

Multi-Branch Fractional Multi-Bit Differential Detection of Continuous Phase Modulation with Decision Feedback

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Abstract

Differential detection of continuous phase modulation suffers from significant intersymbol interference. To reduce bit error rate, multi-branch fractional multi-bit differential detection (MFMDD) with decision feedback is proposed. By introducing decision feedback in multi-bit differential detected signals, severe intersymbol interference can be removed. Simulation results show that the proposed structure can greatly improve the performance compared with MFMDD without decision feedback, and the performance of 9 FMDD is very near to the performance of the coherent detection.

Keywords: Continuous Phase Modulation, Differential Detection, Fractional Multi-Bit Differential Detection, Decision Feedback

1. Introduction

Continuous phase modulations make signal phase continuous by filtering the data sequence with smooth phase functions, such as Gaussian pulse (for Gaussian minimum shift keying and Gaussian frequency shift keying), raised cosine pulse, leading to improved bandwidth efficiency. However, the trade-off of having a more compact power spectrum is the decrease of Euclidean distance in the transmitter signal space. For the detection of GMSK, a coherent detector, a differential detector, or a limit discriminator detector can be employed.

For the coherent detection, the required carrier recovery increases the acquisition time and is also relative complex. Furthermore, in a fading channel the coherent detection exhibits high error floors. Hence, for some burst mode applications requiring fast demodulation, incoherent detection is preferred. The conventional incoherent detection are one-bit differential (1DD) and two-bit differential (2DD). The main disadvantage of the incoherent detection compared with coherent detection is the large loss in signal to noise ratio (SNR). When the normalized 3dB bandwidth of Gaussian filter BT is small or the pulse expand length is long, the performance of conventional one-bit differential detection and two-bit

differential detection degrade greatly. The smallest BT for one-bit differential detection to operate properly is about 0.22. In terms of power efficiency, the two-bit differential detection is about 7dB inferior to the coherent detection [1]. Hence, there is ample room for improvements by introducing additional processing at the receiver. Therefore, Incoherent viterbi demodulation based Laurent decomposition is proposed in [2-4], its performance is about 1.5dB inferior to the optimum coherent detection. However, Laurent representation of CPM signals with small modulation index, long frequency pulse and large cardinality is unfortunately still complex. Multi-symbol differential detection is proposed to greatly improve the system performance by using the memory in CPM [5-7]. Additional, Kee-Hoon Lee [8-9] presents a Fractional Multi-bit Differential Detection (FMDD) technique. The technique improves the performance with a minimum increase in the implementation complexity. When FMDD is applied to GFSK used in Bluetooth systems, a SNR advantage up to 1.8 dB can be achieved compared with the conventional one-bit differential detection (1DD). Reference [10] generalized FMDD to a Multi-branch Fractional Multi-bit Differential Detection (MFMDD), the principle of MFMDD is presented from the physical meaning of phase forming, and theoretical analysis to the upper bound of the fractional bit duration for the maximum performance improvement is given.

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MFMD can obtain above 4 dB gain compared with 1DD. Abbas. Y. demonstrates that decision feedback can significantly reduce the effect of destructive intersymbol interference (ISI) in the conventional differential detectors [11]. The bit error rate (BER) performance is nearer to that of coherent detector while carrier recovery is avoided. This paper considers introducing decision feedback to further improve the performance of MFMD. Firstly, the structure of MFMD is given. Then, we determine how to apply decision feedback to the detector. Finally, computer simulation results for evaluating the BER performance are presented.

2. The Structure of MFMD

Assuming a Gaussian additive white noise channel, the signal at the receiver can be represented as follows

$$x(t) = A(t) \operatorname{Re} \left\{ \sqrt{\frac{2E}{T}} \exp[jw_c t + \phi(t)] \right\} + n(t) \quad (1)$$

where $A(t)$ is the time-varying envelope, E is the symbol energy, T is the symbol duration, w_c is the carrier frequency, $\phi(t)$ is the signal phase forming from the filtering of transmitted data sequence

$\mathbf{a} = \{\dots, a_0, a_1, \dots\}$, $\phi(t) = 2\pi h \sum_{n=-\infty}^{\infty} a_n \int_{-\infty}^t g(\tau - nT) d\tau$ (h is the modulation index, $g(t)$ is the frequency pulse), $n(t)$ is white additive Gaussian noise.

A block diagram of a Λ -fold MFMD receiver is given as **Figure 1**. Assume λ is the branch number, and the total branch number is Λ , $1 \leq \lambda \leq \Lambda$. The oversampling rate of the received signal is f_s ($f_s = m/T$) and the number of delayed samples is λ for the λ th

branch, the delay of fractional-bit differential detector can be expressed as $(\lambda/m)T$. Use $k(=1/m)$ as a notation of the unit fractional number. The λ th branch input signal delayed by λkT and phase shifted by 90° can be represented by

$$\begin{aligned} & x(t - \lambda kT)_{90^\circ} \\ &= A(t - \lambda kT) \cos(w_c(t - \lambda kT) + \phi(t - \lambda kT) + 90^\circ) \end{aligned} \quad (2)$$

The received signal $x(t)$ is multiplied by $x(t - \lambda kT)_{90^\circ}$. Ignoring the second harmonic item, the output of the multiplier (the λ th branch) is

$$\begin{aligned} r_{\lambda k}(t) &= x(t) x(t - \lambda kT)_{90^\circ} \\ &= \frac{A(t) A(t - \lambda kT)}{2} \sin[w_c \lambda kT + \Delta\phi(\lambda kT)] \end{aligned} \quad (3)$$

where $w_c \lambda kT$ is a phase offset contributed by the λ th branch, $\Delta\phi(\lambda kT)$ is

$$\Delta\phi(\lambda kT) = \phi(t) - \phi(t - \lambda kT) \quad (4)$$

which represents the change of signal phase over a fractional bit duration λkT . A phase shift and timing delay adjusting block is introduced in FMDD to compensate the influence of phase offset $w_c \lambda kT$ and delay offset of different fractional differential detection branches. Define the complementary phase shift $\psi_{\lambda k}$ such that $\psi_{\lambda k} + w_c \lambda kT = 2\pi$ and the complementary timing delay $D_{\lambda k}T$ such that different branches are synchronized. All branch outputs are summarized as,

$$f(t) = \sum_{\lambda=1}^{\Lambda} r_{\lambda k}(t - D_{\lambda k}T)_{\psi_{\lambda k}} \quad (5)$$

where $(\cdot)_{\psi_{\lambda k}}$ represents $\psi_{\lambda k}$ phase shift. After the low pass filtering, we have

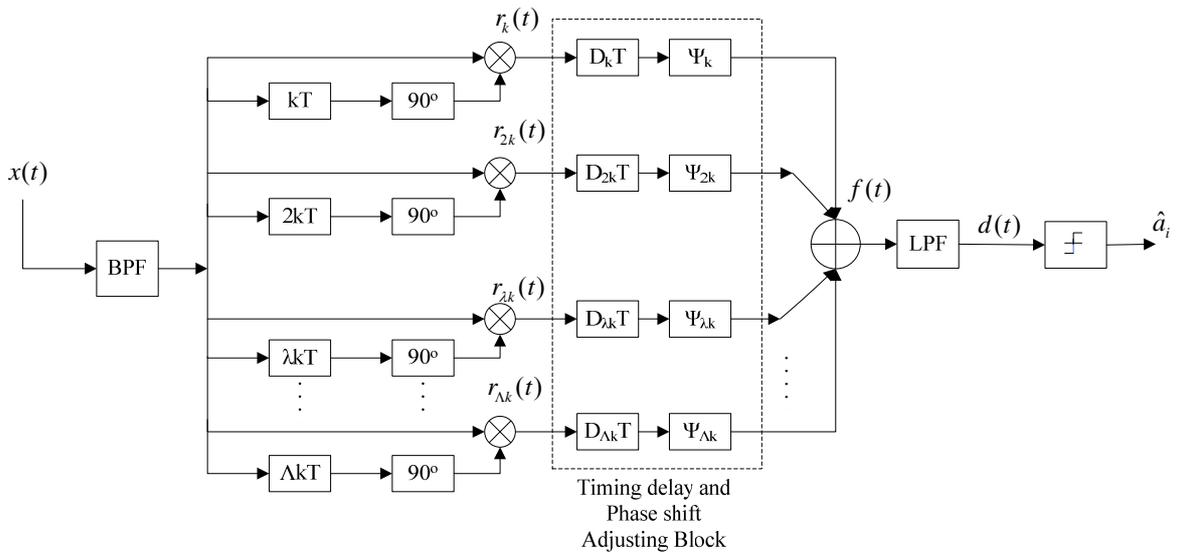


Figure 1. The block diagram of Λ -fold MFMD.

$$d(t) = \sum_{\lambda=1}^{\Lambda} \frac{A(t)A(t-\lambda kT)}{2} \sin(\Delta\phi(\lambda kT) + \Delta\eta_{\lambda k}) \quad (6)$$

where $\Delta\eta_{\lambda k}$ represents the phase shift introduced by AWGN, which can be assumed to be uniformly distributed in $(-\pi, \pi)$. Therefore the combining of different branches can minimize noise and improve BER performance. The decision is made based on the polarity of $d(t)$.

In fact, (3) is based on the processing of bandpass signal. If the processing is made in baseband, $w_c \lambda kT$ is non-existent, and the phase shift adjustment block is unnecessary. The following section presents a further explanation to FMDD from the physical meaning of the phase forming. The phase of CPM is the integral of the output of the premodulation filter, that is, the phase at instant t corresponds to the area of the region enclosed by the output waveform of the filter from $-\infty$ to t and the abscissa axis. **Figure 2** shows the output pulses of the premodulation filter, which corresponds to three symbols a_{n-1} (+1), a_{n+1} (+1), a_n (+1) respectively (Note that the three curves are not superposed).

In **Figure 2**, the phase variation over Δt corresponds to the sum of the areas enclosed by all the curves in this period and the abscissa axis. Taking 5FMDD as example, $k = 1/4$, $\Delta t = \lambda kT$ are $T/4$, $T/2$, $3T/4$, T and $5T/4$ respectively, $D_{\lambda/4}T = (1 - \lambda/8)T$. $D_{\lambda k}T$ makes different branches be synchronized such that the maximum open points of the baseband eyedigram under different fractional bit durations occur all at the same time. The output of different branches corresponds to the area under the intersecting output pulse during λkT , which is $\Delta\phi(\lambda kT)$ after multiplied by $2\pi/T$.

Figure 3 and **Figure 4** present the output waveforms of 1 DD and 5 FMDD under the ideal noiseless channel and additive white Gaussian noise channel respectively. It can be seen that the combination of multi-branch FMDD outputs increases the distance between different phase states, i.e. the detected area is enlarged. So the redundancy between different phase states is increased when the decision is made with zero as the decision threshold.

3. Upper Bound of the Fractional Bit Duration to Achieve the Maximum Performance Gain

From the above analysis, it can be seen that different branch combining is similar to time diversity combining. Each branch signal comes from samples of different duration of one received signal. Since FMDD has good performance, can we obtain an increase performance gain continually by increasing the number of branches?

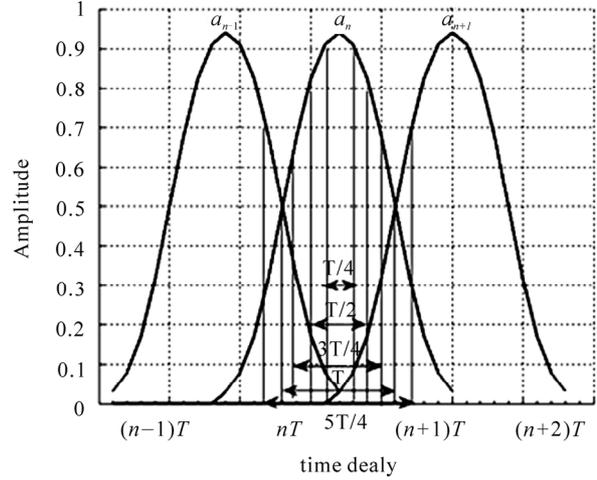


Figure 2. The output of the premodulation filter.

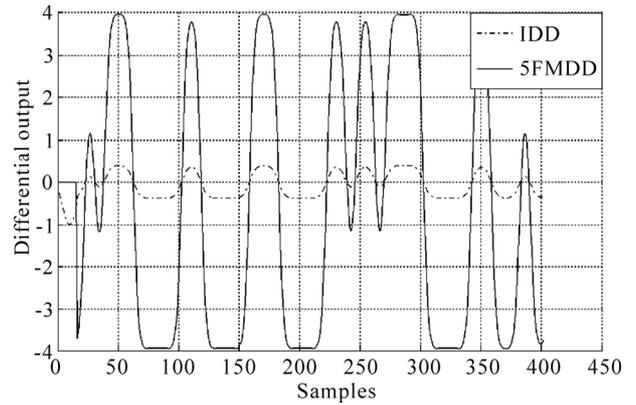


Figure 3. The ideal differential output of 1 DD and 5 FMDD.

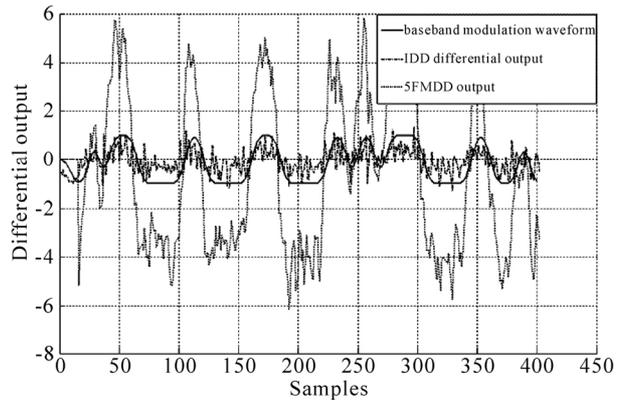


Figure 4. The differential output of 1 DD and 5 FMDD (SNR = 12 dB).

Is there a maximum duration σ such that no performance gain can be obtained when the fractional bit duration exceeds σ ?

Partial response continuous phase modulation introduces intersymbol interference (ISI) in the signal. For the

example that Gaussian pulse lasts two symbol durations, when a_{n-1} and a_{n+1} have the same polarity with the current symbol a_n , a_{n-1} and a_{n+1} play a positive role in phase increase from the instant nT to $(n+1)T$ (as shown in **Figure 2**), otherwise, they play a negative role (as shown in **Figure 5**). Therefore, the fractional bit duration λkT must guarantee that the polarity of $\sin\Delta\phi(\lambda kT)$ is consistent with that of a_n . That is to say, the area under the output of premodulation filter during λkT should be large than zero, which can be represented as $S_{\lambda kT} > 0$. Assume A is the set of all possible input data combinations, and a denotes one possible value of A . Search the maximum value of λkT for each input data combination, $(\lambda kT)_{\max|a}$, then find the minimum one among those $(\lambda kT)_{\max|a}$,

$$(\lambda kT)_{\max} = \min_{a \in A} \left\{ (\lambda kT)_{\max|a} \right\}.$$

When the polarities of the interacting data bits in λkT are the same, λkT is larger, the differential output is larger; when the polarities of the interacting data bits in λkT is not the same, λkT is larger, the output may be smaller. So the minimum $(\lambda kT)_{\max|a}$ should be the one found when the polarities of a_{n-1} and a_{n+1} are different from that of a_n . When L is increased, the number of interacting symbols is also increased. Therefore,

$$\sigma = \min_{a \in A} \left\{ (\lambda kT)_{\max|a}, S_{\lambda kT} > 0 \right\} \quad (7)$$

Figure 5 shows the premodulation filter output phase pulses where the adjacent symbols play a negative role to the phase increase. It can be seen that the integral of the filter output (the part shown in shadow) is zero when $\lambda kT = 2T$. This means that the branch of $\lambda kT = 2T$ will stop making positive contribution to FMDD. When $\lambda kT > 2T$, the output of fractional differential detection is less than zero, the branch will make negative contribu-

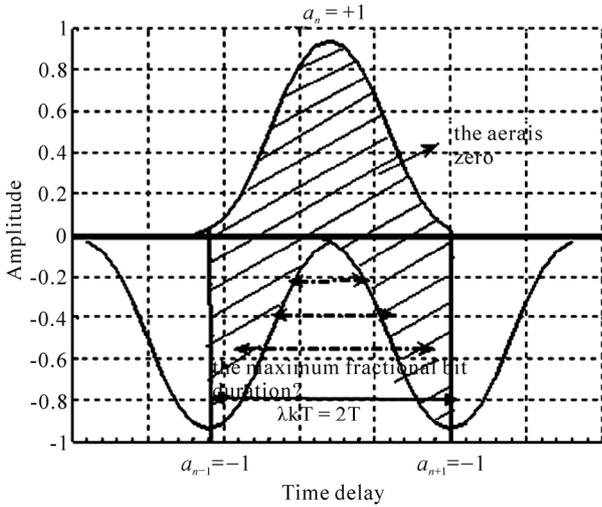


Figure 5. λkT to achieve the maximum performance gain.

tion. Therefore, for Gaussian pulse lasts two symbol durations, $k=1/4$, the value of σ which achieves the maximum performance gain is $7T/4$. In summary, the maximum performance gain can be obtained when the fractional bit duration is the maximal duration in which the symbol decision will not be influenced.

4. Using Decision Feedback in MFMD

In Λ -fold MFMD, the phase change in λkT for each branch is

$$\begin{aligned} \Delta\phi_i(\lambda kT) &= \phi(iT) - \phi(iT - \lambda kT) \\ &= 2\pi h \sum_{j=-\infty}^{\infty} a_j \int_{iT+(-\lambda-1)kT/2}^{iT+(\lambda-1)kT/2} g(\tau - jT) d\tau \end{aligned} \quad (8)$$

The integral in the expression can be understood as the area of $g(t)$ in the duration of λkT , and the summation is the contribution of the adjacent symbols to the signal phase. Let $\theta_{i-j} = 2\pi h \int_{iT+(-\lambda-1)kT/2}^{iT+(\lambda-1)kT/2} g(\tau - jT) d\tau$, the received signal after low pass filter at $t=iT$ can be expressed as

$$d(t) = \sum_{\lambda=1}^{\Lambda} \sin \left(\sum_{j=-\infty}^{\infty} a_j \theta_{i-j} + \Delta\eta_{\lambda k} \right) \quad (9)$$

That is to say the phase change in λkT is

$\Delta\theta_i = \sum_{j=-\infty}^{\infty} a_j \theta_{i-j}$, where θ_0 represents the signal and $\theta_{-2}, \theta_{-1}, \theta_1, \theta_2$ and etc. are the ISI terms. For $BT=0.5$, the values of θ_i for different λ are listed in **Table 1**, $k=1/4$. Where the minimum differential phase angle $\Delta\theta_{\min}$ is the phase separation between the

Table 1. Phase shifts (in degrees) corresponding to signal θ_0 and ISI term as a function of λ .

λ	θ_{-2}	θ_{-1}	θ_0	θ_1	θ_2	$\Delta\theta_{\min}$	$\Delta\theta_{\min}^{DF}$
1	-	0.8	20.8	0.8	-	38.4	40.0
2	-	2.3	40.4	2.3	-	71.6	76.2
3	-	5.1	57.3	5.1	-	94.2	104.4
4	-	9.7	70.6	9.7	-	102.4	121.8
5	-	16.3	79.8	16.3	-	94.4	127.0
6	-	24.8	85.4	24.8	-	71.6	121.2
7	0.1	34.6	88.2	34.6	0.1	37.8	107.2
8	0.3	45.0	89.4	45.0	0.3	-3.4	87.2
9	0.8	55.4	89.8	55.4	0.8	-45.2	67.2
∞	-	-	90	-	-	180	180

closest states and is also tabulated in **Table 1**,

$$\Delta\theta_{\min} = 2\left(\theta_0 - \sum_{i \neq 0} \theta_i\right) \quad (10)$$

Since the adjacent symbols are intersymbol interference terms to the current symbol, and the past symbols are known when deciding the current symbol, we can reduce the intersymbol interference using decision feedback. The block diagram of Λ -fold MFMD using decision feedback is shown in **Figure 6**.

From the above analysis, we can see that $\Delta\theta_k$ is not only related to a_i , but also influenced by a_{i-1} , a_{i-2} , a_{i+1} , and a_{i+2} at instant $t = iT$. When a_i is to be decided, \hat{a}_{i-1} (the estimate of a_{i-1}) and \hat{a}_{i-2} (the estimate of a_{i-2}) are already known. Since the influence of a_{i-2} is very small, we only consider a_{i-1} here. Therefore, we can reduce the influence of a_{i-1} to the received signal by introducing a phase shift of $\hat{a}_{i-1}\theta_1$ in the λkT delay arm. If we do not ignore a_{i-2} , the minimum differential phase using decision feedback is:

$$\Delta\theta_{\min}^{DF} = 2\left(\theta_0 - \sum_{i \leq -1} \theta_i\right) = \Delta\theta_{\min} + 2\sum_{i \geq 1} \theta_i \quad (11)$$

The value of $\Delta\theta_{\min}^{DF}$ is also given in **Table 1**.

Table 2 shows the differential phase angles corresponding to different input data combinations for different branches when $BT = 0.5$. $\Delta\theta_k$ is the differential phase without decision feedback, $\Delta\theta_k'$ is the differential phase with decision feedback. From the data in **Table 2**, we can make the following observations:

1) For state 3 and state 4, since $a_{i-1} \neq a_i$, after introducing phase shift $\hat{a}_{i-1}\theta_1$, the ISI introduced by the pre-

vious symbol is cancelled. The differential phase is increased at the sampling instant, the new differential phase is $\Delta\theta'_{i3,4} = \Delta\theta_{i3,4} + \theta_1$, and the distance from the decision threshold is also increased.

For state 2 and state 5, the effect of the previous symbol may be either constructive or destructive. A phase shift of $\hat{a}_{i-1}\theta_1$ may either increase or decrease the distance from the decision threshold. When $a_{i-1} = a_i$, the new differential phase is $\Delta\theta'_{i2,5} = \Delta\theta_{i2,5} + \theta_1$, and the distance from decision threshold is also increased. When $a_{i-1} = -a_i$, the new differential phase is $\Delta\theta'_{i2,5} = \Delta\theta_{i2,5} - \theta_1$, and the distance from decision threshold is also decreased.

State 1 and state 6 have the widest separation. For these two states, $a_{i-1} = a_i$, the distance from the decision threshold will be decreased by introducing phase shift $\hat{a}_{i-1}\theta_1$, and the new differential phases are $\Delta\theta'_{i1} = \Delta\theta_{i1} - \theta_1$ and $\Delta\theta'_{i6} = \Delta\theta_{i6} + \theta_1$.

State 3 and state 4 are very critical to the overall system performance. They are very vulnerable to be falsely decided. Although the $\Delta\theta_i$ of state 1, 2, 5, and 6 are decreased, the $\Delta\theta_i$ of state 3 and 4 are increased more significantly, leading to improved overall system performance.

2) By adding the differential output for different branches using decision feedback, the overall differential output is increased, and the bit error rate can be decreased. But this does not mean that we should increase the number of branches infinitely. The differential phase without decision feedback in **Table 2** shows that for state 3 and 4, when $\lambda = 8$ and $\lambda = 9$, since the polarity of the current symbol is opposite to $\sin\Delta\theta_i$, the differential output of the system will be decreased. For state 1

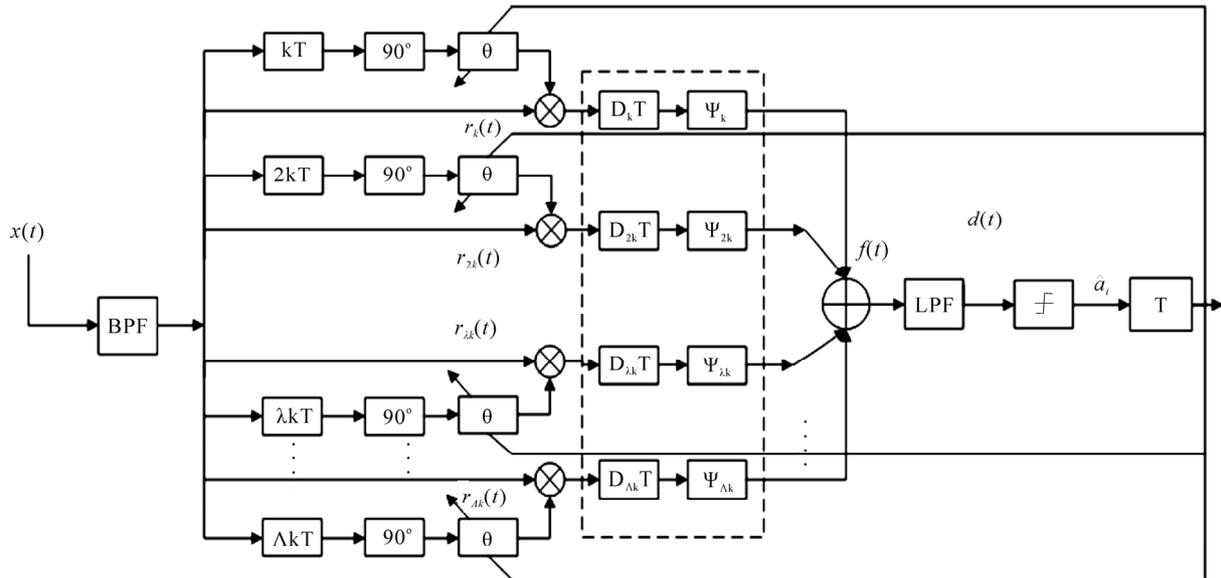


Figure 6. The block diagram of Λ -fold MFMD using decision feedback.

Table 2. Differential phase angles (in degrees) before and after using decision feedback corresponding to different bit combinations as a function of λ .

Differential phase angles for different bit combination								
λ	1 1 1 State 1		1 1 -1 State 2		-1 1 1 State 2		-1 1 -1 State 3	
	$\Delta\theta_k$	$\Delta\theta'_k$	$\Delta\theta_k$	$\Delta\theta'_k$	$\Delta\theta_k$	$\Delta\theta'_k$	$\Delta\theta_k$	$\Delta\theta'_k$
1	22.4	21.6	20.8	20	20.8	21.6	19.2	20
2	45	42.7	40.4	38.1	40.4	42.7	35.8	38.1
3	67.5	62.4	57.3	52.2	57.3	62.4	47.1	52.2
4	90	80.3	70.6	60.9	70.6	80.3	51.2	60.9
5	112.4	96.1	79.8	63.5	79.8	96.1	47.2	63.5
6	135	110.2	85.4	60.6	85.4	110.2	35.8	60.6
7	157.4	122.8	88.2	53.6	88.2	122.8	19	53.6
8	179.4	134.4	89.4	44.4	89.4	134.4	-0.6	44.4
9	200.6	145.2	89.8	34.4	89.8	145.2	-21	34.4

Differential phase angles for different bit combination								
λ	1 -1 1 State 4		1 -1 -1 State 5		-1 -1 1 State5		-1 -1 -1 State 6	
	$\Delta\theta_k$	$\Delta\theta'_k$	$\Delta\theta_k$	$\Delta\theta'_k$	$\Delta\theta_k$	$\Delta\theta'_k$	$\Delta\theta_k$	$\Delta\theta'_k$
1	-19.2	-20	-20.8	-21.6	-20.8	-20	-22.4	-21.6
2	-35.8	-38.1	-40.4	-42.7	-40.4	-38.1	-45	-42.7
3	-47.1	-52.2	-57.3	-62.4	-57.3	-52.2	-67.5	-62.4
4	-51.2	-60.9	-70.6	-80.3	-70.6	-60.9	-90	-80.3
5	-47.2	-63.5	-79.8	-96.1	-79.8	-63.5	-112.4	-96.1
6	-35.8	-60.6	-85.4	-110.2	-85.4	-60.6	-135	-110.2
7	-19	-53.6	-88.2	-122.8	-88.2	-53.6	-157.4	-122.8
8	0.6	-44.4	-89.4	-134.4	-89.4	-44.4	-179.4	-134.4
9	21	-34.4	-89.8	-145.2	-89.8	-34.4	-200.6	-145.2

and 6, $\lambda = 9$ will also decrease the system differential output. Thus, when $\Lambda = 7$, the system has the maximum distance from the decision threshold and best performance. After introducing decision feedback, for state 3 and 4, when $\lambda = 8$, and for state 1, 3, 4, and 6, when $\lambda = 9$, the polarity of the current symbol is the same as $\sin \Delta\theta_i$. After introducing decision feedback, these two branches will improve the system performance. When $\Lambda = 9$, the system will have best performance.

5. Simulation Results

The BER performance and the maximum performance gain of MFMD using decision feedback have been evaluated through computer simulation. In our simulation, the pulse expand length is 3 symbols, the normal-

ized 3 dB bandwidth $BT = 0.5$. The BER performance of Λ -fold MFMD without decision feedback is shown in **Figure 7**, from which we can see that the larger the fractional bit duration the higher the performance gain. The optimal performance gain is reached when $\Lambda = 7$. We can see that for $BT = 0.5$, the performance of 7FMDD is close to the performance of 6 FMDD; the performance of 5FMDD is close to the performance of 8FMDD under high SNR; 7 FMDD outperforms 1 DD by above 4 dB at $BER = 1 \times 10^{-3}$ in an AWGN channel.

We find that after introducing decision feedback, the performance of MFMD is improved. From **Figure 8** we can see that after introducing decision feedback, better BER performance is acquired when we increase the number of branches, reaching the optimum BER in 9 FMDD. When the BER is 1×10^{-3} , after introducing

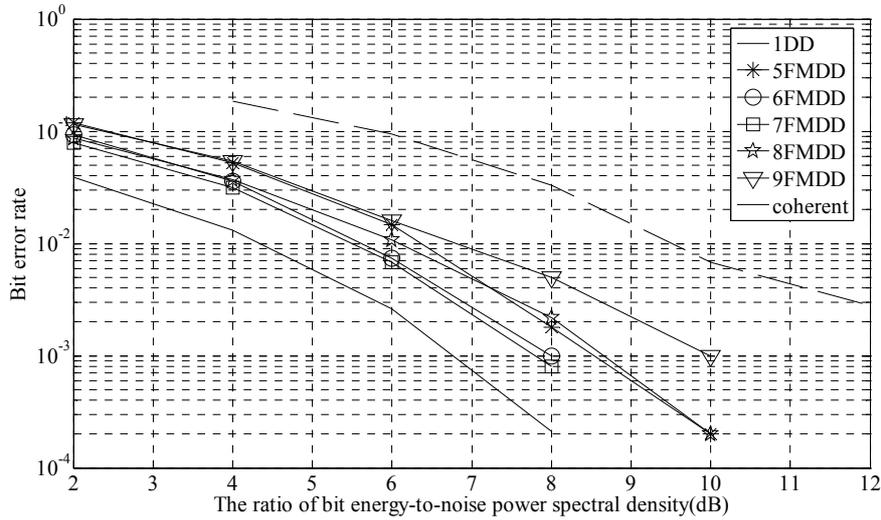


Figure 7. BER of Λ -fold MFMD.

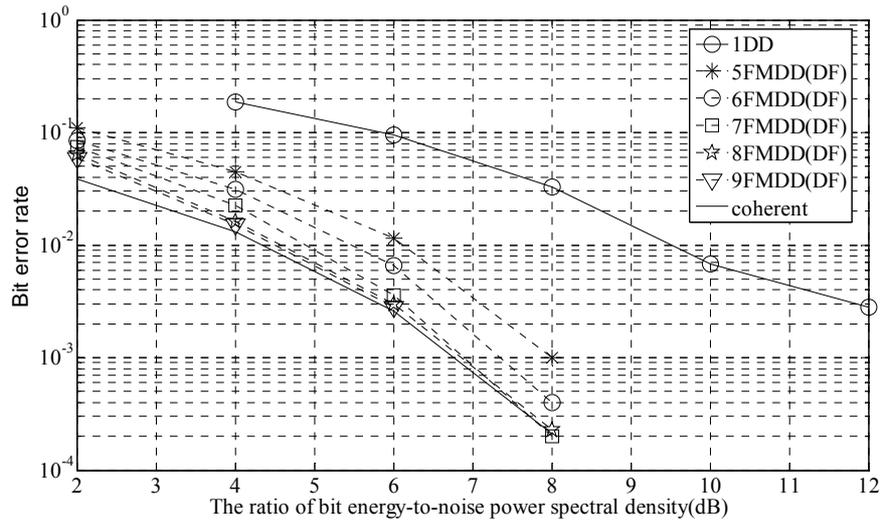


Figure 8. BER of Λ -fold MFMD using decision feedback.

decision feedback, 5 FMDD has a performance gain of 0.5 dB, 6 FMDD has a performance gain of near 0.6 dB, 7 FMDD has a performance gain of about 1 dB, 8 FMDD has a performance gain of 2.2 dB, 9 FMDD has a performance gain of near 4.2 dB than before decision feedback. The performance of 9 FMDD is very near to the coherent detection. We can see that the receiver performance is greatly improved after introducing decision feedback.

6. Conclusions

This paper presents multi-branch fractional multi-bit differential detection technique using decision feedback suitable for continuous phase modulation. By introducing

decision feedback in multi-bit differential detected signals, severe intersymbol interference can be removed. For the $BT = 0.5$, the proposed structure can provide different SNR improvement for different branches compared with MFMD without decision feedback, and the best performance is achieved for 9FMDD which is very near to the performance of the coherent detection.

7. References

- [1] M. K. Simon and C. C. Wang, "Differential Detection of Gaussian MSK in a Mobile Radio Environment," *IEEE Transactions on Vehicle Technology*, Vol. VT-33, No. 4, 1984, pp. 307-320. doi:10.1109/T-VT.1984.24023
- [2] G. Colavolpe and R. Raheli, "Noncoherent Sequence Detection of CPM," *Electronics Letters*, Vol. 34, No. 3,

- 1998, pp. 259-261. doi:10.1049/el:19980225
- [3] M. Wardle and M. Rice, "PAM Approach to Weak CPM and Its Application to Flight Termination Receivers," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 44, No. 2, 2008, pp. 468-480. doi:10.1109/TAES.2008.4560200
- [4] E. Perrins and M. Rice, "PAM Representation of Ternary CPM," *IEEE Transactions on Communications*, Vol. 56, No. 12, 2008, pp. 2020-2024. doi:10.1109/TCOMM.2008.041108
- [5] G. Colavolpe and R. Raheli, "Noncoherent Sequence Detection of Continuous Phase Modulations," *IEEE Transactions on Communications*, Vol. 47, No. 9, 1999, pp. 1303-1307. doi:10.1109/26.789664
- [6] E. Perrins, R. Schober, M. Rice and M. K. Simon, "Multiple-Bit Differential Detection of Shaped-Offset QPSK," *IEEE Transactions on Communications*, Vol. 55, No. 12, 2007, pp. 2328-2340. doi:10.1109/TCOMM.2007.910972
- [7] Q. Zhi, C. H. Zhou and L. M. Xiao, "Non-Coherent Demodulation of Continuous Phase Modulation Signals Based on Multi-Symbol Differential Correlation," *Journal of Tsinghua University*, Vol. 47, No. 4, 2007, pp. 507-510.
- [8] K. H. Lee and J. S. Seo, "Fractional Multi-Bit Differential Detection of Continuous Phase Modulation," *ETRI Journal*, Vol. 26, No. 6, 2004, pp. 635-640. doi:10.4218/etrij.04.0103.0065
- [9] K. H. Lee and J. K. Kim, "Correlative Encoded (CE) FSK for Use in RF Home Networking," *Proceedings of IEEE VTC 2002-Fall*, Vancouver, IEEE Press, 2002, pp. 2400-2404.
- [10] J. H. Sun and X. J. Wu, "Multi-Branch Fractional Multi-Bit Differential Detection of Continuous Phase Modulation," *Journal of Xidian University*, Vol. 37, No. 3, 2010, pp. 423-428.
- [11] Y. Abbas, M. Dimitrios and F. Kamilo, "Differential Detection of GMSK Using Decision Feedback," *IEEE Transactions on Communications*, Vol. 36, No. 6, 1988, pp. 641-649. doi:10.1109/26.2784