*(i) = (n_r - i)(n_t - i), \quad i = 0, \dots, \min\{n_r, n_t\} \quad (9)$$

$n_r, n_t$  are numbers of receive and transmit antennas, respectively. Note that the asymptotic DMT describes situation where SNR approaches infinity. However, for practical system design, it is desirable to characterize the diversity-multiplexing tradeoff at operational SNRs. The finite-SNR definitions for diversity and multiplexing gains can provide useful tool to characterize the DMT at real environment. The finite-SNR multiplexing gain  $r$  is defined as the ratio of  $R$  to the capacity of an AWGN channel at SNR with array gain  $G$  [11],

$$G = \min\{n_r, n_t\} \\ r = \frac{R}{\log(1 + G\rho)} \quad (10)$$

The finite-SNR outage probability  $P_{out}(r, \rho)$  for a given  $r$  and  $\rho$  is given

$$P_{out}(r, \rho) = P\left(W < 2^{(1+G\rho)^r}\right) \quad (11)$$

The finite SNR diversity gain  $d(r, \rho)$  is defined by the negative slope of the plot  $P_{out}(r, \rho)$  versus  $\log \rho$ :

$$d = -\frac{\rho}{P_{out}(r, \rho)} \frac{\partial P_{out}(r, \rho)}{\partial \rho} \quad (12)$$

### 3. Computation of DP finite-SNR DMT

In this section, the DMT for  $2 \times 2$  Dual-polarized Rice channels is examined. First, we derive an exact expression for the mean and variation of mutual information exponent, based on which some discussions on channel parameters are proposed to have a deeper insight into dual-polarization system. Second, using the expressions of statistic information derived in the first step, approximation equations of outage probability are presented. Finally, DMT for both asymptotic and finite-SNR in  $2 \times 2$  Dual-polarized Rice channels at are investigated.

#### 3.1. Statistic Information of Mutual Information Exponent

Consider that channel state information (CSI) is perfectly known at the receiver. The MIMO mutual information  $I$  conditioned on the channel realization is given by

$$\begin{aligned} I &= \log \det \left( I_{n_r} + HH^H \right) \\ &= \log \prod_{i=1}^{\text{rank}(HH^H)} \left( 1 + \frac{\rho}{N_t} \lambda_i \right) \end{aligned} \quad (13)$$

$$\text{where } W = \prod_{i=1}^{\text{rank}(HH^H)} \left( 1 + \frac{\rho}{n_i} \lambda_i \right),$$

and  $\lambda_i$  denotes the eigenvalues of  $HH^H$ . For the case of  $n_r \times 2$  or  $2 \times n_r$  MIMO, mean and variances of  $W$  as a function of k-factor and  $\alpha_f, \alpha$  are expressed below:

$$E(W) = 1 + \frac{\rho}{n} E \left( \sum \lambda_i \right) + \left( \frac{\rho}{n} \right)^2 E \left( \prod \lambda_i \right) \quad (14)$$

$$\begin{aligned} WD(W) &= \left( \frac{\rho}{n} \right)^2 D \left( \sum \lambda_i \right) + \left( \frac{\rho}{n} \right)^4 D \left( \prod \lambda_i \right) \\ &+ 2 \left( \frac{\rho}{n} \right)^3 \cdot R \left( \sum \lambda_i, \prod \lambda_i \right) \end{aligned} \quad (15)$$

where  $n = \min \{ n_r, n_t \}$ , and  $E(\sum \lambda_i)$ ,  $D(\sum \lambda_i)$ ,  $E(\prod \lambda_i)$ ,  $R(\sum \lambda_i, \prod \lambda_i)$  are given in the Appendix 1, for the sake of space saving.

The distribution of the mutual information exponent provides information about the available diversity in the system.  $E(W)$  describes the ergodic mutual information exponent, which can be used to get upper bound of mutual information  $I$ . And  $D(W)$  presents some information about outage probability, i.e., the smaller the variance, the lower the probability of the outage error is when transmitting at a fixed rate [8].

From the analytical expression of  $E(W)$  and  $D(W)$  given in (14)-(15), we find that both of them are influenced by k-factor and SNR. With the existences of po-

larization indicators  $\alpha_f$  and  $\alpha$ , the influence are different. Let  $E_{SP}(W)$ ,  $E_{DP}(W)$  be mean of information exponent of SP and DP, we get

$$E_{SP}(W) - E_{DP}(W) = \frac{1}{(k+1)^2} \frac{\rho}{n} (Ak^2 + Bk + C) \quad (16)$$

Then k-factor for  $E_{SP}(W) = E_{DP}(W)$  is derived:

$$k = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad (17)$$

where

$$\begin{aligned} A &= \frac{\rho}{n} (-1 - \alpha_f^2 + 2\alpha_f) + 2(1 - \alpha_f) \\ B &= \frac{\rho}{n} (2 - 2\alpha_f\alpha) + 2(2 - \alpha_f - \alpha) \\ C &= \frac{\rho}{n} (1 - \alpha^2) + 2(1 - \alpha) \end{aligned} \quad (18)$$

In suburban area, where XPD is measured to be 8-15dB range, let  $\alpha=0.4$ ,  $\alpha_f=0.3$ . We find that in SP, mean of mutual information exponent decrease fast with the increase in k-factor, while for DP, declension is less. At  $\rho=0dB$ , required k-factor to fill the gap between DP and SP is  $k=-0.5754$  or  $-2.4376$ ;  $\rho=10dB$ , required  $k=11.3118$  or  $-0.4546$ ; when  $\rho \rightarrow \infty$ ,  $k=4.0184$  or  $-0.4266$ .

#### 3.2. Approximating of Outage Probability

Motivated by the work [14], in this section, we derive the approximation curve for outage probability at finite SNR for  $2 \times 2$  dual-polarized uncorrelated Rice channels.

The steps begin with the approximation of statistical information of mutual information exponent  $W$ .

If we assume gamma distribution for  $W$ , i.e.

$$f_W(w) = w^{p-1} \frac{e^{-w/\theta}}{\Gamma(p)\theta^p}, w \geq 0 \quad (19)$$

$$\begin{aligned} \theta(\rho, k) &= \frac{D(W)}{E(W)} = \frac{DW(\rho, k)}{EW(\rho, k)} \\ p(\rho, k) &= \frac{[E(W)]^2}{D(W)} = \frac{[EW(\rho, k)]^2}{DW(\rho, k)} \end{aligned} \quad (20)$$

Then outage probability at given multiplexing gain and SNR is

$$P_{out}(r, \rho) = \frac{\gamma \left( p, \frac{(1+G\rho)^r}{\theta} \right)}{\Gamma(p)} \quad (21)$$

where  $\gamma(k, x) = \int_0^x t^{k-1} e^{-t} dt$  is the incomplete gamma function.

If we assume lognormal distribution for  $W$ , i.e.

$$f_w(w) = \frac{1}{w\sqrt{2\pi\sigma^2}} e^{-\frac{(\ln(w)-u)^2}{2\sigma^2}}, w \geq 0 \quad (22)$$

$$u = E(\ln W) = \ln(EW) - \frac{1}{2} \ln\left(1 + \frac{DW}{(EW)^2}\right) \quad (23)$$

$$\sigma^2 = D(\ln W) = \ln\left(1 + \frac{DW}{(EW)^2}\right) \quad (24)$$

Then

$$P_{out}(r, \rho) = \frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{r \ln(1+G\rho) - u}{\sigma\sqrt{2}}\right) \quad (25)$$

where both  $E(W)$  and  $D(W)$  are given in section 2. Note that format parameters of  $W$  are directly related to polarization parameters  $\alpha_f, \alpha$  as well as k-factor, SNR.

Corollary: When  $k \rightarrow \infty$ , for outage probability of DP,

$$P_{out} = 0, 0 \leq r \leq \min\{N_r, N_t\} \quad (26)$$

In contrast, outage probability in SP is given by [12]

$$P_{out} = \begin{cases} 0 & r \leq 1 \\ 1, & r > 1 \end{cases} \quad (27)$$

Proof: As  $k \rightarrow \infty$ , for conventional SP, the Rice fading channel approaches a rank-one AWGN channel, such that the outage probability is 1 for  $r > 1$ , and 0 for  $r < 1$ ; However, for DP Rice fading channels, as  $k \rightarrow \infty$ , thanks to polarization orthogonality, channel matrix remains full rank. Thus, as  $k$  increases, channel approach two rank-one AWGN channels. Therefore as  $k \rightarrow \infty$ , outage probability  $P_{outDP} = 0$  for both  $r < 1$  and  $r > 1$ .

### 3.3. Asymptotic DMT for Rice Dual Polarized Channels

Theorem: The asymptotic DMT curve for dual-polarized channels is independent of  $\alpha$ ,  $\alpha_f$ , which is identical to conventional asymptotic DMT in SP channels as described in (9) [8].

Proof: The proof is given in appendix 2.

### 3.4. Diversity and Multiplexing Trade-off at Finite SNR

Simulated by the method in [11], we get finite-SNR DMT using (11).

$$\begin{aligned} \frac{\partial P_{out}(r, \rho)}{\partial \rho} &= \frac{\partial}{\partial \rho} \left[ \int_0^{(1+G\rho)^r} f(w, \rho, k) dw \right] \\ &= A_1(r, \rho) + A_2(r, \rho) \end{aligned} \quad (28)$$

where for gamma approximation,

$$f(w, \rho, k) = w^{p(\rho, k)-1} \frac{e^{-w/\theta(\rho, k)}}{\Gamma(p(\rho, k))\theta(\rho, k)^{p(\rho, k)}} \quad (29)$$

$$A_1(r, \rho) = rG(1+G\rho)^{r-1} f\left((1+G\rho)^r, \rho, k\right) \quad (30)$$

$$A_2(r, \rho) = \int_0^{(1+G\rho)^r} \frac{\partial}{\partial \rho} f(w, \rho, k) dw$$

For lognormal approximation, calculation step is similar, which is omitted for the sake of space.

## 4. Simulation

### 4.1. Impact of k-factor and SNR on Mutual Information Exponent

As it has been known that DP and SP systems perform rather different in Rice channels. In order to study how such a difference occurs, **Figure 1** plots  $E(W)$  and  $D(W)$  as a function of the k-factor at  $\rho = 0\text{dB}$  and  $\rho = 10\text{dB}$ . Assume that for DP,  $\alpha = 0.4$ ,  $\alpha_f = 0.3$ , and without loss of generality, take  $h_{ij} = 1$  for  $i, j = 1, 2$ .

The theory curves are identical to the ones by Monte Carlo simulations, which validate the derived expression of  $E(W)$  and  $D(W)$  in section 2. As expected, a quantitatively different behavior is observed for DP Rice system. Although either in SP or DP case, expectation and standard deviation of the mutual information exponent drop dramatically with increasing k-factor, especially in low k-factor regime, where k-factor manifests the variation of  $W$ . It is clear that in DP, the drop is far less than that in SP both for  $E(W)$  and  $D(W)$ , since polarization can reduce the channel correlation brought by LOS component. Moreover, at  $\rho = 0\text{dB}$ , no cross points for  $E_{SP}(W) = E_{DP}(W)$  at  $k > 0$  are found. But at  $\rho = 10\text{dB}$ ,  $k \approx 11$ , a cross point appears, matching the previous results from (17). Such a phenomenon can

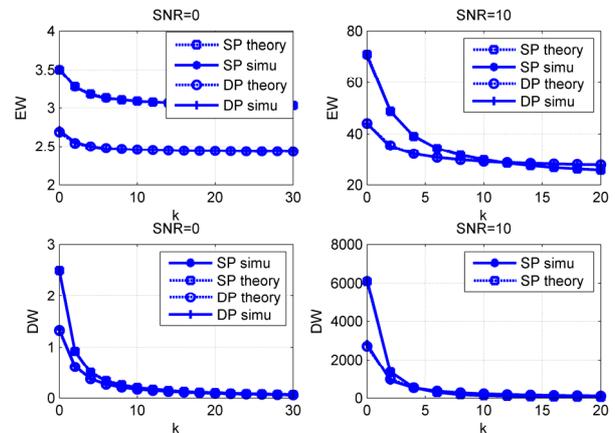
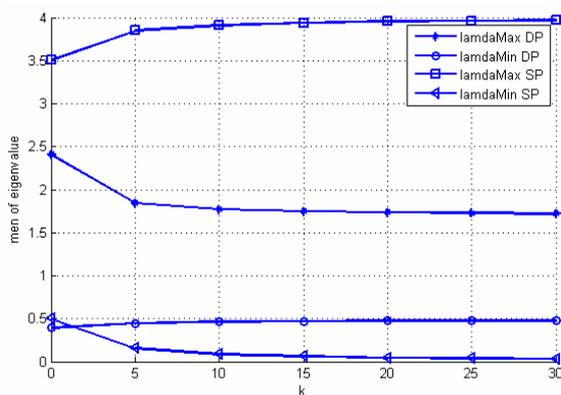


Figure 1. Comparison of mean and variation of  $W$  for SP and DP.

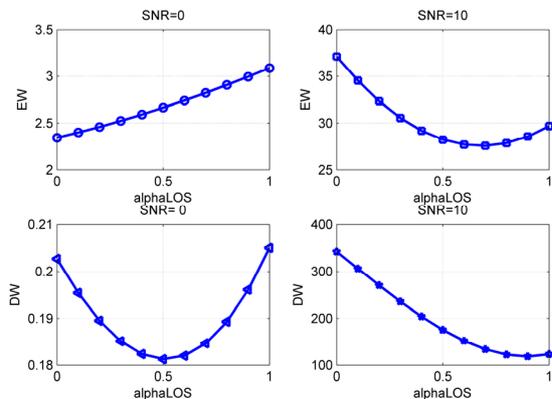
be explained by the effect of eigenvalue of  $HH^H$ . At medium SNR, minimum eigenvalue begins to affect channel information exponent. For conventional SP  $2 \times 2$  Rice systems, when k-factor increases, channel matrix tends to be a rank-deficient matrix, leading the minimum eigenvalue to be smaller even approaching zero. In contrast, eigenvalues of DP systems nearly stays constant, without being hugely affected by varying k-factor. Hence, channel matrix does not become ill conditioned, i.e., not badly affected by LOS component. Thus, in seminars with strong LOS component, we suggested DP be used. To illustrate the different eigenvalues we can see **Figure 2**.

### 4.2. Impact of XPD on Mutual Information Exponent

As a final parameter dependency study, we examine on mutual information exponent as a function of the XPD in LOS component. Using the analytical formation in section 2, **Figure 3** plots plots  $E(W)$  and  $D(W)$  as a function of the  $\alpha_f$  at  $\rho=0\text{dB}$  and  $\rho=10\text{dB}$ , with fixed  $k=10$ ,  $\alpha=0.4$ .



**Figure 2.** Eigenvalues of SP and DP for  $2 \times 2$  uncorrelated Rice fading.



**Figure 3.** XPD influence on mutual information exponent in  $2 \times 2$  Rice uncorrelated channels with  $k=10$ .

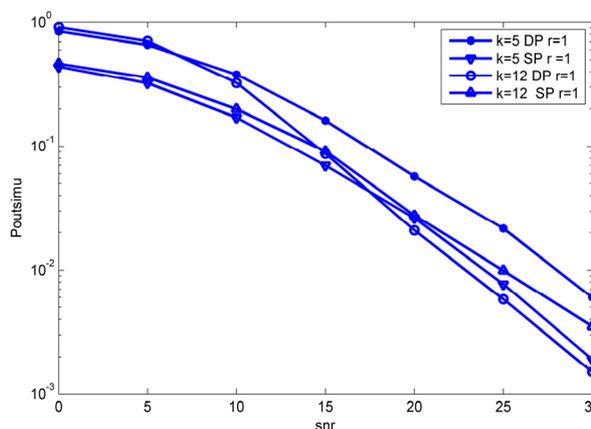
From **Figure 3**, it is clear that at low SNRs  $EW$  increases with  $\alpha_f$ . However, at moderate SNR,  $EW$  starts to drop with improving  $\alpha_f$ . Conclusions can be made that XPD and SNR have impacts on the mutual information at the same time. It is then meaningful to find the optimal SNRs for different DP systems for optimal code design.

### 4.3. Outage Probability in Finite SNR

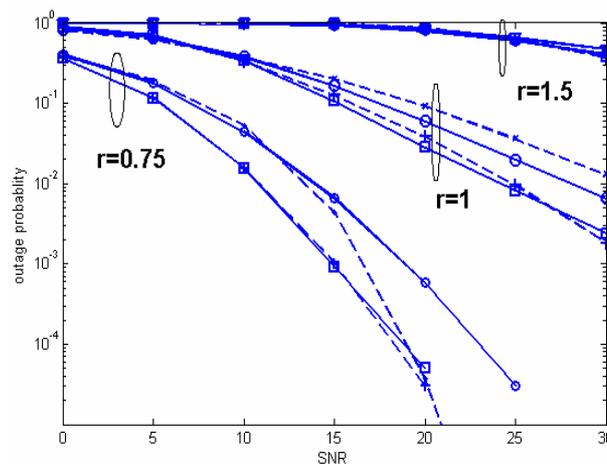
In this part, we study some plots of outage probability versus SNR in uncorrelated Rice fading with  $n_r = n_t = G = 2$ .

In **Figure 4**, given a fixed multiplexing gain  $r=1$ , outage probability versus SNR curves are plotted for SP and DP at  $k=5, 12$ . It is seen that contrary to SP, outage probability of DP always drops as k-factor improves. At some SNR, negative gap of outage probability between SP and DP turns into positive, coinciding with previous analysis.

In **Figure 5**, gamma or lognormal approximation are



**Figure 4.** Outage probability SP VS DP for  $k=5$  and  $k=12$ .



**Figure 5.** Outage probability approximation for different multiplexing gains and k-factor DP.

plotted as well as results of Monte Carlo simulation for outage probability in various multiplexing gain. The dash curves represent the approximation value, while the circle, square symbol represent the DP systems for  $k=5, k=10$  respectively. When  $r > 1$  ( $r=1.5$  in the plot), the gamma approximation matches the simulation well. At  $r < 1$  ( $r=0.75$ ), we can use the lognormal distribution instead, which works well in medium SNR 0-15dB. Note that the higher multiplexing gain, the more accuracy of the gamma approximation. By using this approximation method, it becomes simple to estimate the outage probability of DP Rice channels at  $r > 1$  or medium SNRs without time-cost simulation.

#### 4.4. Diversity Gain at Finite SNR

Figure 6 is a plot of diversity and multiplexing gain tradeoff in finite SNR for DP Rice channels,  $k=10$  and  $r \in [0.75, 2]$ .

Obviously, the approximation curve agrees with the Monte Carlo simulation. For  $r > 1$ , as it has been indicated in [11], the diversity gain in SP Rice fading channels approaches zero rapidly since the rank-one LOS matrix limits the effective degrees of freedom in the channels. However, for DP Rice fading, a relatively high diversity gain can still be observed in  $r \geq 1$ . At  $\rho = 10\text{dB}$ ,  $k=10$ , diversity gain can be as high as one. Explanations can be found from minimum eigenvalue of DP systems, that thanks to the de-correlation effect, minimum eigenvalue of DP do not approach zero despite of the exists of strong LOS component.

For very high k-factor, the channel matrix only depends on the Rice exponent. As  $k \rightarrow \infty$ , the channels tend to be AWGN, and the capacity increases only with SNR. For DP, asymptotic diversity gain becomes infinite.

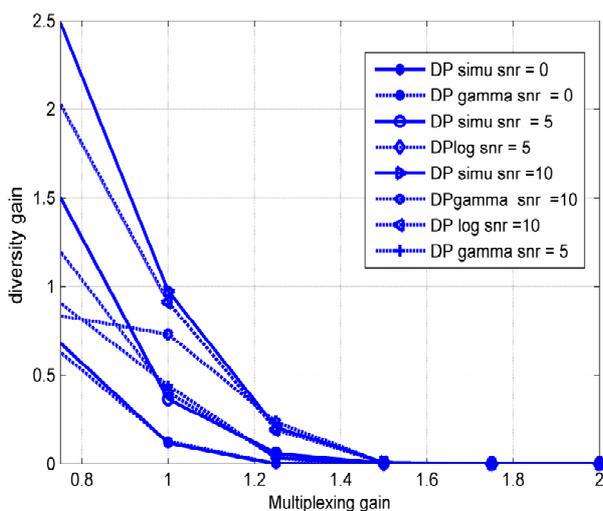


Figure 6. Finite-SNR DMT for DP Rice channels.

## 5. Conclusions

In this paper, outage probability and DMT for asymptotic and finite-SNR are studied in  $2 \times 2$  dual-polarized uncorrelated Rice fading channels. Exact expression mean and variation of mutual information in DP Rice channels are derived, based on which how channel parameters as k-factors, SNR or XPD influence channel information exponent are discussed. Results show that in suburban environments where  $\alpha = 0.4$ ,  $\alpha_f = 0.3$ , at  $\rho = 10\text{dB}$ , a  $k \approx 11$  is required to fill the gap between ergodic mean of mutual information exponent of SP and DP. Outage probability as well as asymptotic and finite-SNR DMT are compared between of SP and DP. Using the gamma or lognormal distribution, their approximation curves for  $2 \times 2$  dual-polarized uncorrelated Rice channels at  $k=10$  are given. The result in this paper, helps in finding the inner difference between DP and SP channels. And the approximation approach for DMT in this paper, although not so accurate in low multiplexing gain, can provide references in practical code design in dual-polarized Rice systems, especially in systems with large amounts of antennas.

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### Appendix 1

According to the distribution of channel elements (5), an

$$\sum \lambda_i = \|HH^H\|_F^2$$

which follows non-central chi-square distribution. Mean and variance of  $\sum \lambda_i$  can be derived as :

$$\begin{aligned} E\left(\sum \lambda_i\right) &= 2\left(\frac{k\alpha_f + \alpha}{k+1} + 1\right) \\ D\left(\sum \lambda_i\right) &= 2\frac{\alpha^2 + 2k\alpha_f\alpha + 2k + 1}{(k+1)^2} \end{aligned} \quad (31)$$

As for  $\prod \lambda_i$ , by  $|H| = \underbrace{h_{11}h_{22}}_{\eta + j\delta_1} - \underbrace{h_{12}h_{21}}_{\tau_2 + j\delta_2} = r + js$ ,

$E(\prod \lambda_i)$  and  $D(\prod \lambda_i)$  calculated as in [14] section 3 (11)-(14).

$$E\left(\prod \lambda_i\right) = 1 + \frac{(k\alpha_f + \alpha)^2 - 2k^2\alpha_f}{(k+1)^2} \quad (32)$$

According to (14),  $E(W)$  is derived. For  $D(W)$ , as

$$\begin{aligned} &R\left(\sum \lambda_i, \prod \lambda_i\right) \\ &= E\left(\sum \lambda_i \cdot \prod \lambda_i\right) - E\left(\sum \lambda_i\right)E\left(\prod \lambda_i\right) \end{aligned} \quad (33)$$

we get

$$\begin{aligned} &E\left(\sum \lambda_i \cdot \prod \lambda_i\right) \\ &= 2 \cdot \left\{ \left[ \left[ \left( \frac{1+2k}{(k+1)^2} + 1 \right) + \left( \frac{k\alpha_f + \alpha}{(k+1)} \right)^2 \right] + \right. \right. \\ &\quad \left. \left[ \frac{k\alpha_f + \alpha}{(k+1)} \left[ 1 + \frac{\alpha^2 + 2k\alpha_f\alpha}{(k+1)^2} + \left( \frac{k\alpha_f + \alpha}{(k+1)} \right)^2 \right] \right] \right\} \\ &\quad \left. - 2 \left[ \left[ \left( \frac{k\alpha_f + \alpha}{k+1} + 1 \right) + \frac{1+\alpha}{k+1} \right] \frac{k^2\alpha_f}{(k+1)^2} \right] \right\} \end{aligned} \quad (34)$$

Finally, using (15), exact expression of  $D(W)$  is given.

### Appendix 2

Using the method prosed in [9] we derive the proof for asymptotic DMT curve in dual-polarized uncorrelated

Rice channels. The proof begins with Rayleigh fading cases.

According to [9], let  $R = \log_2(\rho)$ . Firstly, we decompose  $H$  as:

$$H = QR \odot X, R = \begin{bmatrix} r_{11} & r_{12} \\ 0 & r_{22} \end{bmatrix} \odot X = \begin{bmatrix} r_{11} & \sqrt{\alpha} r_{12} \\ 0 & r_{22} \end{bmatrix} \quad (35)$$

Since  $r_{11} = \|H(:,1)\|^2$ , and  $r_{11}^2$  is approximated as  $r_{11}^2 \sim \chi_4^2$ , similarly,  $r_{12}^2 \sim \chi_2^2$ ,  $r_{22}^2 \sim \chi_2^2$ . Thus, for  $1 \leq r \leq 2$ , keep quadratic term  $\rho^2$  and neglect the other lower terms, we have

$$P_{out} \doteq P\left(\left(\frac{\rho}{n_t}\right)^2 (r_{11}^2 r_{22}^2) < (\rho)^r\right) \quad (36)$$

As  $r_{11}^2$  is a variable with a higher order than  $r_{22}^2$ , and that small  $r_{11}^2 r_{22}^2$  is mainly due to small  $r_{22}^2$ , i.e., the main event that causes

$$\left(\frac{\rho}{n_t}\right)^2 (r_{11}^2 r_{22}^2) < (\rho)^r \text{ is } (r_{11}^2 < 1) \cup (r_{22}^2 < \rho^{r-2})$$

occurring.

$$\text{Thus } P_{out} \doteq P\left((r_{11}^2 < 1) \cup (r_{22}^2 < \rho^{r-2})\right) \doteq \rho^{r-2}.$$

When  $1 \leq r \leq 2$ , diversity gain is derived as

$$d(r, \rho \rightarrow \infty) = r - 2 \quad (37)$$

And when  $0 \leq r < 1$ , neglecting the constant term, we

get

$$P_{out} \doteq P\left(\frac{\rho}{N_t} (r_{11}^2 + \alpha^2 |r_{12}^2|^2 + r_{22}^2) + \left(\frac{\rho}{N_t}\right)^2 (r_{11}^2 r_{22}^2) < \rho^r\right) \quad (38)$$

Here, main events are

$$\begin{aligned} & (r_{11}^2 < \rho^{r-1}) \cup (\alpha^2 |r_{12}^2|^2 < \rho^{r-1}) \cup (r_{22}^2 < \rho^{-1}) \\ & \doteq \frac{\rho^{2(r-1)} \rho^{(r-1)} \rho^{-1}}{\alpha^2} = \frac{\rho^{3(r-4)}}{\alpha^2} \end{aligned} \quad (39)$$

So that for  $0 \leq r < 1$  the diversity gain is derived as  $d(r, \rho \rightarrow \infty) = 3r - 4$ , without any relationship with  $\alpha$ . Eventually, as LOS component do not affect the high-SNR diversity gain [12], the asymptotic DMT analysis here hold on for Rice channels. Therefore, asymptotic DMT curve for DP Rice channels at infinite SNR is the same as the conventional conclusion(9) in [8].