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OFDMA Uplink Frequency Offset Estimation with Multi-Access Interference Mitigation*

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ABSTRACT

In this paper, we consider the frequency offset estimation for Orthogonal Frequency-Division Multiple Access (OFDMA) uplink (UL) transmissions. We first analyze the negative effect of Multi-Access-Interference (MAI) on OFDMA UL, and then propose two interference reduction/elimination methods, *i.e.*, the Reduced-Rank-Projector (RRP) and Shift-Sampling-Projector (SSP) methods, to eliminate/reduce the heavy MAI due to the frequency offsets. Finally, we propose a new training sequence group named the Round-Robin Training Sequence Group (RRTSG), which has a high interference mitigation capabilities for OFDMA UL transmission. The Cramer-Rao Lower Bound (CRLB) for an unbiased frequency offset estimator in a Multiple Access (MA) system is also derived. Numerical results show that the proposed methods are suitable to eliminate/mitigate the effect of the frequency offset on OFDMA UL transmission.

Keywords: Synchronization; Timing Offset; Frequency Offset; OFDMA

1. Introduction

Orthogonal Frequency-Division Multiple Access (OFDMA) is an effective Multiple Access (MA) scheme that divides the total signal bandwidth into multiple orthogonal subcarrier groups, with each group being allocated to one user [1]. In uplink (UL) transmission, different users have different frequency offsets and result an interference-limited system [2]. Multi-Access Interference (MAI) may appear between neighboring users due to the frequency offsets.

The Bit Error Rate (BER) of Orthogonal Frequency-Division Multiplex (OFDM) systems impaired by the frequency offset is analyzed in [3]. The effect of the frequency offset on OFDM/OFDMA has been analyzed considerably (see [4-6]). Performance improvement of OFDMA UL transmission through cooperative relaying is studied in [7,8], with either ergodic or outage information rate is analyzed in the presence of the frequency offset.

Although several classical frequency offset estimators [9-13] are proposed, their performance will degrade in an interference-limited environment. Since the frequency offset estimations are imperfect, MAI arises between different users. MAI cancelation in an OFDMA system is discussed in [14], where MAI can be eliminated correctly based on the perfect knowledge of the frequency offset of each user. A conventional estimator, such as that used in [15], is considered as a candidate for the frequency offset estimation for each user. Carrier Frequency Offset and I/Q Imbalances compensations in OFDM Systems by considering subcarrier allocation are studied [16]. Synchronization in OFDMA considering the effect of MAI has been studied [17-22]. OFDMA UL synchronization by exploiting the statistical character of the frequency offset is studied in [23], where the frequency offsets can be modeled as either uniform or Gaussian distributed random variables (RVs). For more references about OFDMA synchronization, please refer to, e.g., [24-26].

In this paper, we consider OFDMA UL frequency offset estimation based on the proposed training sequence. Subcarrier allocation for one user need not be contiguous in the frequency-domain. The MAI introduced by the non-zero frequency offset is analyzed first. The Reduced-Rank-Projector (RRP) and Shift-Sampling-Projector (SSP) methods are proposed to eliminate/reduce MAI due to the frequency offset. A new training sequence group named the Round-Robin Training Sequence Group (RRTSG) is also proposed for interference randomization. The quasi-synchronized transmission of

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accessing users is considered in this paper; *i.e.*, both the time and frequency of these users have been aligned to the base station. Perfect timing synchronization is considered, and only UL frequency offset estimation is discussed in this paper.

The remainder of this paper is organized as follows. Section 2 proposes the OFDMA UL signal model. MAI analysis is provided in Section 3. Sections 4 and 5 discuss the interference elimination/reduction methods, where the heavy MAI elimination/reduction methods are proposed in Section 4, and an interference randomization method is introduced in Section 5. Section 6 analyzes the proposed schemes and discusses some numerical results. Finally, Section 7 concludes the paper.

Notation: $(\cdot)^T$ and $(\cdot)^H$ denote transpose and conjugate transpose of a matrix, respectively. $(\cdot)^{-1}$ represents the inverse of a matrix. The imaginary unit is $j=\sqrt{-1}$. $\Re\{x\}$, and $\Im\{x\}$ are the real and imaginary part of x, respectively. $(\cdot)^*$ denotes complex conjugate. A circularly symmetric complex Gaussian variable with mean m and variance σ^2 is denoted by $z\sim CN(m,\sigma^2)$. $\mathbf{x}[i]$ represents the ij-th element of vector $\mathbf{x}$. $[\mathbf{A}]_{ij}$ represents the j-th element of matrix $\mathbf{A}$. $[\cdot]_N$ means "Mod N". The $N\times N$ identity matrix is $\mathbf{I}_N$, and the $N\times N$ all-zero matrix is $\mathbf{O}_N$. $E\{x\}$ and $\mathrm{Var}\{x\}$ denote the mean and the variance of x, respectively.

2. OFDMA UL Signal Model

In OFDMA, each user is modulated by complex data symbols from a signal constellation, e.g., phase-shift keying (PSK) or quadrature amplitude modulation (QAM). In a frequency-selective fading channel, the received signal at the base station can be represented as

$$\mathbf{y}=\sum_k \mathbf{y}_k=\underbrace{\sum_k \mathbf{E}_k \underbrace{\mathbf{F}_k\mathbf{H}_k\mathbf{\Phi}_k\mathbf{x}_k}_{\mathbf{m}_k}}_{\mathbf{v}_k}+\underbrace{\sum_k \mathbf{w}_k}_{\mathbf{w}}, \tag{1}$$

where $\mathbf{\Phi}_k=\mathrm{diag}\left\{\sqrt{P_i}:i\in G_k\right\}$ represents the power allocation to each subcarrier of user k,

$$\mathrm{H}_k=\mathrm{diag}\left\{H_k^i:i\in G_k\right\}$$

with H_k^i denoting the channel attenuation at the i-th subcarrier, and G_k stands for the subcarriers allocated to user k. Note that $\bigcap_k G_k=\phi$ and $\bigcup_k G_k=\{0,1,\cdots,N-1\}$,

$$\mathbf{E}_k=\mathrm{diag}\left\{e^{j\psi_k},e^{j(2\pi\varepsilon_k/N+\psi_k)},\cdots,e^{j(2\pi\varepsilon_k(N-1)/N+\psi_k)}\right\}, \tag{2}$$

with ψ_k and ε_k representing the initial phase and normalized frequency offset of user k, respectively. $\mathbf{F}_k$ denotes the *Inverse Discrete Fourier Transform* (IDFT) matrix for the k-th user (where only the subcarriers allocated to user k are modulated). N is the *Discrete Fourier Transform* (DFT) length. $\mathbf{x}_k$ is the transmit vector of user k, and without loss of generality, we assume that $\mathbf{x}_k[m]$ are independent and identically distributed (i.i.d.) complex RVs with zero mean and unit variance. $\mathbf{m}_k$ represents the time-domain transmit training of user k, $\mathbf{w}$ in (1) is a vector of additive white Gaussian noise (AWGN) with $\mathbf{w}[i]\sim CN(0,\sigma_w^2)$, and $\mathbf{w}_k$ represents the additive noise added to user k.

UL synchronization for each user can be performed based on the received training sequence $\mathbf{y}$. Without loss of generality, we assume that the frequency offsets of different users are i.i.d. RVs. For an OFDMA UL with M users accessing a base station, the Fisher Information Matrix (FIM) $\mathbf{\Gamma}_M$ is given by

$$\mathbf{\Gamma}_M=\begin{bmatrix}\mathbf{\Gamma}_{M-1} & \mathbf{b}_{M-1}\\ \mathbf{b}_{M-1}^H & [\mathbf{\Gamma}_M]_{MM}\end{bmatrix}, \tag{3}$$

where $\mathbf{\Gamma}_{M-1}$ represents the northwestern $(M-1)\times(M-1)$ sub-matrix of $\mathbf{\Gamma}_M$, and $\mathbf{b}_{M-1}$ is a $(M-1)\times 1$ vector. The CRLB of user M is given by

$$\mathrm{CRLB}\{M\}=\left[\mathbf{\Gamma}_M^{-1}\right]_{MM}=\left([\mathbf{\Gamma}_M]_{MM}-\mathbf{b}_{M-1}\mathbf{\Gamma}_{M-1}^{-1}\mathbf{b}_{M-1}^H\right)^{-1}. \tag{4}$$

From Appendix I, for an unbiased estimator $\hat{\varepsilon}_k$, the CRLB is derived as

$$\begin{aligned}\mathrm{Var}\{\hat{\varepsilon}_k\}&\ge\left[\mathbf{\Gamma}^{-1}\right]_{kk}\ge[\mathbf{\Gamma}]_{kk}^{-1}\\ &=\frac{1}{\sum_{i=1}^{N_k}\frac{\left(\frac{\partial\lambda_{k,i}}{\partial\varepsilon_k}\right)^2}{(\lambda_{k,i}+z_{k,i})^2}+\sum_{l\ne k}\sum_{j=1}^{N_l}\frac{\left(\frac{\partial z_{l,i}}{\partial\varepsilon_k}\right)^2}{(\lambda_{l,j}+z_{l,j})^2}}\\ &=\frac{1}{\alpha_k\cdot\mathrm{SNR}_k^2}+\frac{1}{\beta_k\cdot\mathrm{SNR}_k}+\frac{1}{\varpi_k\cdot\mathrm{SIR}_k^2},\end{aligned} \tag{5}$$

where $\lambda_{k,i}$ and $z_{k,i}$ are defined as in Appendix I, α_k, β_k and ϖ_k are constants specified by the structure of the training sequence used by user k and SNR_k and SIR_k stand for the Signal-to-Noise-Ratio (SNR) and Signal-to-Interference-Ratio (SIR) of user k, respectively. (5) suggests that MAI will degrade the estimation performance of the UL synchronization, and the MAI elimination/reduction is critical in increasing the frequency offset estimation accuracy.

3. Interference Analysis in OFDMA UL

Note that in an OFDMA UL, the subcarrier groups allocated to different users are orthogonal, and this allocation eases the decomposing of the transmitted signal of each user. We consider two types of subcarrier allocation schemes: Block-based and Interleaved, as illustrated in **Figures 1(a)** and **(b)**.

In an OFDMA UL, the receiver (base station) usually

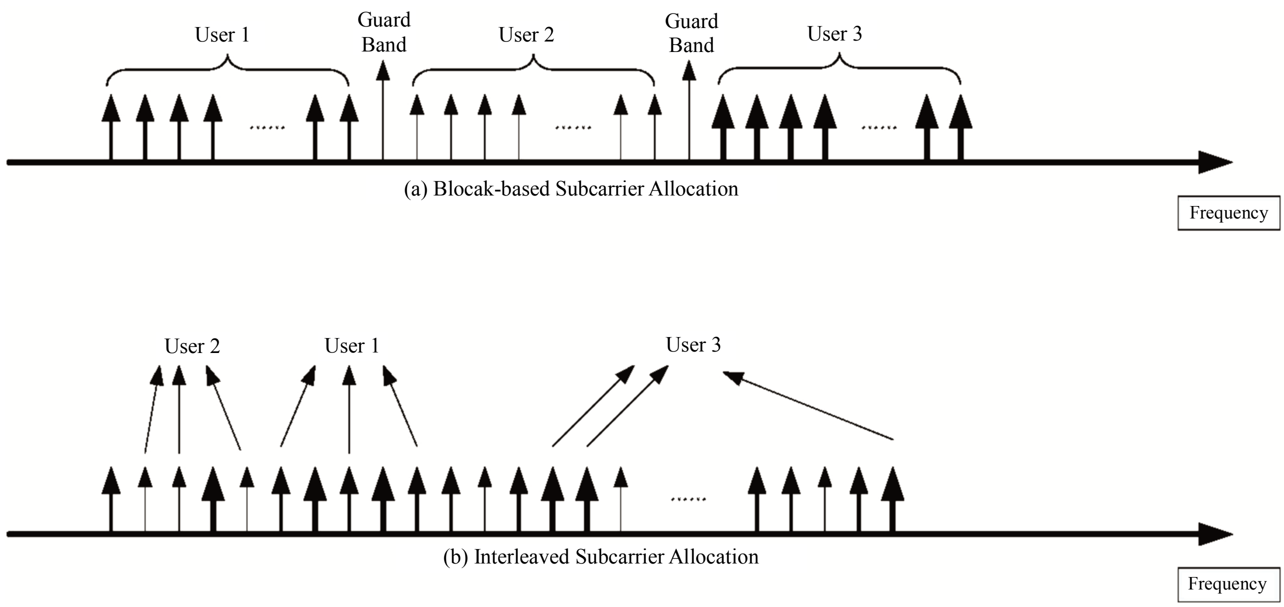

Figure 1. Subcarrier allocation in OFDMA.

pre-demodulates the signal of each user, and only signals projected into the signal space of interest will be demodulated. Since the pre-demodulation improves the effective Signal-to-Interference-plus-Noise Ratio (SINR), an optimum CRLB for the k-th user is achieved as

$$\mathrm{Var}\left\{\hat{\varepsilon}_k\right\} \ge \frac{1}{\sum_{i=1}^{N_k} \frac{\left(\frac{\partial \lambda_{k,i}}{\partial \varepsilon_k}\right)^2}{\left(\lambda_{k,i} + z_{k,i}\right)^2}}. \quad (6)$$

The pre-demodulation of each user's signal can be performed as

$$\begin{aligned} \mathbf{r}_k &= \mathbf{P}_k \mathbf{y} = \mathbf{P}_k \sum_l \underbrace{\mathbf{E}_l \mathbf{F}_l \mathbf{H}_l \mathbf{\Phi}_l \mathbf{x}_l}_{\mathbf{v}_l} + \mathbf{P}_k \mathbf{w} \\ &= \mathbf{P}_k \mathbf{v}_k + \mathbf{P}_k \sum_{l \ne k} \mathbf{v}_l + \mathbf{P}_k \mathbf{w}, \end{aligned} \quad (7)$$

where $\mathbf{P}_k = \mathbf{F}_k \left(\mathbf{F}_k^H \mathbf{F}_k\right)^{-1} \mathbf{F}_k^H = \mathbf{F}_k \mathbf{F}_k^H$ and $\mathbf{P}_k \sum_{l \ne k} \mathbf{v}_l$ represents the interference from other users, *i.e.*, MAI.

In order to analyze the MAI contributed by users other than user k (the user of interest), we first assume that M users access the base station, and then we make some presuppositions:

1) $\bigcap_{m=1}^{M} G_m G_{n \ne m} = \phi$ and $\bigcup_{m=1}^{M} G_m \subseteq \{0, 1, \cdots, N-1\}$, where G_m represents the set of subcarriers allocated to user m.

2) $N_m \ll N$, where N_m represents the cardinality of G_m.

The average SINR of user k can be represented as

$$\mathrm{SINR}_k = \frac{E\left\{\left\|\mathbf{F}_k^H \mathbf{v}_k\right\|^2\right\}}{E\left\{\sum_{n \in G_k} \left|\mathrm{I}_k(n) + \mathrm{I}_{l \ne k}(n)\right|^2\right\} + E\left\{\left\|\mathbf{F}_k^H \mathbf{w}_k\right\|^2\right\}}, \quad (8)$$

where

$$\mathrm{I}_k(n) = \sum_{i \in G_k, i \ne n} \sqrt{P_i} \mathbf{x}_k[i] H_k^i \frac{\sin\left[\pi\left(i - n + \varepsilon_k\right)\right]}{N \sin\left[\frac{\pi\left(i - n + \varepsilon_k\right)}{N}\right]}$$

is the interference of the subcarrier n contributed by the other subcarriers of user k (ICI), and

$$\mathrm{I}_{k \ne l}(n) = \sum_{l \ne k} \sum_{i \in G_l} \sqrt{P_i} \mathbf{x}_l[i] H_l^i \frac{\sin\left[\pi\left(i - n + \varepsilon_l\right)\right]}{N \sin\left[\frac{\pi(i - n + \varepsilon_l)}{N}\right]}$$

represents the interference of the subcarrier n contributed by the users other than k.

When $\varepsilon_l \ll 1$, the average SINR of user k can be simplified as

$$\mathrm{SINR}_k = \frac{E\left\{\sum_{n \in G_k} \left|\sqrt{P_n} \mathbf{x}_k[n] H_k^n\right|^2\right\} \cdot E\left\{\left|\frac{\sin\left(\pi \varepsilon_k\right)}{N \sin\left(\frac{\pi \varepsilon_k}{N}\right)}\right|^2\right\}}{\sum_{n \in G_k} E\left\{\left|\sum_{l=1}^{M} \rho_l(n) \cdot \varepsilon_l\right|^2\right\} + E\left\{\left\|\mathbf{F}_k^H \mathbf{w}\right\|^2\right\}}, \quad (9)$$

where $\rho_l(n)$ is a random variable (not necessarily Gaussian) with $E\left\{\rho_l(n)\right\} = 0$.

For a system with all the users running at the tracking phase (the frequency offset acquisition has been performed, and the remaining frequency offset is small), it is reasonable to assume that ε_m for each m is an i.i.d. RV uniformly distributed in $(-\varepsilon,\varepsilon)$, where $0<\varepsilon\ll 1$ [23]. The averaged SINR of user k can be further approximated as

$$\mathrm{SINR}_k=\frac{\overline{\gamma}_k}{\frac{\pi^2\varepsilon^2\overline{\gamma}_k}{9}+1}\cdot\left(1-\frac{\pi^2\varepsilon^2}{9}+\frac{\pi^4\varepsilon^4}{180}\right), \quad (10)$$

where $\overline{\gamma}_k=\frac{E\{K_n\}}{\sigma_n^2}$ is used to represent the average received SNR of user k.

The SINR reduction due to MAI is shown in **Figure 2**. A considerable MAI of one user will be contributed by the interfering users because of their non-zero frequency offsets. For example, when $\varepsilon=0.1$, the averaged SINR loss at the objective user may be up to 36.7%.

4. Heavy MAI Elimination

In Section 3, we analyzed the MAI introduced by interfering users. Since a new accessing user's instantaneous frequency offset may contribute a heavy MAI to its neighbors, the base station should first perform UL synchronization for that user before its totally access. The accessing user will then correct its synchronization errors based on the feedback information from the base station.

In order to illustrate the distribution ofInter-Carrier-Interference (ICI) with respect to the distance between the object subcarrier (subcarrier n) and the interfering subcarrier (subcarrier i), we define

$$F_{\mathrm{ICI}}(n,i)=\frac{1}{N^2\sin^2\left[\frac{\pi(i-n)}{N}\right]}$$

as the "ICI factor" between subcarriers i and n, and we know that the ICI between subcarrier i and n is proportional to $F_{\mathrm{ICI}}(n,i)$. The upper figure in **Figure 3** shows the distribution of $F_{\mathrm{ICI}}(n,i)$ as a function of $(i-n)/N$, from which we know that the ICI factor diminishes as $[i-n]_N$ increases. The lower figure in **Figure 3** illustrates the concentration of ICI energy, and it shows that most of the ICI energy added to one subcarrier is concentrated within a small range around the subcarrier. For a given ICI energy, the ratio of the concentration range to N becomes smaller as N increases.

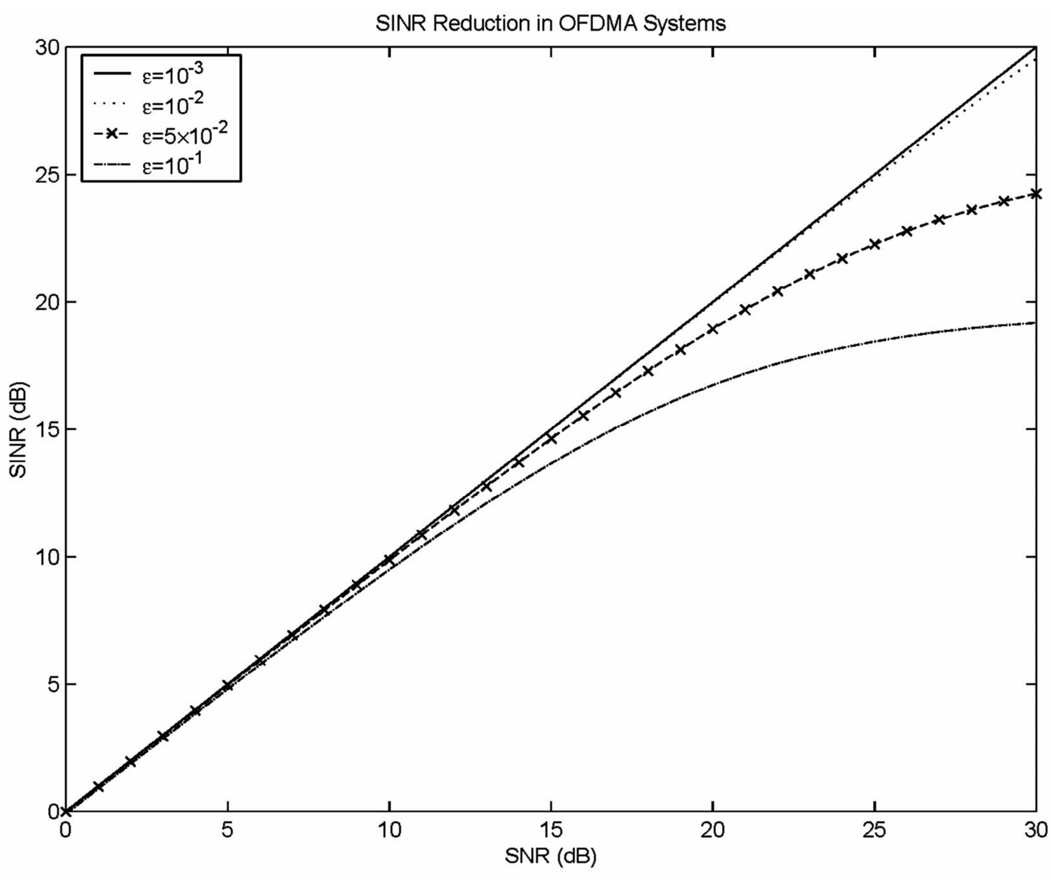

Figure 2. SINR reduction introduced by non-zero frequency offset in OFDMA systems.

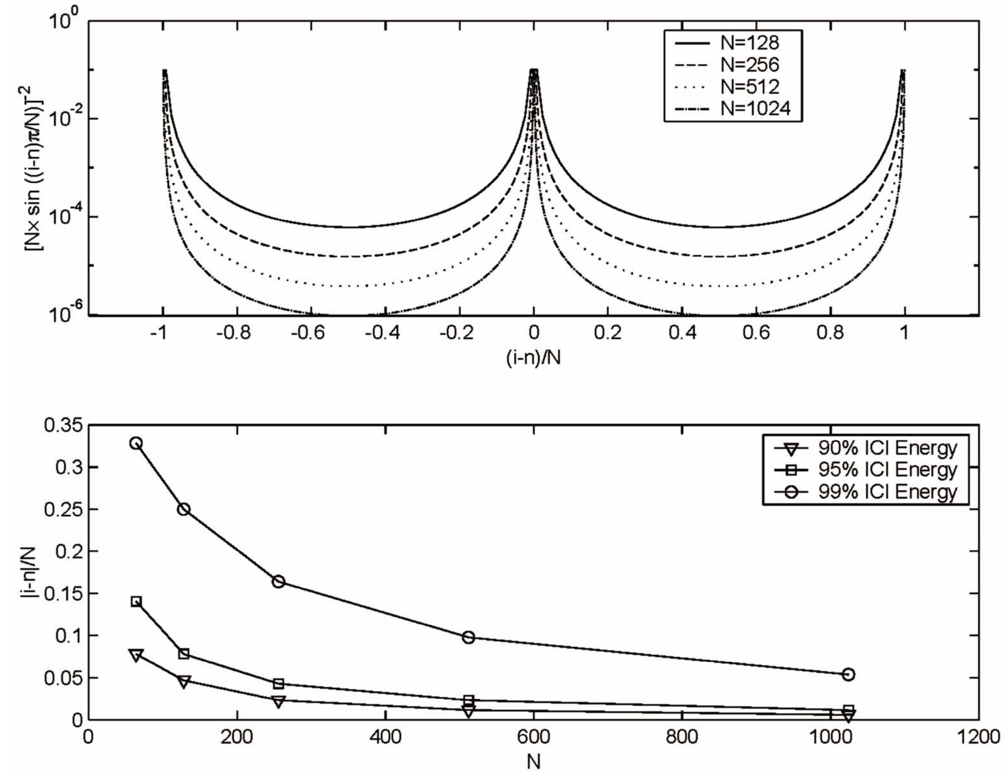

Figure 3. Inter-carrier-interference factors in OFDMA systems.

4.1. Heavy MAI Elimination in Block-Based Subcarrier Allocation Scheme

In the Block-based allocation scheme, user k is allocated a set of contiguous subcarriers, *i.e.*, G_k, and its marginal subcarriers suffer most of the MAI. If we optimize $\mathbf{P}_k$ of user k to demodulate only its less-interfered subcarriers (the marginal subcarriers of heavy MAI are left un-demodulated, or in other method, we leave wide enough guard band between neighboring users), most of its MAI can be eliminated. Based on this princeple, we propose a method named the Reduced-Rank-Projector (RRP) method, *i.e.*, $\tilde{\mathbf{P}}_k$. In order to maximize the SINR of user *k*, we have

$$\tilde{\mathbf{P}}_k = \arg\max_{\tilde{\mathbf{P}}_k} \frac{\left\|\tilde{\mathbf{P}}_k \mathbf{v}_k\right\|^2}{\left\|\tilde{\mathbf{P}}_k \sum_{l \neq k} \mathbf{v}_l + \tilde{\mathbf{P}}_k \mathbf{w}\right\|^2}, \qquad (11)$$

where $\tilde{\mathbf{P}}_k = \tilde{\mathbf{F}}_k \tilde{\mathbf{F}}_k^H$. $\tilde{\mathbf{F}}_k$ here is defined as follows: without loss of generality, we first assume that the central frequency of G_k is higher than that of G_{k-1} and lower than that of G_{k+1}. We also assume that user k is interfered by user $(k-1)$ and user $(k+1)$ with its left-most L_k and right-most U_k subcarriers, respectively. $\mathbf{F}_k$ can be derived by deleting the left-most L_k columns and the right-most U_k columns from $\mathbf{F}_k$. Here, we also assume that the guard band between user $(k-1)$ and user k is $B_{k-1,k}$ subcarriers, and that between user k and user $(k+1)$ is $B_{k,k+1}$ subcarriers. Therefore,

$$L_k = \left(\left\lfloor \hat{\varepsilon}_{k-1} - B_{k-1,k} + 1 \right\rfloor\right)^{\dagger}$$

and

$$U_k = \left(\left\lfloor -\hat{\varepsilon}_{k+1} - B_{k,k+1} + 1 \right\rfloor\right)^{\dagger},$$

where $\lfloor x \rfloor$ means the maximum integer part of x, and $(x)^{\dagger} = \max\{x, 0\}$.

Figure 4 illustrates the frequency offset estimation with the help of the RRP method. Three frequency-domain neighbors, *i.e.*, user $(k-1)$, user k and user $(k+1)$, have instantaneously large frequency offsets. Since user k suffers the highest MAI, we perform a synchronization for user $(k-1)$ and user $(k+1)$ first. At that time, the signals of user $(k-1)$ and user $(k+1)$ are demodulated by using projectors $\mathbf{P}_{k-1}$ and $\mathbf{P}_{k+1}$, respectively. Using the estimation results $\hat{\varepsilon}_{k-1}$ and $\hat{\varepsilon}_{k+1}$, L_k and U_k can be derived to optimize $\tilde{\mathbf{P}}_k$. After that, $\hat{\varepsilon}_k$ is used to calculate U_{k-1} and L_{k+1}, and then $\tilde{\mathbf{P}}_{k-1}$ and $\tilde{\mathbf{P}}_{k+1}$ are optimized. The optimized $\tilde{\mathbf{P}}_{k-1}$ and $\tilde{\mathbf{P}}_{k+1}$ then can then be used to further improve the performance of $\hat{\varepsilon}_{k-1}$ and $\hat{\varepsilon}_{k+1}$ [1].

[1]This kind of operation can be applied in the scenario of multiple users transmitting and receiving jointly, e.g., multiuser Multiple-Input Multiple-Output (MIMO).

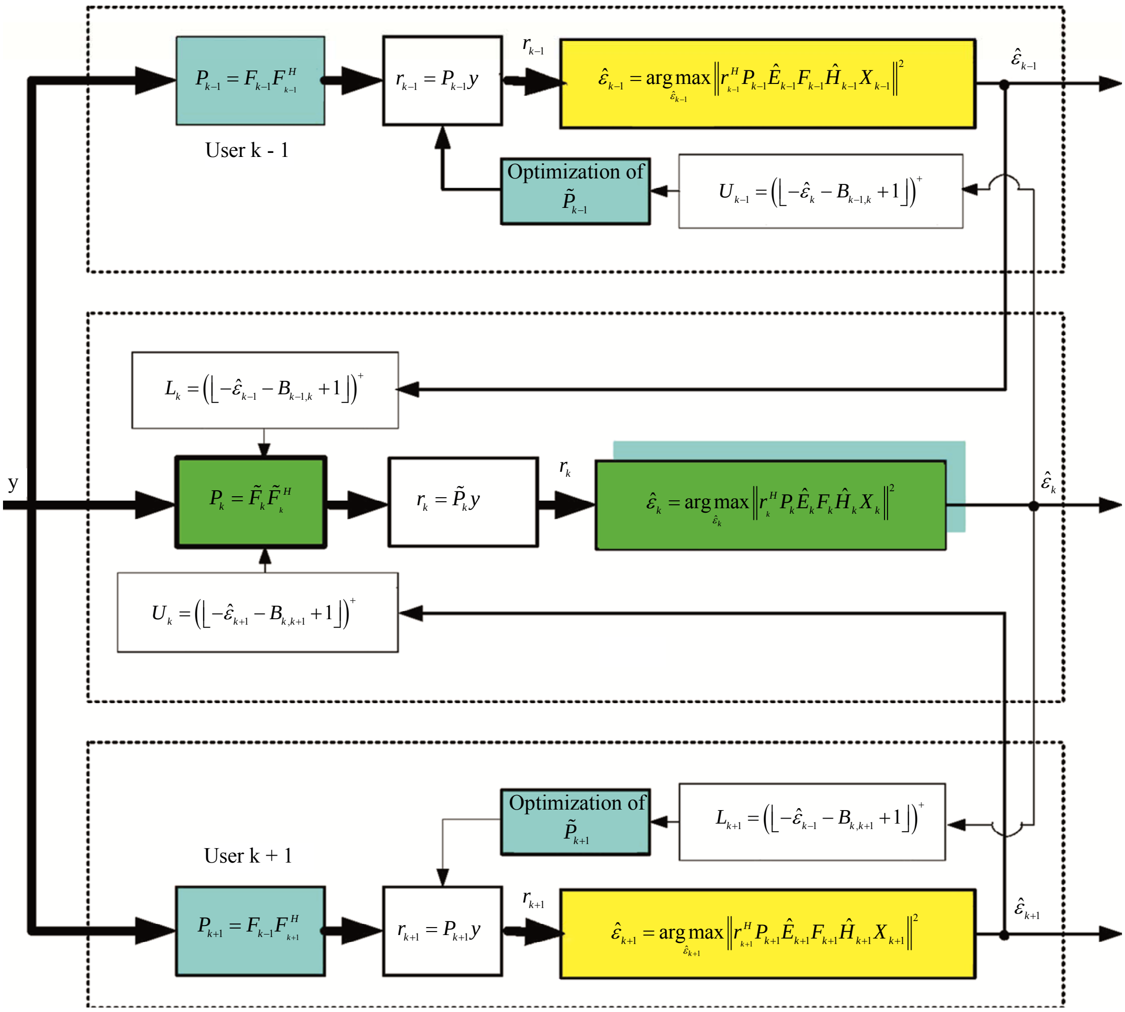

Figure 4. Interference elimination and frequency offset estimation in block-based subcarrier allocation scheme by using RRP method.

4.2. Heavy MAI Elimination in Interleaved Subcarrier Allocation Scheme

In Interleaved subcarrier allocation scheme (as illustrated in **Figure 1(b)**), there is no enough guard band to separate the different users, and the frequency offset immune capability will be degraded as a result. In this case, without loss of generality, we assume user k to be the user of interest and that user l acts as an interfering user to it. We also assume that the frequency offset estimation result of user l is $\hat{\varepsilon}_l = \mu_l + \xi_l \left(0 \le \xi_l \le 0.5\right)$, where μ_l and ξ_l represent the integer part and the fractional part of $\hat{\varepsilon}_l$, respectively. Here, we assume that μ_l has already been corrected and that MAI is contributed only by ξ_l. In order to eliminate this kind of MAI, we propose a method named the Shift-Sampling-Projector (SSP) method. We first perform the frequency offset estimation for user l to obtain ξ_l. The Shift-Sampling-Projector for user k can be represented as

$$\mathbf{P}_{k+\xi_l} = \mathbf{F}_k \mathbf{F}_{k+\xi_l}^H = \mathbf{P}_k \Theta_l ,$$

where

$$\mathbf{F}_{k+\xi_l}^H = \mathbf{F}_k^H \Theta_l$$

and

$$\Theta_l = \text{diag}\left\{1, e^{-j2\pi\xi_l/N}, \cdots, e^{-j2\pi\xi_l(N-1)/N}\right\},$$

as illustrated in **Figure 5**. Note that $\hat{\mathbf{E}}_{k+\xi_l}$ in **Figure 5** is given by (2) with ε_k being replaced by $\hat{\varepsilon}_{k+\xi_l}$. If $\hat{\varepsilon}_l$ is accurate enough, the interference from user l can be eliminated. The estimation result, *i.e.*, $\hat{\varepsilon}_k$, then will be used to optimize $\mathbf{P}_l$ in order to further increase the accuracy of $\hat{\varepsilon}_l$. By using the SSP method, the variance error of $\hat{\varepsilon}_k$ will satisfy

$$\text{Var}\left\{\hat{\varepsilon}_k\right\} \propto \left(\frac{\left\|\mathbf{P}_{k+\xi_l}\mathbf{v}_k\right\|^2}{\left\|\mathbf{P}_{k+\xi_l}\mathbf{w}_k\right\|^2}\right)^{-1} .$$

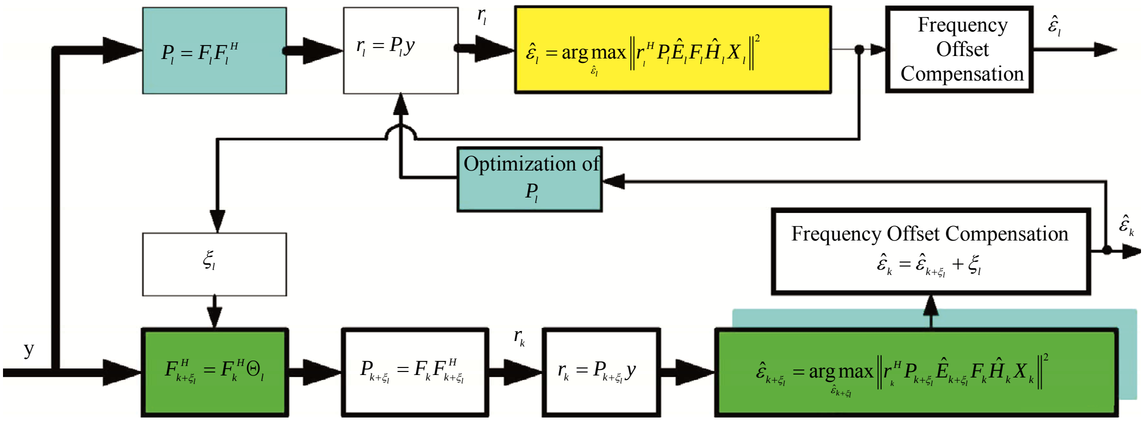

Figure 5. Interference elimination and frequency offset estimation in interleaved subcarrier allocation scheme by using SSP method.

5. Frequency Offset Estimation with MAI Reduction/Randomization

As mentioned earlier, instantaneously heavy MAI introduced by frequency-domain neighboring users should be eliminated before a user totally access the base station. In this section, we first discuss interference elimination by using the Successive Interference Cancellation (SIC) method, and then propose a frequency offset estimation algorithm based on an interference reduction scheme.

5.1. SIC with MAI Elimination

Let us assume that M users access a base station, and that $\mathbf{H}_1\mathbf{x}_1, \mathbf{H}_2\mathbf{x}_2, \cdots, \mathbf{H}_M\mathbf{x}_M$ are perfectly known at the receiver (base station). Based on the joint PDF of $\{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_M\}$ and $\mathbf{y}$, *i.e.*, $f(\hat{\mathbf{v}}_M, \hat{\mathbf{v}}_{M-1}, \cdots, \hat{\mathbf{v}}_1; \mathbf{y})$, (as well as $\{\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_M\}$) can be jointly estimated as

$$\begin{aligned}
&\{\hat{\mathbf{v}}_1, \hat{\mathbf{v}}_2, \cdots, \hat{\mathbf{v}}_M\} \\
&= \arg \max_{\{\hat{\mathbf{v}}_1, \hat{\mathbf{v}}_2, \cdots, \hat{\mathbf{v}}_M\}} \left\{ \ln f(\hat{\mathbf{v}}_M, \hat{\mathbf{v}}_{M-1}, \cdots, \hat{\mathbf{v}}_1; \mathbf{y}) \right\} \\
&= \sum_{m=1}^{M} \arg \max_{\hat{\mathbf{v}}_m} \left\{ \ln f\left(\hat{\mathbf{v}}_m \middle| \hat{\mathbf{v}}_{m-1}, \cdots, \hat{\mathbf{v}}_1; \mathbf{y} - \sum_{l=1}^{m-1} \hat{\mathbf{v}}_l \right) \right\} \\
&+ \ln f(\mathbf{y}) \qquad (12) \\
&= \sum_{m=1}^{M} \arg \max_{\hat{\mathbf{v}}_m} \left\{ \ln f\left(\hat{\mathbf{v}}_m \middle| \mathbf{y} - \sum_{l=1}^{m-1} \hat{\mathbf{v}}_l \right) \right\} + \ln f(\mathbf{y}) \\
&= \sum_{m=1}^{M} \arg \max_{\hat{\mathbf{v}}_m} \left\| \hat{\mathbf{v}}_m \left(\mathbf{y} - \sum_{l=1}^{m-1} \hat{\mathbf{v}}_l \right) \right\|^2 .
\end{aligned}$$

(16) provides a feasible way to estimate the signal/frequency offset of each user: the signals of different users can be estimated one by one based on $\mathbf{y}$. After performing a synchronization for the first l users, user $(l+1)$ will estimate $\mathbf{v}_{l+1}$ based on $\left(\mathbf{y} - \sum_{m=1}^{l} \hat{\mathbf{v}}_m \right)$. In theory, if $\hat{\mathbf{v}}_1, \hat{\mathbf{v}}_2, \cdots, \hat{\mathbf{v}}_l$ are accurate enough, $\mathbf{v}_{l+1}, \cdots, \mathbf{v}_M$ can be estimated successfully[2]. However, if one previous user has a large estimation error, it will propagate to $\hat{\mathbf{v}}_{l+1}, \hat{\mathbf{v}}_{l+2}, \cdots, \hat{\mathbf{v}}_M$; *i.e.*, the estimation accuracy of $\hat{\mathbf{v}}_m$ depends on the estimation accuracy of $\hat{\mathbf{v}}_{l \neq m}$ v. For a large M, estimation performance by using the SIC method will degrade because of the error propagation. **Figure 6** shows the SIC-based frequency offset estimation performance. In this figure, a total of 16 users access a base station simultaneously. In order to reduce the error propagation in SIC, more than one iteration is required when performing MAI elimination, and the algorithm converges after 5 iterations.

5.2. RRTSG for MAI Randomization

The SIC-based method aforementioned requires both CSI and initial phase information, and in addition, more interfering users imply more iterations being required. Therefore, SIC is inefficient in a multiuser environment. Some differential algorithms that are independent of the initial phase and the wireless channel provide a new way for a solution [9,15]. In these differential algorithms, the training sequence contains repetition information, and the frequency offset can be estimated at the receiver by estimating the phase rotations between these repetitions. However, these conventional algorithms were originally proposed for DL synchronization, and they are not immune to MAI.

In this section, a new training sequence group named the Round-Robin Training Sequence Group (RRTSG) is proposed to perform OFDMA UL synchronization. A total of N training sequences are included in RRTSG, with each training sequence being composed of two

[2]SIC algorithm requires a knowledge of channel state information (CSI) at the base station, and the CSI information can be obtained through channel estimation (please refer to, e.g., [26]), which is beyond the scope of this paper.

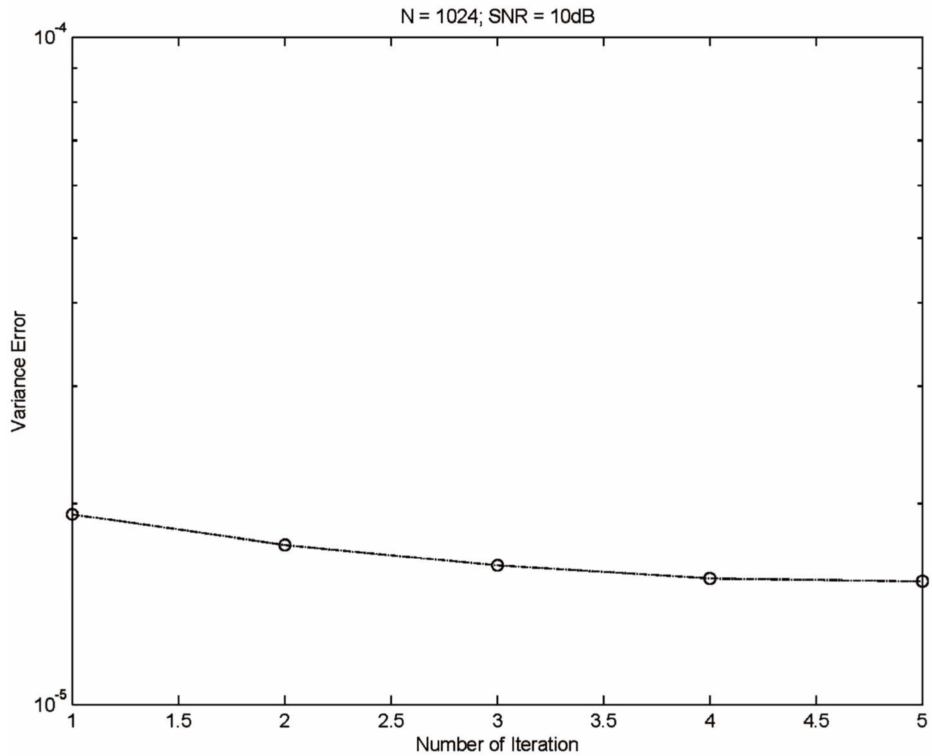

Figure 6. Frequency offset estimation with MAI elimination by using SIC.

training symbols, where the second symbol is generated by copying data from the first one (but at a different order). When generating the second training symbol of the d-th ($0\le d \le N-1$) training sequence, we simply copy data from the already generated first symbol with its last d samples shifted to the front. After padding each symbol with its corresponding CP, the d-th training sequence is generated as:

$$s_d = \Bigg[\underbrace{\left(\mathbf{F}_k\mathbf{x}_k\left[N-L:N-1\right]\right)^T}_{CP1}, \underbrace{\left(\mathbf{F}_k\mathbf{x}_k\right)^T}_{symbol1}, \underbrace{\left(\mathbf{F}_k\mathbf{D}_k^d\mathbf{x}_k\left[N-L:N-1\right]\right)^T}_{CP2}, \underbrace{\left(\mathbf{F}_k\mathbf{D}_k^d\mathbf{x}_k\right)^T}_{symbol2}\Bigg]^T$$
$$= \Bigg[\underbrace{x(N-L)\cdots x(N-1)}_{CP1}\underbrace{x(0)\cdots x(N-1)}_{symbol1} \underbrace{x(N-L-d-1)\cdots x(N-d-1)}_{CP2} \underbrace{x(N-d)\cdots x(N-1)x(0)\cdots x(N-d-1)}_{symbol2}\Bigg]^T, \quad (13)$$

where L represents the CP length, and

$$\mathbf{D}_k^d = \text{diag}\left\{e^{-j2\pi dk_1/N}, e^{-j2\pi dk_2/N}, \cdots, e^{-j2\pi dk_{N_k}/N},\right\}$$

is a pre-coding matrix for user k in generating the d-th training sequence.

RRTSG can randomize MAI when performing UL synchronization, and an unbiased frequency offset estimator is always achieved, provided that each user is allocated a unique training sequence. A Best Linear Unbiased Estimator (BLUE) of ε_k based on the d-th training sequence is provided by:

$$\hat{\varepsilon}_{k|d} = \frac{\mathbf{1}^T\mathbf{\Lambda}_d^{-1}\boldsymbol{\varepsilon}_d}{\mathbf{1}^T\mathbf{\Lambda}_d^{-1}\mathbf{1}}, \quad (14)$$

where

$$\mathbf{\Lambda}_d = \text{diag}\left\{\text{Var}\left\{\hat{\varepsilon}_{k,d}^1\right\}, \text{Var}\left\{\hat{\varepsilon}_{k,d}^2\right\}\right\},$$

$$\boldsymbol{\varepsilon}_d = \left[\hat{\varepsilon}_{k,d}^1, \hat{\varepsilon}_{k,d}^2\right]^T = \left[\frac{N\cdot\arg\left\{\sum_{z=0}^{N-d-1}\zeta_{d,z}\right\}}{2\pi(N+L+d)}, \frac{N\cdot\arg\left\{\sum_{z=0}^{d-1}\wp_{d,z}\right\}}{2\pi(L+d)}\right]^T,$$

$$\mathbf{1} = [11]^T, \quad \zeta_{d,z} = r_k^*(L+z)\cdot r_k(N+2L+d+z),$$

$$\wp_{d,z} = r_k^*(N+L-d+l)\cdot r_k(N+2L+l),$$

and $r_k(n)$ represents the n element of $\mathbf{r}_k$. We can easily prove that the BLUE estimator provided by (18) is conditionally unbiased, and that its CRLB is found to be

$$\mathrm{Var}\left\{\hat{\varepsilon}_{k|d}^{1}\right\} \geq \frac{N}{4\pi^2\left(N^2+L^2-d^2+2NL+Nd\right)\cdot \mathrm{SINR}_k}. \quad (15)$$

Among all the training sequences in the proposed RRTSG, the minimum CRLB is achieved when $d = N/2$.

In order to make the estimator work properly,

$$\left|\frac{2\pi\varepsilon_k\left(N+L+d\right)}{N}\right| < \pi$$

should be satisfied for each d, and this requires the normalized frequency offset estimation range of the d-th estimator to be limited within

$$\left(-\frac{N}{2\left(N+L+d\right)}, \frac{N}{2\left(N+L+d\right)}\right).$$

6. Numerical Results

In our simulation, an OFDMA UL transmission with a bandwidth of 10 MHz and DFT length of 1024 is considered. A length-64 cyclic-prefix is padded to the front of each symbol or training sequence. Wireless channel parameters are defined in **Tables 1** and **2**.

Figure 7 illustrates the performance improvement achieved in RRP by eliminating the instantaneously heavy MAI. In this simulation, we assume that 10 users access a base station simultaneously, and users #1, #2 and #3 are assumed to have instantaneously large frequency offsets. The frequency offsets for the other 6 users are assumed to have already been estimated and corrected, and their residual frequency offsets are assumed to be uniformly distributed within (–0.01, 0.01). This simulation parameters for users #1, #2 and #3 are presented in **Table 1**. User #2 is assumed to be the user of interest. The frequency offset estimation can be improved by using the RRP method. At low SNR, a variance error of less than 10^{-3} (or 10^{-4}) can be obtained if we reduce the demodulated subcarrier range to [–22, 23] (or [–20, 21]).

Although the RRP method can achieve a considerable performance improvement with respect to MAI elimination, this method is not applicable in the Interleaved

Table 1. Subcarrier allocation of heavy-interference users in block-based subcarrier allocation scheme (bandwidth = 10 MHz, DFT length = 1024, CP = 64).

	User 1	User 2	User 3
Subcarrier Allocated	[–63,–25]	[–24,25]	[26,64]
Normalized CFO	3.2	0.1	–3.7
Algorithm Used	Moose [15]	Moose [15]	Moose [15]
Multipath Delays (μs)	0; 0.2; 0.4	0; 0.4; 0.8	0; 0.6; 1.0
Averaged Power (dB)	0; –3; –6	0; –6; –9	0; –8; –12
Guard Band		No	
Initial Phase		$[0, 2\pi)$	

Table 2. Performance comparison between RRTSG-based scheme and Moose's algorithm (bandwidth = 10 MHz, DFT length = 1024, CP = 64; No guard band between different users).

		User 1	User 2	User 3
Scenario I	Subcarrier Allocated	[–63,–41] [1,20]	[–40,–10] [21,60]	[–9,0] [61,90]
	Normalized CFO	3.2	0.1	–3.7
	Algorithm Used	Moose [15]	Moose [15]	Moose [15]
	Multipath Delays (μs)	0; 0.2; 0.4	0; 0.4; 0.8	0; 0.6; 1.0
	Averaged Power (dB)	0; –3; –6	0; –6; –9	0; –8; –12
	Initial Phase		$[0, 2\pi)$	
Scenario II	Subcarrier Allocated	[–63,–41] [1,20]	[–40,–10] [21,60]	[–9,0] [61,90]
	Normalized CFO	3.2	0.1	–3.7
	Algorithm Used	RRTSG	RRTSG	RRTSG
	Multipath Delays (μs)	0; 0.2; 0.4	0; 0.4; 0.8	0; 0.6; 1.0
	Averaged Power (dB)	0; –3; –6	0; –6; –9	0; –8; –12
	Initial Phase		$[0, 2\pi)$	

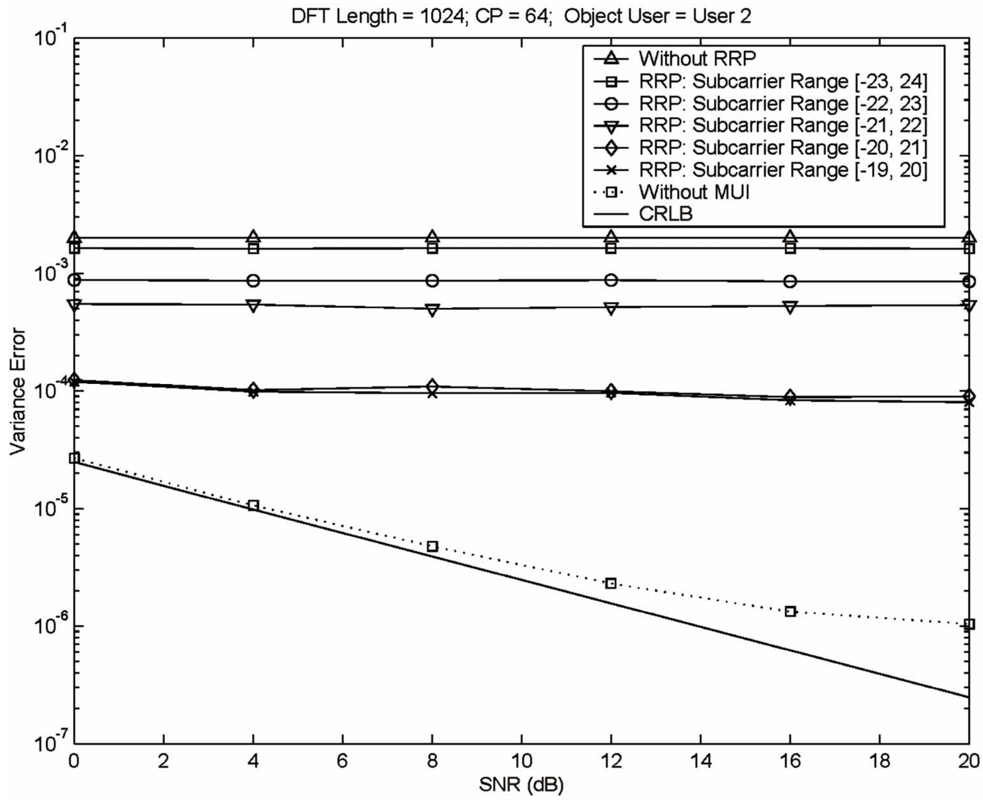

Figure 7. Frequency offset estimation with the help of heavy MAI elimination by using RRP.

subcarrier allocation scheme, which relies on the SSP method. **Table 3** illustrates the parameters of this simulation. The simulation results are illustrated in **Figure 8**. Without using SSP, the variance error will always be higher than 3×10^{-4}, no matter how high the SNR is. When the SSP method is applied, a considerable performance improvement can be obtained, even with imperfect frequency offset estimation in the interfering users. For example, at an SNR of 16dB, when the SSP method is applied with a 30% compensation error being assumed in the interfering user, a variance error smaller than 3×10^{-5} can be obtained. The variance error can be further reduced to about 1×10^{-5} if the compensation error is reduced to 20%.

When the number of accessing users is large, MAI elimination is impractical to perform. As a feasible MAI reduction scheme, MAI randomization is usually utilized. The proposed RRTSG has a high ability in performing MAI randomization in OFDMA UL transmission. **Figure 9** illustrates the performance comparison between Moose's algorithm and the proposed RRTSG-based algorithm. The simulated environment is defined by **Table 2**. The simulation results show that the proposed RRTSG-based algorithm outperforms Moose's algorithm with respect to interference mitigation capability. Without using any heavy MAI elimination method (e.g., RRP or SSP), a biased frequency offset estimator is obtained by using Moose's algorithm, so that its variance error is much larger than that of the proposed RRTSG-based algorithm. For example, a variance error of 8×10^{-4} can be achieved at a high SNR by using the proposed RRTSG-based algorithm, as compared to the variance error of 1×10^{-3} obtained in Moose's algorithm. The proposed MAI elimination methods proposed can improve the performance of both algorithms considerably. When most of the heavy MAI is eliminated by using RRP, e.g., the subcarriers ranging from $[-35,-15]$ & [26,55] are demodulated, a variance error of about 9×10^{-6} is achieved by using the proposed RRTSG-based algorithm at an SNR

Table 3. Subcarrier allocation of heavy-interference users in interleaved subcarrier allocation scheme (bandwidth = 10 MHz, DFT length = 1024, CP = 64; no guard band between different users).

	User 1	User 2
Subcarrier Allocated	Even Numbered	Odd Numbered
Normalized CFO	0.1	–0.2
Algorithm Used	Moose [15]	Moose [15]
Multipath Delays (μs)	0; 0.2; 0.4	0; 0.4; 0.8
Averaged Power (dB)	0; –3; –6	0; –6; –9
Initial Phase	$[0, 2\pi)$	

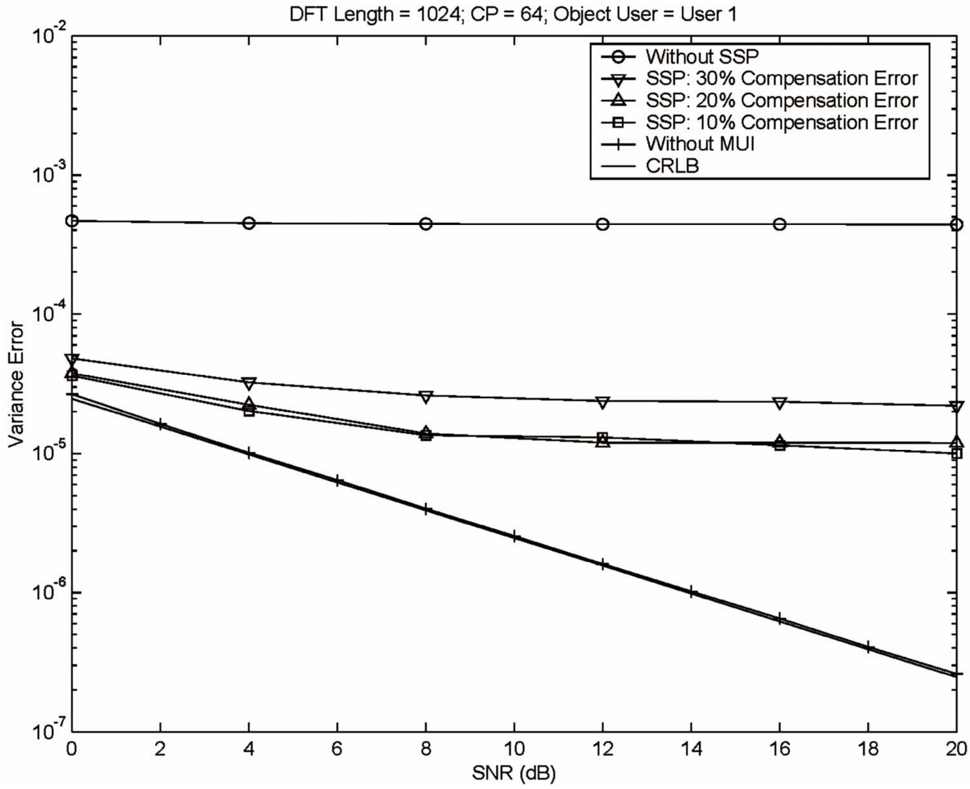

Figure 8. Frequency offset estimation with the help of heavy MAI elimination by using SSP.

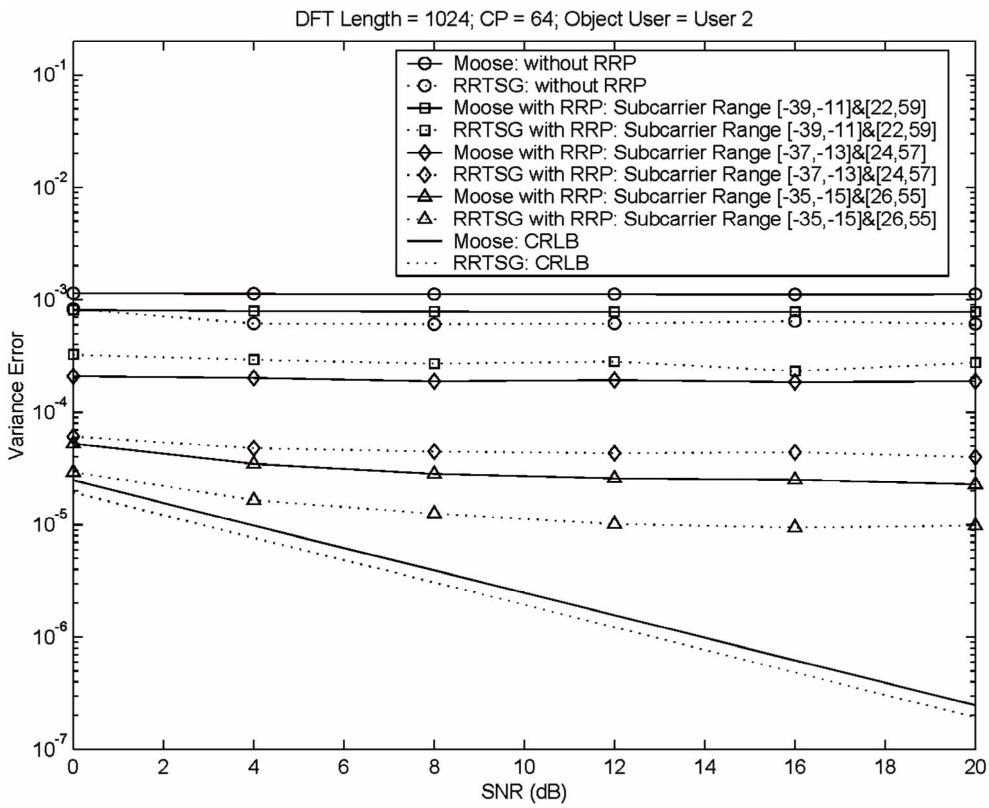

Figure 9. Performance comparison between Moose algorithm and the proposed RRTSG-based algorithm in heavy MAI environment.

of 15 dB. This error is still much smaller than that achieved by using Moose's algorithm, *i.e.*, 3×10^{-5}.

7. Conclusion

This paper discussed OFDMA UL frequency offset estimation and showed that the performance improvement in estimation benefits from MAI elimination/reduction results in improved estimation accuracy. Two MAI elimination/reduction methods, *i.e*., the RRP method and the SSP method, were proposed for a Block-based subcarrier allocation scheme and interleaved subcarrier allocation scheme, respectively. In order to randomize the MAI contributed by the other synchronized users (because of their non-zero residual frequency offsets), a new training sequence group named the RRTSG was also proposed. Numerical results proved that the proposed RRTSG-based algorithm outperforms the conventional algorithm considerably in terms of frequency offset estimation accuracy because of its MAI randomization/mitigation capability.

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Appendix

Derive the Fim

From [15], the kl-th element of FIM $\mathbf{\Gamma}$ can be represented as

$$[\mathbf{\Gamma}] = \text{trace}\left(\mathbf{C}^{-1}\frac{\partial \mathbf{C}}{\partial \varepsilon_k}\mathbf{C}^{-1}\frac{\partial \mathbf{C}}{\partial \varepsilon_l}\right), \quad (16)$$

where $\mathbf{C} = E\left\{(\mathbf{y}-\mathbf{m})(\mathbf{y}-\mathbf{m})^H\right\}$ and $\mathbf{m} = \sum_{\mathbf{k}} \mathbf{m_k}$.

Note that

$$\begin{aligned}\mathbf{C} &= E\left\{(\mathbf{y}-\mathbf{m})(\mathbf{y}-\mathbf{m})^H\right\} = E\left\{\sum_k (\mathbf{y}_k-\mathbf{m}_k)(\mathbf{y}_k-\mathbf{m}_k)^H\right\} + E\left\{\sum_{k\neq l}(\mathbf{y}_k-\mathbf{m}_k)(\mathbf{y}_l-\mathbf{m}_l)^H\right\} \\ &= \sum_k\left(\mathbf{C}_k + \sum_{k\neq l}\mathbf{Z}_{kl}\right) = \sum_k(\mathbf{C}_k+\mathbf{Z}_k),\end{aligned} \quad (17)$$

where

$$\mathbf{C}_k = E\left\{(\mathbf{y}_k-\mathbf{m}_k)(\mathbf{y}_k-\mathbf{m}_k)^H\right\}$$

represents the covariance matrix of user k, and

$$\mathbf{Z}_{kl} = E\left\{(\mathbf{y}_k-\mathbf{m}_k)(\mathbf{y}_l-\mathbf{m}_l)^H\right\}$$

is the MAI matrix of user k contributed by user l. For the non-zero frequency offsets, we have $\sum_{k\neq l}\mathbf{Z}_{kl} \neq \mathbf{O}_N$.

$\mathbf{C}$ is decomposed as

$$\mathbf{C} = \sum_k(\mathbf{C}_k+\mathbf{Z}_k) = \sum_k \mathbf{U}\left(\mathbf{D}_k+\tilde{\mathbf{D}}_k\right)\mathbf{U}^H, \quad (18)$$

where $\mathbf{U}$ is a $N\times N$ Unitary matrix,

$$\mathbf{D}_k = diag\left\{0,\cdots,\underbrace{\lambda_{k,1},\cdots,\lambda_{k,N_k}}_{G_k},\cdots,0\right\}$$

with $\lambda_{k,i}$ representing the i-th eigenvalue of $\mathbf{C}_k$, and

$$\tilde{\mathbf{D}}_k = diag\left\{0,\cdots,\underbrace{z_{k,1},\cdots,z_{k,N_k}}_{G_k},\cdots,0\right\}$$

with $z_{k,i}$ representing the i-th eigenvalue of $\mathbf{Z}_k$. Therefore, we also have

$$\begin{aligned}\frac{\partial \mathbf{C}}{\partial \varepsilon_k} &= \frac{\partial(\mathbf{C}_k+\mathbf{Z}_k)}{\partial \varepsilon_k} + \sum_{l\neq k}\frac{\partial(\mathbf{C}_l+\mathbf{Z}_l)}{\partial \varepsilon_k} = \mathbf{U}\cdot\text{diag}\left(0,\cdots,\underbrace{\frac{\partial(\lambda_{k,1}+z_{k,1})}{\partial \varepsilon_k},\cdots,\frac{\partial(\lambda_{k,N_k}+z_{k,N_k})}{\partial \varepsilon_k}}_{G_k},\cdots 0\right)\cdot\mathbf{U}^H \\ &+\sum_{l\neq k}\mathbf{U}\cdot\text{diag}\left(0,\cdots,\underbrace{\frac{\partial(\lambda_{l,1}+z_{l,1})}{\partial \varepsilon_k},\cdots,\frac{\partial(\lambda_{l,N_l}+z_{l,N_l})}{\partial \varepsilon_k}}_{G_l},\cdots 0\right)\cdot\mathbf{U}^H.\end{aligned} \quad (19)$$

From the above discussion,

$$\frac{\partial \mathbf{C}}{\partial \varepsilon_k} = \frac{\partial \mathbf{C}_k}{\partial \varepsilon_k} + \sum_{l\neq k}\frac{\partial \mathbf{Z}_l}{\partial \varepsilon_k} = \mathbf{U}\cdot\text{diag}\left(0,\cdots,\underbrace{\frac{\partial\lambda_{k,1}}{\partial \varepsilon_k},\cdots,\frac{\partial\lambda_{k,N_k}}{\partial \varepsilon_k}}_{G_k},\cdots 0\right)\cdot\mathbf{U}^H + \sum_{l\neq k}\mathbf{U}\cdot\text{diag}\left(0,\cdots,\underbrace{\frac{\partial z_{l,1}}{\partial \varepsilon_k},\cdots,\frac{\partial z_{l,N_l}}{\partial \varepsilon_k}}_{G_l},\cdots 0\right)\cdot\mathbf{U}^H. \quad (20)$$

when $l\neq k$, the kl-th element in FIM can be represented as

$$\begin{aligned}[\mathbf{\Gamma}]_{kl} &= \sum_{i=1}^{N_k}\frac{\frac{\partial\lambda_{k,i}}{\partial\varepsilon_k}\frac{\partial z_{k,i}}{\partial\varepsilon_l}}{\left(\lambda_{k,i}+z_{k,i}\right)^2} + \sum_{j=1}^{N_l}\frac{\frac{\partial\lambda_{l,j}}{\partial\varepsilon_l}\frac{\partial z_{l,j}}{\partial\varepsilon_k}}{\left(\lambda_{l,j}+z_{l,j}\right)^2} \\ &+\sum_{n\neq k}\sum_{p=1}^{N_n}\frac{\frac{\partial^2 z_{n,p}}{\partial\varepsilon_k\partial\varepsilon_l}}{\left(\lambda_{n,p}+z_{n,p}\right)^2},\end{aligned} \quad (21)$$

and the kk-th element in FIM is given by

$$[\mathbf{\Gamma}]_{kk} = \sum_{i=1}^{N_k}\frac{\left(\frac{\partial\lambda_{k,i}}{\partial\varepsilon_k}\right)^2}{\left(\lambda_{k,i}+z_{k,i}\right)^2} + \sum_{l\neq k}\sum_{j=1}^{N_l}\frac{\left(\frac{\partial z_{l,j}}{\partial\varepsilon_k}\right)^2}{\left(\lambda_{l,j}+z_{l,j}\right)^2}. \quad (22)$$

