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Conditional Diagnosability of the Locally Twisted Cubes under the PMC Model*

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Abstract

In a multiprocessor systems, it is important to local and to replace the faulty processors to maintain systempsilas high reliability. The fault diagnosis, which is the process of identifying fault processors in a multiprocessor system through testing. The conditional diagnosis requires that for each processor u in a system, all the processors that are directly connected to u do not fail at the same time. In this paper, we study the conditional diagnosability of the n-dimensional locally twisted cubes. After showing some properties of the locally twisted cubes, we prove that it under the PMC model is $4n - 7$ for $n \geq 5$.

Keywords: Locally Twisted Cubes, Diagnosability, Conditional Diagnosability, PMC Mode

1. Introduction

In recent years, the number of the processors in a multiprocessor system increases as fast as the technology development. Thus some processors may fail in such a multiprocessor operating system. So locating the faulty processors is important for system maintenance and dependable computing. System level diagnosis is an important approach for fault diagnosis in a multiprocessor system. Many different models for system level diagnosis in multiprocessor systems have been proposed, e.g., the PMC (the Perfect Minicomputer Corporation) model [1], the comparison model [2] and the BGM model [3]. So far, the well-studied mode is the PMC model introduced by Preparata, Metze, and Chien [1].

A multiprocessor system is an interconnected collection of processors and can be represented by an undirected graph $G = (V, E)$, where each vertex of the vertex set V represents a processor and each edge of the edge set E represents a communication link between a pair of processors. Two processors interact with each other by sending messages over the communication link. Under the PMC model, two processors can test each other if and only if there is a link between them. The processor which tests the status of the other is called a tester. It is assumed that the test result is reliable if and only if the tester is fault free; otherwise, the test result is unreliable. The collection of all test results is called a syndrome $\sigma \cdot r(u, v)$ denotes the test result of processor u testing processor v. If v pass the test executed by u, $r(u, v) = 0$; otherwise, $r(u, v) = 1$. **Table 1** shows all possible test results of the test $r(u, v)$.

For a given syndrome σ, a subset of vertices $F \subseteq V$ is said to be consistent with σ if σ can arise from the circumstance that all nodes in F are faulty and all nodes in *V-F* are fault free. It is worth pointing out that a given set *F* of faulty vertices may be consistent with different syndromes. Let $\sigma(F)$ be the set containing all syndromes which can be produced by *F*. Two distinct sets F_1, $F_2 \subseteq V$ are said to be distinguishable if $\sigma(F_1) \cap \sigma(F_2) = \varnothing$ otherwise, F_1, F_2 are said to be indistinguishable.

Table 1. Test results.

u	*v*	*r* (*u*, *v*)
Fault-free	Fault-free	0
Fault-free	Fault	1
Fault	Fault-free	0 or 1
Fault	Fault	0 or 1

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A system is said to be t-diagnosable if given a syndrome σ, all processors can correctly be identified faulty or faulty free, provided that the number of faulty processors present in the system does not exceed t. The diagnosability of a system is the maximal number of faulty processors that the system can guarantee to diagnose. The diagnosability of some interconnection networks have been discussed under the *PMC* model, see [4-6].

Lai *et al*. in [7] introduced conditional diagnosability by restricting that for each processor u in a system, all processors adjacent to u are not faulty at the same time, and showed that conditional diagnosability of the n-dimensional hypercube (Q_n) is $4n – 7$ for $n \geq 5$, which is about four times as large as its classical diagnosability [8]. Zhu *et al.* in [9] presented that under PMC-model the conditional diagnosability of FQn ($t_c(FQ_n)$) was 4n – 3 when $n \geq 5$ or $n > 8$; $t_c(FQ_3) = 3$, $t_c(FQ_4) = 7$.

In recent years, conditional diagnosability of several interconnection networks has also been explored under the *PMC* model [7,9-12].

In the paper, we prove that conditional diagnosability of locally twisted cubes under the *PMC* model is $t_c(LTQ_n) = 4n-7$ for $n \geq 5$.

The rest of paper is organized as follows: Preliminary knowledge is provided in Section 2; The main results of this paper are presented and proven in Section 3; The conclusions are made in Section 4.

2. Preliminaries

For all the terminologies and notations not defined here, we follow [13]. For a graph $G = (V, E)$ and $S \subset V(G)$ or $S \subset G$, we use $N_G(S)$ to denote the set of neighboring vertices of S in G-S, when it is easy to know from the context what G denotes, it is usually simplified with $N(S)$. We use $A_G(S)$ to denote the union of S and $N_G(S)$. And similarly $A_G(S)$ can be simplified with $A(S)$.

That is, $N_G(S) = \{v \in V(G)\text{-}S \mid \exists\ u \in S$ such that $(u, v) \in E(G)\}$, $A_G(S) = N_G(S) \cup S$.

We use $d_G(u)$ to denote the degree of u in G and $d_G(u)$ can be simplified with $d(u)$.

Definition 1. [14] For $n \geq 2$, an n-dimensional locally twisted cube, denoted by LTQ_n, is defined recursively as follows:

1) LTQ_2 is a graph consisting of four nodes labeled with 00, 01, 10 and 11, respectively, connected by four edges {00, 01}, {01, 11},{11, 10} and {10, 00}.

2) For $n \geq 3$, LTQ_n is built from two disjoint copies of LTQ_{n-1} according to the following steps: Let $0LTQ_{n-1}$ denote the graph obtained from one copy of LTQ_{n-1} by prefixing the label of each node with 0. Let $1LTQ_{n-1}$ denote the graph obtained from the other copy of LTQ_{n-1} by prefixing the label of each node with 1. Connect each node $0x_2x_3\cdots x_n$ of 0LTQn 1 to the node $1(x_2+x_n)x_3\cdots x_n$ of $1LTQ_n$ 1 with an edge, where "+" represents the modulo 2 addition.

Figure 1 shows two examples of locally twisted cubes. The locally twisted cubes can also be equivalently defined in the following non-recursive fashion.

Definition 2. [14] For $n \geq 2$, the n-dimensional locally twisted cube, LTQ_n, is a graph with $\{0, 1\}^n$ as the node set. Two nodes $x = x_1x_2\cdots x_n$ and $y = y_1y_2\cdots y_n$ of LTQ_n are adjacent if and only if either one of the following conditions are satisfied.

1) $x_i = \overline{y_i}$ and $x_{i+1} = y_{i+1} + y_n$ for some $1 \leq i \leq n – 2$, and $x_j = y_j$ for all the remaining bits;

2) $x_i = \overline{y_i}$ for $i \in \{n – 1, n\}$, and $x_j = y_j$ for all the remaining bits.

The definition of the conditional diagnosability is as follows.

Definition 3. [7] A faulty set $F \subseteq V$ is called a conditional faulty set if $N(v) \not\subset F$ for any vertex $v \in V$. A system $G(V, E)$ is conditionally t-diagnosable if F_1 and F_2 are distinguishable, for each pair of conditional faulty sets $F_1, F_2 \subseteq V$, and $F_1 \neq F_2$ with $|F_1|; |F_2| \leq t$. Conditional diagnosability of a system G, written as $t_c(G)$ is defined to be the maximum value of t such that G is conditionally t-diagnosable.

Let F_1, F_2 be two distinct sets, the symmetric difference of F_1 and F_2 is denoted by $F_1\Delta F_2$, that is, $F_1\Delta F_2 = (F_1 – F_2) \cup (F_2 – F_1)$. The following lemma proposed by Dahbura and Masson [15] gives a necessary and sufficient condition for a system to be t-diagnosable.

Lemma 1. [16] A system $G(V, E)$ is t-diagnosable if and only if, for each pair F, $F_2 \subset V$ with$|F_1|$, $|F_2| \leq t$ and $F_1 \neq F_2$, there is at least one test from $V – F_1 \cup F_2$ to $F_1\Delta F_2$.

Lemma 2. [14] $k(LTQ_n) = n$ for $n \geq 2$.

Lemma 3. [17] $k(LTQ_n) = 2n – 2$ for $n \geq 3$.

Lemma 4. [17] Let S be a set of vertices $S \subset V(LTQ_n)$ with $|S| = n$, if LTQ_n-S is disconnected, there exists a vertex $u \in V(V(LTQ_n))$ such that $N(u) = S$ for $n \geq 2$.

The following lemma is derived based on [18,19].

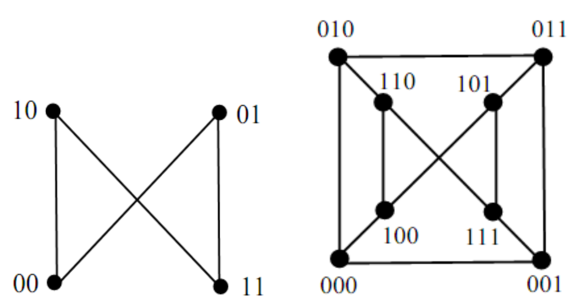

Figure 1. Example of LTQ_n: LTQ_2 and LTQ_3.

Lemma 5. Let F be a subgraph of LTQ_n with $4 \leq |V(F)| \leq 3n-5$, we have $|N_{LTQ_n}(F)| \geq 4n-8$.

3. Conditionally Diagnosability

Lemma 6. Let S be a set of vertices $S \subset V(LTQ_n)$ and $n \geq 3$. Suppose that $LTQ_n - S$ is disconnected. The following two conditions hold:

(1) $|S| \geq n$;

(2) If $n \leq |S| \leq 2n - 3$, then LTQ_n-S has exactly two components, one is trivial and the other is nontrivial. The nontrivial component of LTQ_n-S contains $2^n - |S| - 1$ vertices.

Proof: By lemma 2 $k(LTQ_n) = n$, so condition (1) holds. We need to prove that condition (2) is true. Since $LTQ_n - S$ is disconnected, there are at least two components in $LTQ_n - S$. We will prove that $|S| \geq 2n - 2$ when $LTQ_n - S$ contains at least two trivial components or two nontrivial components. It implies that $n \leq |S| \leq 2n - 3$ when $LTQ_n - S$ contains a trivial components and nontrivial components.

Case 1. $LTQ_n - S$ contains at least two trivial components. Let $u_1, u_2 \in V(LTQ_n)$ and $\{u_1\}$, $\{u_2\}$ be two trivial components. Then $N(u_1) \subset$ S and $N(u_2) \subset$ S. Since any two distinct vertices of a LTQ_n have at most two common neighbors, we have $|N(V_1) \cap |N(V_2)| \leq 2$.

Hence, $|S| \geq |N(V_1)| + |N(V_2)| - |N(V_1) \cap N(V_2)| \geq 2n + 2n - 2 = 2(2n - 1)$.

Case 2. LTQ_n-S contains at least two nontrivial components. We prove condition (2) by induction on n. Suppose $n \leq |S| \leq 2n - 3$, it is easy to see that $|S| = 3$ for $n = 3$. The connectivity of LTQ_3 is 3. By Lemma 4, there exist a vertex $u \in V(LTQ_3)$ such that $S = N(u)$ Thus LTQ_3-S has exactly two components: one is trivial and the other is nontrivial. Therefore, if LTQ_3-S has at least two nontrivial components, $|S| \geq 2n - 2$, where $n = 3$. Assume that the result holds for all $n - 1$, $n - 1 \geq 3$. In the following we show that it holds for n.

Let $S_0 = S \cap V(0LTQ_{n-1})$ and $S_1 = S \cap V(1LTQ_{n-1})$, F and F' be two nontrivial component of LTQ_n-S. So $|V(F)| \geq 2$ and $|V(F')| \geq 2$.

We consider the following three cases:

Case 2.1. $F, F' \subseteq 0LTQ_{n-1}$ or $F, F' \subseteq 1LTQ_{n-1}$. Without loss of generality, let F, F $\subseteq 0LTQ_{n-1}$, then $0LTQ_{n-1}$-S_0 is disconnected and $|S_1| \geq |F| + |F'| \geq 4$. So $|S_0| \geq k_2 = 2n - 4$ by lemma 3. Thus $|S| = |S_0| + |S_1| \geq 2n - 2$.

Case 2.2. $F \subseteq 0LTQ_{n-1}$ and $F' \subseteq 1LTQ_{n-1}$, or $F' \subseteq 1LTQ_{n-1}$ and F $\subseteq 1LTQ_{n-1}$. Without loss of generality, let $F \subseteq 0LTQ_{n-1}$ and $F' \subseteq 1LTQ_{n-1}$. If both $0LTQ_{n-1} - S_0$ and $1LTQ_{n-1} - S_1$ are connected, then $|S_0| \geq 2n - 4$ and $S_1| \geq 2n - 4$. So $|S| = |S_0| + |S_1| \geq 2n - 4 + 2n - 4 \geq 2n - 2$ for n ≥ 3. If exactly one of $0LTQ_{n-1} - S_0$ and $1LTQ_{n-1} - S_1$ is disconnected, let $0LTQ_{n-1} - S_0$ be disconnected, then $N_{0LTQ_n}(F) \subseteq S_0$. So

$$|S| \geq |N_{0LTQ_n}(F)| + 2 \geq 2n - 4 + 2 = 2n - 2$$

Case2.3. $0LTQ_{n-1} \cap F \neq \varnothing$ and $1LTQ_{n-1} \cap F \neq \varnothing$, or $0LTQ_{n-1} \cap F' \neq \varnothing$ and $1LTQ_{n-1} \cap F' \neq \varnothing$. Without loss of generality, let $0LTQ_{n-1} \cap F \neq \varnothing$ and $1LTQ_{n-1} \cap F \neq \varnothing$. Since there is another component F' of $LTQ_n - S$, at least one of the two graphs $0LTQ_{n-1} - S_0$ and $1LTQ_{n-1} - S_1$ is disconnected. So we drive the result by consider two Subcase.

Case 2.3.1. Both $0LTQ_{n-1} - S_0$ and $1LTQ_{n-1} - S_1$ are disconnected. Since $k(LTQ_{n-1}) = n - 1$, $|S_0| \geq n - 1$ and $|S_1| \geq n - 1$. Then $|S| = |S_0| + |S_1| \geq 2n - 2$.

Case 2.3.2. Exactly one of the two subgraphs $0LTQ_{n-1} - S_0$ and $1LTQ_{n-1} - S_1$ is disconnected. Without loss of generality, assume that $0LTQ_{n-1} - S_0$ is connected and $1LTQ_{n-1} - S_1$ is disconnected. Then $|S_1| \geq$ n $- 1$ and $N_{0LTQn}(F) \subseteq S_0$. Hence, $|S_0| \geq |V(F')| \geq 2$. If $|S_1| \geq 2n - 4$, then $|S| = |S_0| + |S_1| \geq 2 + (2n - 4) = 2n - 2$. Otherwise, n $- 2 \leq |S_1| \leq 2n - 5$. By induction hypothesis, $1LTQ_{n-1} - S_1$ has exactly two components: one is trivial and the other is nontrivial. We know that $1LTQ_{n-1} \cap F$ and F' are two components of $1LTQ_{n-1} - S_1$, and F' is a nontrivial component. Thus $1LTQ_{n-1} \cap F$ must be a trivial component of $11LTQ_{n-1} - S_1$, and $|V(F')| = 2^{n-1} - |S_1| - 1$. Note that $N_{0LTQn}(F') \subseteq S_0$. Hence, $|S| = |S_0| + |S_1| \geq |V(F')| + |S_1| = 2^{n-1} - |S_1| - 1 + |S_1| \geq 2n - 2$ for $n \geq 4$.

Consequently, condition (2) holds.■

Lemma 7: Let S be a set of vertices $S \subset V(LTQ_n)$ and n ≥ 5. Suppose that LTQ_n-S is disconnected and every component of LTQ_n-S is nontrivial, and there exists one component F of LTQ_n-S such that $d_F(v) \geq 2$ for any vertex $v \in F$. Then one of the following two conditions must hold:

(1) $|S| \geq 4n - 8$;

(2) $|V(F)| \geq 4n - 9$.

Proof: Let $F_0 = 0LTQ_{n-1} \cap F$, $F_1 = 1LTQ_{n-1} \cap F$, $S_0 = S \cap V(0LTQ_{n-1})$ and $S_1 =$ S $\cap V(1LTQ_{n-1})$. We consider two cases: (a) $F \subset 0LTQ_{n-1}$ or $F \subset 1LTQ_{n-1}$. (b) $0LTQ_{n-1} \cap F \neq \varnothing$ and $1LTQ_{n-1} \cap F \neq \varnothing$.

Case 1. $F \subset 0LTQ_{n-1}$ or $F \subset 1LTQ_{n-1}$. Without loss of generality, let $F \subset 0LTQ_{n-1}$. Then F $\subset S_1$. In the following we consider two cases.

Case 1.1. $0LTQ_{n-1}$-F is connected. Then $|S| = |S_0| + |S_1| \geq |S_0| + |V(F)| = 2^{n-1} \geq 2n - 2$ for $n \geq 4$ and conditional (a) holds.

Case 1.2. $0LTQ_{n-1} - F$ is disconnected. If $4 \leq |V(F)| \leq 3n - 5$, by Lemma 5, we have $|S_0| \geq |N_{LTQ_n}(F)| \geq 4n - 8$.Therefore, $|S| \geq 4n - 8$ and conditional (a) holds. If $3n - 4 \leq |V(F)| \leq 4n - 10$, then $|S_0| \geq n - 1$ since $0LTQ_{n-1} - F$

is disconnected and $|S_1| \geq |V(F)| \geq 3n - 4$. Thus $|S| = |S_0| + |S_1| \geq n - 1 + 3n - 4 = 4n - 5$ and conditional (a) holds. Otherwise, $|V(F)| \geq 4n - 9$ and conditional (b) holds.

Case 2. $0LTQ_{n-1} \cap F \neq \emptyset$ and $1LTQ_{n-1} \cap F \neq \emptyset$. Since every vertex x in F_0 (resp. y in F_1) has at most one neighbor in F_1(resp. F_0), we have $d_{F_0}(x) \geq 1$ and $d_{F_1}(x)$. Since $LTQ_n - S$ is disconnected, there are at least two components in $LTQ_n - S$. At least one of the two graphs $0LTQ_{n-1} - S_0$ and $1LTQ_{n-1} - S_1$ is disconnected since both LTQ_n and LTQ_{n-1} contain some non-empty part of the component F.

In the following we drive the result by consider two cases.

Case 2.1. Exactly one of the two graphs $0LTQ_{n-1} - S_0$ and $1LTQ_{n-1} - S_1$ is disconnected. Without loss of generality, assume that $0LTQ_{n-1} - S_0$ is connected and $1LTQ_{n-1} - S_1$ is disconnected. Let F' be another non-trivial component of $LTQ_n - S$ other than F. Then $F' \subset (1LTQ_{n-1} - S_1)$ and $N_{0LTQ_{n-1}}(F') \subset S_0$. Note that both F' and F_1 are nontrivial component. By Lemma 6, $|S_1| \geq 2n - 4$. If j$|S_0| \geq 2n - 4$, then $|S| = |S_0| + |S_1| \geq 4n - 8$ and condition (1) holds. Otherwise, $|S_0| \leq 2n - 5$. Then $|V(F_0)| = 2^{n-1} - |S_0|$ since $V(LTQ_{n-1}) = S_0 \cup V(F_0)$.

Thereby, $|V(F)| = |V(F_0)| + |V(F_1)| \geq (2^{n-1} - (2n-5)) + 2 \geq 4n - 9$ for $n \geq 4$ and condition (2) holds.

$0LTQ_{n-1} - S_0$ and $1LTQ_{n-1} - S_1$ are disconnected. We consider the following three subcases.

Case 2.1.1. $|S_0| \geq 2n - 4$ and $|S_1| \geq 2n - 4$.Clearly, $S| = |S_0| + |S_1| \geq 8n - 8$. Therefore, condition (1) holds.

Case 2.2.2. Either $n - 1 \leq |S_0| \leq 2n - 5$, $|S_1| \geq 2n - 4$ or $|S_0| \geq 2n - 4$, $n - 1 \leq |S_1| \leq 2n - 5$. Without loss of generality, assume that $S_0| \geq 2n - 4$, $n - 1 \leq |S_1| \leq 2n - 5$. Then we have $|V(F_1)| = 2^{n-1} - |S_1| - 1$ by the lemma 6. Since $d_{F_0}(u) \geq 1$ for any vertex $u \in V(F_0)$,$|V(F_0)| \geq 2$. Thus, $|V(F)| = |V(F_0)| + |V(F_1)| \geq 2 + (2^{n-1} - 2n - 4) \geq 4n - 9$ for $n \geq 5$. Hence, condition (2) holds.

Case 2.2.3. $n - 1 \leq |S_0| \leq 2n - 5$ and $n - 1 \leq |S_1| \leq 2n - 5$. By the lemma 6, we have $|V(F_0)| = 2^{n-1} - |S_0| - 1$ and $|V(F_1)| = 2^{n-1} - |S_1| - 1$. So $|V(F)| = |V(F_0)| + |V(F_1)| = 2^n - |S| - 2$. If $|S| \geq 4n - 8$, then condition (1) holds. Otherwise, $|S| \leq 4n - 9$, then $|V(F)| = 2^n - (4n - 9) - 2 \geq 4n - 9$ for $n \geq 4$. Hence, condition (2) holds.

Consequently, the lemma holds. ■

Theorem 1. Let $F_1, F_2 \subset V(LTQ_n)$ be two indistinguishable conditional faulty sets, then either $|F_1| \geq 4n - 6$ or $|F_2| \geq 4n - 6$ for $n \geq 5$.

Proof: Let $S = F_1 \cap F_2$, according to $LTQ_n - S$ is connected or not, we consider the following two cases.

Case 1. $LTQ_n - S$ is connected. We assert that $F_0 \Delta F_1 = V(LTQ_n) - S$. Otherwise, suppose $u \in V(LTQ_n - S) - F_1 \Delta F_2 = V(LTQ_n) - F_1 \cup F_2$.Then u is connected to $F_1 \Delta F_2$ since $LTQ_n - S$ is connected. That is, there is an edge between $F_1 \Delta F_2$ and $V - F_1 \cup F_2$. This is a contradiction to the fact F_1 and F_2 are an indistinguishable. Since $|F_1| + |F_2| = |F_1|\Delta|F_2| = |V(LTQ_n)| = 2^n \geq 8n - 13$ for $n \geq 5$, either $|F_1| \geq 4n - 6$ or $|F_2| \geq 4n - 6$. Then the result follows.

Case 2. $LTQ_n - S$ is disconnected. Since F_1 and F_2 is indistinguishable, there is no edge between $F_1 \Delta F_2$ and $V(LTQ_n) - F_1 \cup F_2$ by Lemma 1.That is, for any vertex $u \in F_1 \Delta F_2$, $N_{LTQ_n}(u) \subset F_1 \cup F_2$. Since both $F_1$and F_2 are conditional faulty set, $N_{LTQ_n}(u) \not\subset F_1$ and $N_{LTQ_n}(u) \not\subset F_2$. So $|N_{LTQ_n}(u) \cap (F_2 - F_1)|$ and $|N_{LTQ_n}(u) \cap (F_2 - F_1)| \geq 1$.

Thus for any vertex $u \in F_1 \Delta F_2$, $|N_{F_1 \Delta F_2}(u)| \geq 2$. So $LTQ_n - S$ has a component P with $V(P) \subset F_1 \Delta F_2$ such that $d_P(u) \geq 2$ for any vertex $u \in V(P)$. By Lemma 7, we have $|S| \geq 4n - 8$ or $|V(P)| \geq 4n - 9$ for $n \geq 5$. So we consider the following two subcases.

Case 2.1. $|S| \geq 4n - 8$. Let C be a cycle in P. Since $d_P(u) \geq 2$ for each vertex $u \in V(F)$, and $V(LTQ_n) \geq 4$, the cycle C of length is not less than 4. Because $V(C) \subset V(P) \subset F_1 \Delta F_2$, either $|F_1 - F_2| \geq 2$ or $|F_2 - F_1| \geq 2$. Thereby, either $|F_1| = |S| + |F_1 - F_2| \geq 4n - 6$ or $|F_2| = |S| + |F_1 - F_2| \geq 4n - 6$.

Case 2.2. $|V(P)| \geq 4n - 9$. Since $|V(P)| \geq 4n - 9$ and $V(P) \subset F_1 \Delta F_2$, either $|F_1 - F_2| \geq 2n - 4$ or $|F_2 - F_1| \geq 2n - 4$. And since there is no isolated vertex in LTQ_n (both F_1 and F_2 are conditional faulty set) and $LTQ_n - S$ is disconnected, $|S| \geq 2n - 2$ by lemma 3. Thereby, either$|F_1| = |S| + |F_1 - F_2| \geq 4n - 6$ or $|F_2| = |S| + |F_2 - F_1| \geq 4n - 6$.

Consequently, the theorem holds. ■

The theorem 1 shows that the conditional diagnosability of LTQ_n is not less than $4n - 7$ for $n \geq 5$. In the following we will show that the conditional diagnosability of LTQ_n is not more than $4n - 7$ for $n \geq 5$.

Theorem 2. $t_c(LTQ_n) \leq 4n - 7$ for $n \geq 3$.

Proof: (See **Figure 2**) Let $C = (u_1, u_2, u_3, u_4)$ be a cycle of length 4 in LTQ_n. u_1, u_2, u_3, u_4 are the four consecutively vertices in the cycle C. Let $F_1 = N_{LTQ_n}(C) \cup \{u_1, u_2\}$ and $F_2 = N_{LTQ_n}(C) \cup \{u_3, u_4\}$. It is easy to verify that $F_1$and F_2 are two indistinguishable conditional faulty

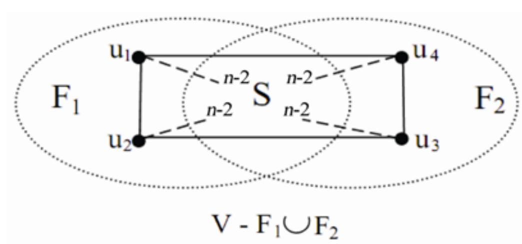

Figure 2. An illustration of the proof of Theorem 2.

set. It is easy to see that there exists no triangle in LTQ_n and any two distinct vertices in LTQ_n have at most two common neighbors. Thus we have $|F_1 \cap F_2| = N_{LTQ_n}(C) = 4n - 8$ and $|F_1 - F_2| = |F_2 - F_1|$. So $|F_1| = |F_2| = 4n - 6$. Hence, LTQ_n is not conditionally $(4n - 6)$ diagnosable. We are done. ■

By Theorems 1 and 2, the following corollary holds.

Corollary 1. $t_c(LTQ_n) = 4n - 7$ for $n \geq 5$.

4. Conclusions

Since the probability that any faulty set contains all the neighbors of some processor is very small, conditional diagnosability, requiring that each processor of a system is incident with at least one fault-free processor, can better measure the diagnosability of interconnection. In this paper, the main contribution is the determination of the conditional diagnosability of the locally twisted cubes. We obtain that the conditional diagnosability of a locally twisted cube under the PMC model is $t_c(LTQ_n) = 4n - 7$ for $n \geq 5$.

5. References

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