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The Tight Bound for the Number of Pilots in Channel Estimation for OFDM Systems

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ABSTRACT

Coherent detection in OFDM systems requires accurate channel state information (CSI) at the receiver. Channel estimation based on pilot-symbol-assisted transmissions provides a reliable way to obtain CSI. Use of pilot symbols for channel estimation, introduces overhead and it is desirable to keep the number of pilot symbols as minimum as possible. This paper introduces a new tight bound for the number of pilots in channel estimation using adaptive scheme in OFDM systems. We calculate the minimum number of necessary pilots using two approaches. The first approach for the number of pilots is obtained based on Doppler frequency shift estimation and the second approach is acquired based on channel length estimation using second order statistics of received signal. Finally we obtain the tight bound for the number of pilots using attained values.

Keywords: Doppler Frequency Shift; Second Order Statistics; Channel Estimation; Pilot; OFDM

1. Introduction

Orthogonal frequency division multiplexing (OFDM) represents a valid choice for a variety of wireless applications since it has been shown to outperform single-carrier schemes especially in high data rate applications [1]. To correctly demodulate the received data symbols, the channel estimation plays an important role to the system performance greatly. The most common technique to obtain channel information is via pilot aided channel estimation [2]. Pilot symbols consume both power and spectrum and thus it is desirable to keep the number of pilot symbols as minimum as possible. Some approaches for reduction of pilot overhead were proposed in [3-5]. On the other hand the spacing between pilot symbols should be small enough to make channel estimates reliable. Adaptive resource allocation in order to achieve a proper trade-off between utilized resources and quality of channel estimates based on CSI is necessary. Some authors discussed the number of used pilots in channel estimation and represent some methods for reduction them in OFDM systems. Authors in [6] use deviation of corresponding signals as a parameter to predict the channel characteristics in the transmitter end. They transmit fixed length of test signal to the channel whenever connection setup initiation takes place. Then by using the response, number of pilot signals to insert is determined and correspondingly appended to the data signal. Thus by varying the number of pilots depending upon the channel environment, it certainly improves the channel estimation and reduction in number of pilots used for channel estimation and feedback. The proposed estimator in [7] employs a specific pilot structure which consists of two types of pilot symbols with different pilot density. The combination of interference alleviation and pilot rearrangement not only makes the channel estimation robust to the time selectivity of the channel but also reduces the number of pilot subcarriers needed to estimate the channel. Authors in [8] develop a method to obtain the optimal number of known pilot symbols for pilot-aided linear channel estimation in OFDM. They derive an expression for channel estimation error covariance from which performance measures can be numerically evaluated to obtain the optimal number of pilot symbols. In [9], the shortest preamble that attains a given mean square error in a sequence that under-samples the channel spectrum is found. A high mobile wireless channel is time-variant due to Doppler frequency shift. Then, the number of used pilots in channel estimation depends on the Doppler frequency shift. So, we represent a lower bound for the number of pilots based on it. Also, the number of used pilots depends on the length of channel. Thus, we evaluate channel length estimation problem for obtaining another lower bound for the number of pilots. Finally the tight bound is obtained.

The reminder of this paper is structured as follows: in Section 2 the system model is introduced. Bounds for

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number of used pilots in channel estimation are presented in Section 3. Some simulation results are given in Section 4 and finally Section 5 concludes the paper.

2. OFDM System Model

The basic building block for OFDM transceiver is shown in **Figure 1**. In an OFDM transmitter, at the *n*th symbol time, *K* data symbols $\{X[n,k]\}, k = 0,1,2,\cdots,K-1$, are converted into time-domain signals using the inverse fast Fourier transform (FFT). A cyclic prefix (CP) is inserted to preserve the orthogonality between the subcarriers and to eliminate the interference between the adjacent OFDM symbols. At the receiver, the CP is removed before the FFT process. Assuming the ideal synchronization at the receiver, the received symbol of the *k* th subcarrier at the *n* th symbol time can be represented by:

$$Y[n,k] = X[n,k]H[n,k] + Z[n,k] \tag{1}$$

where $H[n,k]$ is the frequency response of the channel at the *k* th subcarrier and the *n* th symbol time, and $Z[n,k]$ is the background noise, which can be approximated as zero-mean additive white Gaussian noise (AWGN) with variance σ_z^2 [10].

3. Bounds for the Number of Pilots

We study the number of used pilots for channel estimation in OFDM systems, and achieve the bounds for it. Here, we recall that unnecessary overhead is not desirable in terms of the resource allocation for the pilot signals. An appropriate pilot distribution is necessary using adaptive pilot allocation based on obtained bounds.

3.1. The First Lower Bound for the Number of Used Pilots in OFDM Systems

For proper channel estimation, it is necessary to sample channel according to the Nyquist theorem. The parameters of used in this section are listed in **Table 1**. Let us denote the spacing between pilot symbols in the time domain by N_t and in the frequency domain by N_f. The sampling theorem states that [11]:

$$N_t \le \frac{1}{2 f_d T_s} \text{ and } N_f \le \frac{1}{2\tau_m \Delta F} \tag{2}$$

where f_d and τ_m denote maximum Doppler frequency shift and maximum delay respectively. T_s denotes the OFDM symbol duration and the carrier spacing, ΔF, is the spacing in the frequency domain. As illustrated in **Figure 2**, the transmitter sends pilot symbols scattered in the time and frequency domains in a regular pattern. Assuming that power spectrum density (PSD) of channel is based on jakes spectrum [12], f_d can be estimated as follows:

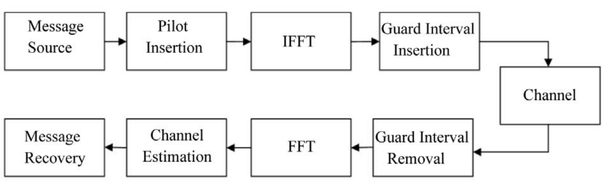

Figure 1. Baseband OFDM transceiver.

Table 1. The list of used parameters in Section 3.1.

Parameter	Specification
N_t	Pilot spacing in the time domain
N_f	Pilot spacing in the frequency domain
f_d	Maximum Doppler frequency shift
τ_m	Maximum delay
T_s	OFDM symbol duration
ΔF	Carrier spacing
N_p	The number of pilot tones
N_s	The number of OFDM symbols
K	The number of subcarriers

$$f_d = \frac{1}{2\pi} J_0^{-1}\left(r_t\left(\Delta n\right)\right) \tag{3}$$

where J_0 is the 0th order Bessel function of the first kind and $r_t(\Delta n)$ denotes the correlation function in the time domain. Also τ_m can be estimated with some approaches such as [13]. The N_p pilot tones are uniformly inserted into OFDM block. N_p is calculated as follows:

$$N_p = \left\lceil \frac{K}{N_f} \right\rceil \times \left\lceil \frac{N_s}{N_t} \right\rceil \tag{4}$$

where N_s is the number of OFDM symbols in the time domain.

Using Equations (2) and (4), the first lower bound for the number of used pilots is obtained as follows:

$$N_p \ge 4KN_s f_d T_s \tau_m \Delta F \tag{5}$$

3.2. The Second Lower Bound for the Number of Used Pilots in OFDM Systems

We assume that channel impulse response is as follows:

$$h[n] = \begin{cases} h_n, n = 0,1,\cdots,L \\ 0, \quad otherwise \end{cases} \tag{6}$$

where L is the length of channel. Authors in [14] proposed an algorithm for estimation of the channel length without the need of pilots based on the second-order statistics of the received signal. Here, we represent a terse description of it.

Assuming an arbitrary positive integer N ($N > L$), we may construct an $(N+1)\times 1$ autocorrelation vector $\boldsymbol{r}_{N+1}$ for the received signal such that:

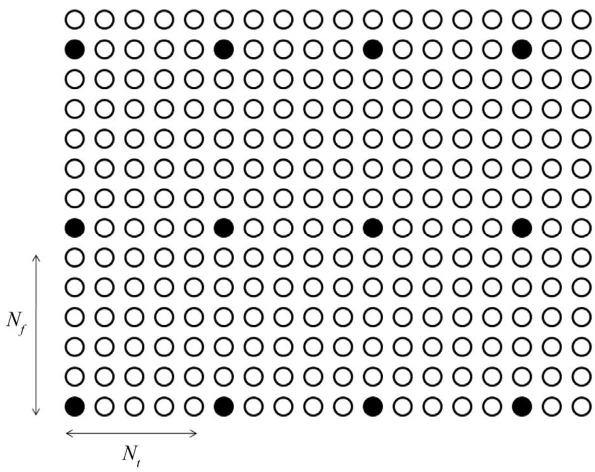

Figure 2. Pilot symbols in the OFDM signals.

$$\boldsymbol{r}_{N+1} = \left[\rho_0 \rho_1 \cdots \rho_N\right]^T \tag{7}$$

where

$$\rho_l \equiv \frac{1}{M}\sum_{n=0}^{L} y[n]\, y^*[n+l] \tag{8}$$

M denotes the sample size. Assume that the maximum lag N satisfies that $N > L$. A threshold ζ can be established as follows:

$$\zeta = \min\left[\frac{M\psi}{(N+1)^2} + \theta\left(\frac{|\rho_0|^2}{|\rho_N|^2} - \frac{M\psi}{(N+1)^2}\right), M\psi\right] \tag{9}$$

where $0 < \theta < 1$ is the parameter to be chosen and ψ is denotes as the *start-terminal-coefficients-to-channel-gain ratio*, which is related to the detectable start and terminal channel coefficients, such that:

$$\psi \equiv \frac{|h_0|^2 |h_L|^2}{\left(\sum_{p=0}^{L} |h_p|^2\right)^2} \tag{10}$$

The steps of proposed algorithm for channel length estimation in [14] are as follows:

Step 1: Choose maximum lag N and sample size M. Set the threshold ζ according to Equation (9).

Step 2: Compute $\boldsymbol{r}_{N+1}$ as given by Equation (7). Set $l = N$.

Step 3: Compute $\frac{|\rho_{l-1}|^2}{|\rho_l|^2}$ with the threshold ζ. Determine $\hat{L} = (l-1)$ if $\frac{|\rho_{l-1}|^2}{|\rho_l|^2} \geq \zeta$. Otherwise, decrement l by 1 and redo Step 3 until $l = 0$.

Step 4: If $l = 0$, set $\hat{L} = 0$. Where $\hat{L}$ is estimated channel length.

To control the detectability, $\psi = 0.0001$ (high detectability) and $\psi = 0.1$ (low detectability) are chosen. The performance of presented channel length estimation method is not sensitive to the values of the parameter θ and is robust over all kinds of constellations such as BPSK and QPSK.

The following theorem states that, with knowing length of channel, we can have a bound for the number of pilots.

Theorem: assuming equi-spaced and equal power pilot tones with the number of pilot tones, N_p, greater than or equal to the channel length, implies:

1) The covariance matrix of the channel estimation error , Σ, is a diagonal matrix.

2) If matrix Σ is diagonal, the MSEs of channel prediction are identical for all subcarriers.

3) Increasing the number of pilot tones greater than channel length will not enhance the performance of the channel predictor.

The proof of this theorem is given in [15]. Then, we have:

$$N_p \geq \hat{L} \tag{11}$$

3.3. Adaptive Algorithm for Pilot-Aided Channel Estimation Based on Tight Bound

We rename right hand of inequalities (5) and (11), as follows:

$$a = 4KN_s f_d T_s \tau_m \Delta F, b = \hat{L} \tag{12}$$

Then

$$B_1 = \min(a,b),\ B_2 = \max(a,b) \tag{13}$$

Thus, we have a tight bound for the number of necessary pilots for channel estimation in OFDM systems as follows:

$$B_1 \leq N_p \leq B_2 \tag{14}$$

The receiver, after calculating the tight bound for the number of pilots, feeds these values back to the OFDM transmitter such that these values will be used in the next transmission period. By exploiting CSI, The proposed system minimizes the pilot overhead while maintaining desired channel estimation accuracy. In other words, receiver by consideration the tight bound for the number of necessary pilots and environment conditions, decide that how many pilots should be used for channel estimation. **Figure 3** illustrates proposed adaptive algorithm for pilot aided channel estimation in the OFDM systems.

4. Simulation Results

In this section, we evaluate the performance of proposed adaptive algorithm for channel estimation. Simulation parameters are shown in **Table 2**.

BER vs. SNR curves with $L = 8$, 12 are shown in

Table 2. Simulation parameters.

PARAMETER	VALUE
Number of total subcarriers	$K = 128$
Cyclic prefix length	$N_g = 16$
Bandwidth	1 MHz
OFDM symbol duration	$T_s = 128\mu s$
Carrier frequency	2 GHz
Modulation	16QAM
Channel type	Jakes Rayleigh fading [12]
Type of estimation	Least square
Sample size	$M = 1000$
Chosen parameter	$\theta = 0.01$
Start-terminal-coefficient-to-channel-gain ratio	$\psi = 0.0001$

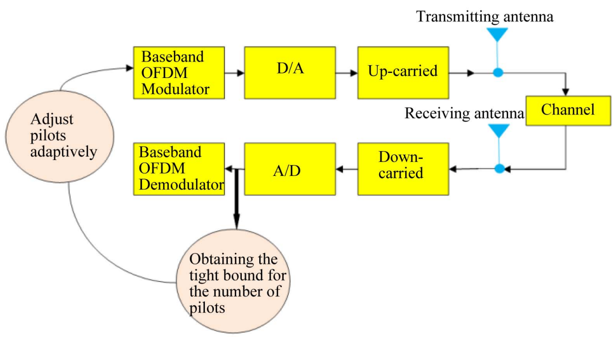

Figure 3. Proposed OFDM system with adaptive scheme for number of used pilots.

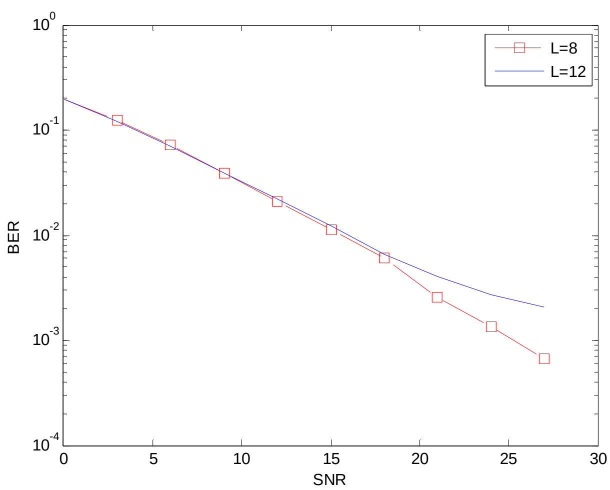

Figure 4. BER vs. SNR with L = 8, 12.

Figure 4. The adaptive pilot system based on the first lower bound is compared to fixed pilot system in **Figures 5** and **6** in $f_d = 50,\ 150$ Hz respectively.

The fixed number of pilots is calculated according to the average situation of the environment and the number of adaptive pilots is equal to $a = 4KN_s f_d T_s \tau_m \Delta F$.

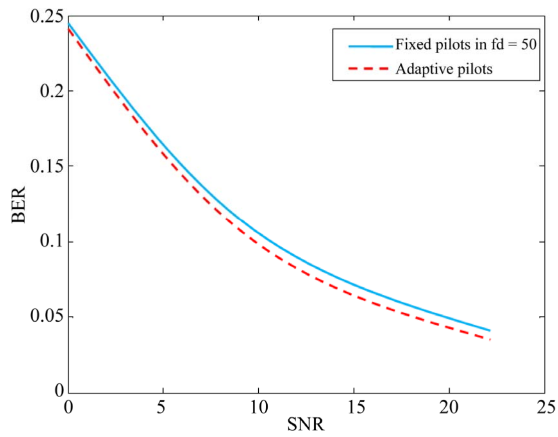

Figure 5. Performance improvement using adaptive resource allocation based on the first lower bound for the number of pilots, $N_p = 4KN_s f_d T_s \tau_m \Delta F$. Channel estimation using fixed pilots is implemented in f_d = 50 Hz.

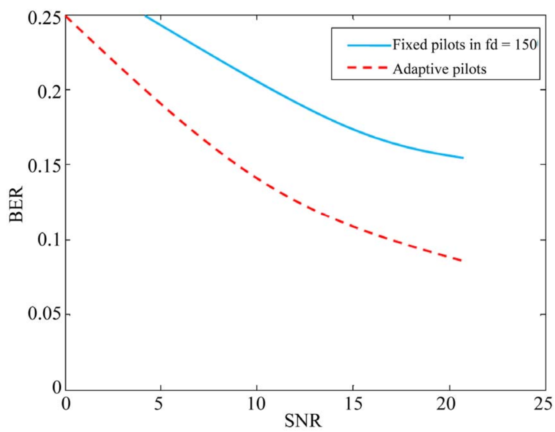

Figure 6. Performance improvement using adaptive resource allocation based on the first lower bound for the number of pilots, $N_p = 4KN_s f_d T_s \tau_m \Delta F$. Channel estimation using fixed pilots is implemented in f_d = 150 Hz.

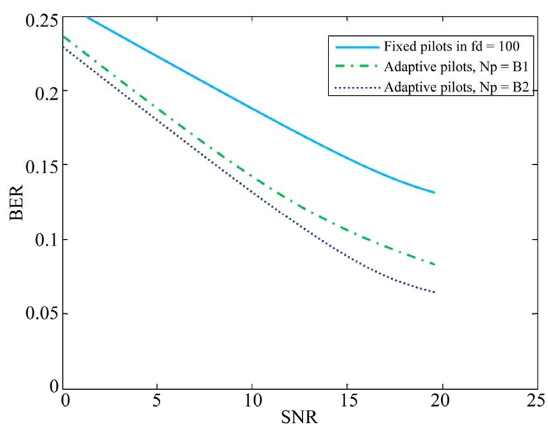

Figure 7. BER performance of the OFDM system with fixed pilots and adaptive pilots based on obtained tight bound with $N_p = B_1$ and $N_p = B_2$.

Figures 5 and **6** illustrate that adaptive pilot improves performance of OFDM system especially under high Doppler frequency shift. **Figure 7** shows the bit error rate (BER) performance of the OFDM system with fixed pilots and adaptive pilots with $N_p = B_1$ and $N_p = B_2$. Proximity of curves for $N_p = B_1$ and $N_p = B_2$ demonstrates that presented bound in Section 3 is really tight.

5. Conclusion

In this paper, we drive a new tight bound for the number of used pilots in channel estimation for OFDM systems. This bound is fed back to the transmitter adaptively to improve the system performance with low pilot overhead compared to the conventional schemes.

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